Tool Kits in Multi-regional and Multi-sectoral General Equilibrium Modeling for Chile

*International Workshop on General Equilibrium Modeling, Universidad Adolfo Ibañez Viña del Mar, December 4-5, 2018*

Eduardo A. Haddad
Research team – NEREUS

Eduardo Amaral Haddad (coordinator)
Ademir Antônio Moreira Rocha
Bruno Proença Pacheco Pimenta
Denise Leyi Li
Karina Simone Sass
Keyi Ando Ussami
Lucas Cardoso Correa Dias
Raphael Pinto Fernandes
Sofia Marques Arantes
Background

**Interregional Input-Output Adjustment System (IIOAS)**

- Colombia, Ecuador, Azores, Lebanon, Egypt, Morocco, Brazil, Greece, **Chile**, Mexico

Mediterranean (partnerships for modeling)

- **Portugal**, Spain, France, **Italy**, **Greece**, Turkey, Lebanon, Egypt, Tunisia, **Morocco**

REAL/RSAI network (Geoffrey J. D. Hewings)

“Nereids Program” and “CONICYT-FAPESP Project”
EAE 5918 – “Applied General Equilibrium Models”

- Project 2017: Greece
- Part 1 – Input-Output models
- Part 2 – CGE models
- Modeling Marathons (2x)
- Publication process

Enhance broader scientific communication skills

- **Project 2018: Chile**
- Prof. Patricio Aroca
I Modeling Marathon

Date: September 13, 2018
Time: 8:00 – 18:00
Place: NEREUS meeting room
Outcomes

Interregional Input-Output System for Chile

National CGE Model for Chile

Interregional CGE Model for Chile

Eight different applications

“International Workshop on General Equilibrium Modeling: Applications for the Chilean Economy”
Interregional Input-Output System for Chile

Input-output flows

Final Demand
- Output
- Capital
- Households
- Exports
- Government

Are demanded from:
- Domestic Goods
- Imported Goods

Are supplied to:
- Domestic Inputs
- Labor
- Capital
- Land
- Imported Inputs

Primary Inputs

Department of Economics, University of Sao Paulo
Input-output table

The Input-Output Matrix

Sector i
Selling Sectors

Sector j
Buyng Sectors

Intermediate Consumption

Imports
Sales Taxes
Value Added
Total Output

Exports
Households
Government
Investments
Stocks

Final Demand

Total Outp.
# Interregional IO models

<table>
<thead>
<tr>
<th>Buying Sectors</th>
<th>Selling Sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Region L</strong></td>
<td><strong>Region L</strong></td>
</tr>
<tr>
<td>Interindustry Inputs LL</td>
<td>Interindustry Inputs LM</td>
</tr>
<tr>
<td>Sales Taxes</td>
<td>Value Added</td>
</tr>
<tr>
<td>Total Output L</td>
<td>Total Output L</td>
</tr>
<tr>
<td><strong>Region M</strong></td>
<td><strong>Region M</strong></td>
</tr>
<tr>
<td>Interindustry Inputs ML</td>
<td>Interindustry Inputs MM</td>
</tr>
<tr>
<td>Imports from the World</td>
<td>Imports from the World</td>
</tr>
<tr>
<td>Sales Taxes</td>
<td>Value Added</td>
</tr>
<tr>
<td>Total Output M</td>
<td>Total Output M</td>
</tr>
</tbody>
</table>

---

Department of Economics, University of Sao Paulo
Interregional IO models

Large amount of data
Problem?!
Interregional IO models
Methodology
ORANI-Chile Model

National Computable General Equilibrium Model for Chile

The University of Sao Paulo Regional and Urban Economics Lab - NEREUS

November 2018
What is an applied CGE model?

Computable, based on data
It has many sectors
And perhaps many regions, primary factors and households
A big database of matrices
Many, simultaneous, equations (hard to solve)
Prices guide demands by agents
Prices determined by supply and demand
Trade focus: elastic foreign demand and supply
What is a CGE model good for?

Analyzing policies that affect different sectors in different ways

The effect of a policy on different:
- Sectors
- Regions
- Factors (Labor, Capital)

Policies that help one sector a lot, and harm all the rest a little
What-if questions

What if productivity in the grape sector decreased due to a temporary climate shock?
What if the climate shock also affected the quality of Chilean wine?
What if productivity in mining decreased by 1%?
What if government imposed a 3% production tax on total output of copper sector?
What if a drought hit Central Chile decreasing productivity in agriculture by 10%?
What if Chile faced (uncertain) commodity price shocks?
What if Central government increased its expenditures?
ORANI-Chile like other GE models

Equations typical of an AGE model, including:

✓ market-clearing conditions for commodities and primary factors;
✓ producers' demands for produced inputs and primary factors;
✓ final demands (investment, household, export and government);
✓ the relationship of prices to supply costs and taxes;
✓ a few macroeconomic variables and price indices.

Neo-classical flavor:

✓ demand equations consistent with optimizing behavior (cost minimization, utility maximization).
✓ competitive markets: producers price at marginal cost.
# Model Database – Structural coefficients

(miles de millones de pesos de 2014)

<table>
<thead>
<tr>
<th></th>
<th>User (1j)</th>
<th>User (2j)</th>
<th>User (3)</th>
<th>User (4)</th>
<th>User (5)</th>
<th>User (6)</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i \in G, s \in S$</td>
<td>(\text{BAS}(i,s,(1j)))</td>
<td>(\text{BAS}(i,s,(2j)))</td>
<td>(\text{BAS}(i,s,(3)))</td>
<td>(\text{BAS}(i,s,(4)))</td>
<td>(\text{BAS}(i,s,(5)))</td>
<td>(B(i,s,(6)))</td>
<td>(B(i,s,(\bullet)))</td>
</tr>
<tr>
<td>$i \in G, s \in S$</td>
<td>(\text{MAR}(i,s,(1j)))</td>
<td>(\text{MAR}(i,s,(2j)))</td>
<td>(\text{MAR}(i,s,(3)))</td>
<td>(\text{MAR}(i,s,(4)))</td>
<td>(\text{MAR}(i,s,(5)))</td>
<td>-</td>
<td>(\text{MAR}(i,s,(\bullet)))</td>
</tr>
<tr>
<td>$i \in G, s \in S$</td>
<td>(\text{TAX}(i,s,(1j)))</td>
<td>(\text{TAX}(i,s,(2j)))</td>
<td>(\text{TAX}(i,s,(3)))</td>
<td>(\text{TAX}(i,s,(4)))</td>
<td>(\text{TAX}(i,s,(5)))</td>
<td>-</td>
<td>(\text{TAX}(i,s,(\bullet)))</td>
</tr>
<tr>
<td>$s \in F$</td>
<td>(\text{VA}(g+1,s,(1j)))</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(\text{VA}(g+1,s,(\bullet)))</td>
</tr>
<tr>
<td>TOTAL</td>
<td>(\text{VTOT}(\bullet,\bullet,(1j)))</td>
<td>(\text{VTOT}(\bullet,\bullet,(2j)))</td>
<td>(\text{VTOT}(\bullet,\bullet,(3)))</td>
<td>(\text{VTOT}(\bullet,\bullet,(4)))</td>
<td>(\text{VTOT}(\bullet,\bullet,(5)))</td>
<td>-</td>
<td>(\text{VTOT}(\bullet,\bullet,(\bullet)))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>User (1j)</th>
<th>User (2j)</th>
<th>User (3)</th>
<th>User (4)</th>
<th>User (5)</th>
<th>User (6)</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i \in G, s \in S$</td>
<td>124,761</td>
<td>32,235</td>
<td>71,195</td>
<td>47,567</td>
<td>18,982</td>
<td>-864</td>
<td>293,877</td>
</tr>
<tr>
<td>$i \in G, s \in S$</td>
<td>5,753</td>
<td>2,102</td>
<td>12,271</td>
<td>1,646</td>
<td>3</td>
<td>-</td>
<td>21,776</td>
</tr>
<tr>
<td>$i \in G, s \in S$</td>
<td>3,109</td>
<td>1,108</td>
<td>10,269</td>
<td>0</td>
<td>95</td>
<td>-</td>
<td>14,581</td>
</tr>
<tr>
<td>$s \in F$</td>
<td>136,170</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>136,170</td>
</tr>
<tr>
<td>TOTAL</td>
<td>269,793</td>
<td>35,445</td>
<td>93,735</td>
<td>49,213</td>
<td>19,080</td>
<td>-864</td>
<td>466,403</td>
</tr>
</tbody>
</table>
Features of database

Commodity flows are valued at “basic prices” (do not include user-specific taxes or margins)

For each user of each imported good and each domestic good, there are numbers showing:

✓ tax levied on that usage
✓ usage of several margins (trade, transport)

MAKE multi-production:

✓ each commodity may be produced by several industries
✓ each industry may produce several commodities

For each industry the total cost of production is equal to the total value of output (column sums of MAKE).

For each commodity the total value of sales is equal to the total value of output (row sums of MAKE).

No data regarding direct taxes or transfers. Not a full SAM.
Industries and commodities

• 111 industries:
  – 12 industries - Agriculture and fishing
  – 6 industries – Mining
  – 45 industries – Manufacturing
  – 6 industries – Public Utilities
  – 42 industries – Services

• 179 products
  – 26 products - Agriculture and fishing
  – 10 products – Mining
  – 82 products – Manufacturing
  – 6 products - Public Utilities
  – 55 products - Services
Model Database – Behavioral parameters

SIGMA1PRIM – CES between primary factors: 0.9

SIGMA1, SIGMA2, SIGMA3 – Armington elasticities:
- Agriculture and fishing: 1.2
- Mining and manufacturing: 0.8
- Services: 0.6

SIGMA1OUT – CET transformation elasticities:
- Agriculture and fishing: 1.2
- Mining and manufacturing: 0.8
- Services: 0.6
<table>
<thead>
<tr>
<th>Grupo de alimentos y productos que los componen</th>
<th>Grupo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aceitunas</td>
<td>Carne de cerdo</td>
</tr>
<tr>
<td>Alimentos colados y picados</td>
<td>Carne de cordero</td>
</tr>
<tr>
<td>Arroz</td>
<td>Carne de vacuno</td>
</tr>
<tr>
<td>Avena, maíz y sus derivados</td>
<td>Menudencias de ave</td>
</tr>
<tr>
<td>Cereales preparados</td>
<td>Pavo</td>
</tr>
<tr>
<td>Fortificante para leche</td>
<td>Pollo</td>
</tr>
<tr>
<td>Galletas de agua y soda</td>
<td>Atún en conserva</td>
</tr>
<tr>
<td>Galletas dulces</td>
<td>Jurel en conserva</td>
</tr>
<tr>
<td>Harina y otros derivados</td>
<td>Marisco en conserva</td>
</tr>
<tr>
<td>del trigo</td>
<td>Marisco fresco y congelado</td>
</tr>
<tr>
<td>Marini</td>
<td></td>
</tr>
<tr>
<td>Otros alimentos para niños</td>
<td>Otros pescados en conserva</td>
</tr>
<tr>
<td>Pan</td>
<td></td>
</tr>
<tr>
<td>Pastas</td>
<td></td>
</tr>
<tr>
<td>Pre-pizza y otras masas saladas preparadas</td>
<td>Pescado congelado, apanado y congelado</td>
</tr>
<tr>
<td>Tortas, pasteles y queques</td>
<td>Pescado fresco</td>
</tr>
</tbody>
</table>
CUADRO A.5
Elasticidades precio compensadas y elasticidades gasto

<table>
<thead>
<tr>
<th></th>
<th>Grupo 1</th>
<th>Grupo 2</th>
<th>Grupo 3</th>
<th>Grupo 4</th>
<th>Grupo 5</th>
<th>Grupo 6</th>
<th>Grupo 7</th>
<th>Grupo 8</th>
<th>Grupo 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grupo 1</td>
<td>-0.9497141*</td>
<td>0.0623478</td>
<td>0.076896</td>
<td>0.6684487*</td>
<td>-0.0497305</td>
<td>0.1406558</td>
<td>0.0223432</td>
<td>0.0303922</td>
<td>-0.0016291</td>
</tr>
<tr>
<td></td>
<td>(0.1601491)</td>
<td>(0.0593656)</td>
<td>(0.0483894)</td>
<td>(0.0581545)</td>
<td>(0.0912752)</td>
<td>(0.0880997)</td>
<td>(0.0443766)</td>
<td>(0.0350805)</td>
<td>(0.1200206)</td>
</tr>
<tr>
<td>Grupo 2</td>
<td>0.0581937</td>
<td>-1.006515*</td>
<td>0.0751686*</td>
<td>0.671997*</td>
<td>0.1080661*</td>
<td>-0.0047645</td>
<td>-0.0126273</td>
<td>0.0841292*</td>
<td>0.0263519</td>
</tr>
<tr>
<td></td>
<td>(0.0552598)</td>
<td>(0.0442234)</td>
<td>(0.0345295)</td>
<td>(0.0275487)</td>
<td>(0.0332465)</td>
<td>(0.0440765)</td>
<td>(0.033907)</td>
<td>(0.0258371)</td>
<td>(0.0343825)</td>
</tr>
<tr>
<td>Grupo 3</td>
<td>0.1665837</td>
<td>0.172516*</td>
<td>-1.041851*</td>
<td>0.6211044*</td>
<td>0.0023026</td>
<td>-0.0412164</td>
<td>0.0539479</td>
<td>0.0884859</td>
<td>-0.0218729</td>
</tr>
<tr>
<td></td>
<td>(0.1038494)</td>
<td>(0.0792835)</td>
<td>(0.0926305)</td>
<td>(0.0482005)</td>
<td>(0.0623415)</td>
<td>(0.0940439)</td>
<td>(0.0591497)</td>
<td>(0.0502529)</td>
<td>(0.0495205)</td>
</tr>
<tr>
<td>Grupo 4</td>
<td>0.06864*</td>
<td>0.0743639*</td>
<td>0.0301886*</td>
<td>-0.3386191*</td>
<td>0.0181034*</td>
<td>0.0363424*</td>
<td>0.0486253*</td>
<td>0.0520556*</td>
<td>0.012099*</td>
</tr>
<tr>
<td></td>
<td>(0.006005)</td>
<td>(0.003025)</td>
<td>(0.002265)</td>
<td>(0.0048472)</td>
<td>(0.0034653)</td>
<td>(0.0036027)</td>
<td>(0.0026836)</td>
<td>(0.0017231)</td>
<td>(0.0048505)</td>
</tr>
<tr>
<td>Grupo 5</td>
<td>-0.1485971</td>
<td>0.3550615*</td>
<td>0.0036194</td>
<td>0.5309373*</td>
<td>-0.7836784*</td>
<td>-0.0461161</td>
<td>-0.083142</td>
<td>0.1776787*</td>
<td>-0.0057633</td>
</tr>
<tr>
<td></td>
<td>(0.2788861)</td>
<td>(0.1091271)</td>
<td>(0.0891279)</td>
<td>(0.1034712)</td>
<td>(0.2339404)</td>
<td>(0.1892983)</td>
<td>(0.0950944)</td>
<td>(0.0667375)</td>
<td>(0.2235709)</td>
</tr>
<tr>
<td>Grupo 6</td>
<td>0.2628648</td>
<td>-0.0092489</td>
<td>-0.0359062</td>
<td>0.6527503*</td>
<td>-0.028156</td>
<td>-0.9761824*</td>
<td>0.153559*</td>
<td>-0.1027723</td>
<td>0.0830918</td>
</tr>
<tr>
<td></td>
<td>(0.1634168)</td>
<td>(0.0880677)</td>
<td>(0.0818344)</td>
<td>(0.0655191)</td>
<td>(0.115126)</td>
<td>(0.1857242)</td>
<td>(0.0667661)</td>
<td>(0.0607816)</td>
<td>(0.081703)</td>
</tr>
<tr>
<td>Grupo 7</td>
<td>0.03996</td>
<td>-0.0191067</td>
<td>0.0357704</td>
<td>0.6605733*</td>
<td>-0.037874</td>
<td>0.1161563*</td>
<td>-1.010518*</td>
<td>0.2174435*</td>
<td>0.0066489</td>
</tr>
<tr>
<td></td>
<td>(0.0625085)</td>
<td>(0.0512835)</td>
<td>(0.0589423)</td>
<td>(0.036976)</td>
<td>(0.0367649)</td>
<td>(0.0504347)</td>
<td>(0.0775507)</td>
<td>(0.0525917)</td>
<td>(0.0339081)</td>
</tr>
<tr>
<td>Grupo 8</td>
<td>0.0284388</td>
<td>0.109957*</td>
<td>0.0502787</td>
<td>0.6112714*</td>
<td>0.070369*</td>
<td>-0.0672534</td>
<td>0.1879528*</td>
<td>-1.036212*</td>
<td>0.0351528*</td>
</tr>
<tr>
<td></td>
<td>(0.3441296)</td>
<td>(0.0337749)</td>
<td>(0.0286966)</td>
<td>(0.0206078)</td>
<td>(0.0265961)</td>
<td>(0.0397867)</td>
<td>(0.0453017)</td>
<td>(0.0446212)</td>
<td>(0.0158269)</td>
</tr>
<tr>
<td>Grupo 9</td>
<td>-0.0080426</td>
<td>0.175699</td>
<td>-0.063745</td>
<td>0.6168926*</td>
<td>-0.0120674</td>
<td>0.2770866</td>
<td>0.0296594</td>
<td>0.1793745*</td>
<td>-1.194197</td>
</tr>
<tr>
<td></td>
<td>(0.7441296)</td>
<td>(0.2290938)</td>
<td>(0.1436894)</td>
<td>(0.2923435)</td>
<td>(0.4536233)</td>
<td>(0.2938811)</td>
<td>(0.1497257)</td>
<td>(0.0838536)</td>
<td>(0.7064214)</td>
</tr>
</tbody>
</table>

Elasticidades

|                  | 0.6768144*       | 0.9386683*       | 0.9202147*       | 1.049464*        | 0.9754854*       | 0.9591301*       | 1.092167*        | 0.9165859*       | 0.8800257*       |
|                  | (0.0109929)      | (0.0123508)      | (0.0149915)      | (0.0037972)      | (0.0145283)      | (0.0139224)      | (0.0236223)      | (0.0121372)      | (0.0182508)      |

Nota: * significativo al 5%. Errores estándar entre paréntesis.
### Cuadro 1. – Elasticidades precio de las exportaciones chilenas

<table>
<thead>
<tr>
<th>Producto</th>
<th>Modelo</th>
<th>Elasticidad precio de corto plazo</th>
<th>Elasticidad precio de largo plazo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobre, Gran Minería</td>
<td>I</td>
<td>0,10</td>
<td>0,14</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0,06</td>
<td></td>
</tr>
<tr>
<td>Cobre, Mediana y Pequeña Minería</td>
<td>II</td>
<td>0,06</td>
<td></td>
</tr>
<tr>
<td>Molibdeno a/</td>
<td>I</td>
<td>0,15</td>
<td>0,22</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0,10</td>
<td></td>
</tr>
<tr>
<td>Hierro a/</td>
<td>I</td>
<td>0,24</td>
<td>0,58</td>
</tr>
<tr>
<td>Celulosa</td>
<td>I</td>
<td>0,12</td>
<td>0,47</td>
</tr>
<tr>
<td>Harina de pescado a/</td>
<td>I</td>
<td>0,34</td>
<td>0,64</td>
</tr>
<tr>
<td>Sector industrial b/</td>
<td>III</td>
<td>0,33</td>
<td>1,77</td>
</tr>
<tr>
<td>Sector industrial excluidos</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>algunos productos b/</td>
<td>III</td>
<td>0,44</td>
<td>2,34</td>
</tr>
<tr>
<td>Sector agropecuario y del mar b/</td>
<td>III</td>
<td>0,59</td>
<td>3,56</td>
</tr>
</tbody>
</table>
Primary factor mix

\[ X_{1PRIM}(i) = CES \left( \frac{X_{1LAB\_O}(i)}{A_{1LAB\_O}(i)}, \frac{X_{1CAP}(i)}{A_{1CAP}(i)}, \frac{X_{1LND}(i)}{A_{1LND}(i)} \right) \]
Intermediate sourcing

\[ X_{1\_S}(c,i) = \text{CES( All,s,SRC: } X_1(c,s,i)/A_1(c,s,i)) \]
Top nest of industry inputs

\[
X1_{TOT}(i) = \text{MIN}( \text{All, } c, \text{COM: } X1_{S}(c,i)/[A1_{S}(c,s,i)\times A1_{TOT}(i)], \\
X1_{PRIM}(i)/[A1_{PRIM}(i)\times A1_{TOT}(i)], \\
X1_{OCT}(i)/[A1_{OCT}(i)\times A1_{TOT}(i)])
\]
Industry output mix

One Industry:
- \(\text{MAKE}(c,i)\)
- \(p0\text{com}(c)\)
- \(q1(c,i)\)

Activity Level:
- \(V1\text{TOT}(i)\)
- \(p1\text{tot}(i)\)
- \(x1\text{tot}(i)\)

CET

All-Industry:
- \(\text{SALES}(c)\)
- \(p0\text{com}(c)\)
- \(x0\text{com}(c)\)
Composition of investment

New Capital for Industry $i$

Leontief

Good 1 — up to — Good C

CES

Domestic Good 1

Imported Good 1

Domestic Good C

Imported Good C

V2TOT($i$)
$p2tot(i)$
$x2tot(i)$

V2PUR_S($c,i$)
$p2_s(c,i)$
$x2_s(c,i)$

V2PUR($c,s,i$)
$p2(c,s,i)$
$x2(c,s,i)$
Household demand

\[ V_{3TOT} = p_{3tot} \times x_{3tot} \]

Household Utility

Klein-Rubin

\[ V_{3PUR\_S(c)} = p_{3\_s(c)} \times x_{3\_s(c)} \]

V3SUB(c)

Subsistence

\[ V_{3LUX(c)} = p_{3\_s(c)} \times x_{3lux(c)} \]

Luxury

\[ V_{3PUR(c,s)} = p_{3(c,s)} \times x_{3(c,s)} \]

Domestic

Imported

Good 1

Good C

CES

Good 1

Imported Good 1

Domestic Good C

Imported Good C
Consumption function

**Variable**

- `w1prim_i` # Aggregate primary factor payments #;
- `w3tot` # Nominal total household consumption #;
- `f3tot` # Ratio, factor income/ consumption #;

**Equation**

\[ w3tot = w1prim_i - f3tot; \]

! note: normally ONE of \( f3tot \) and \( x3tot \) is exogenous

\( f3tot \) exogenous gives \( x3tot \) ... hou cons follows factor income

\( x3tot \) exogenous gives \( f3tot \) ... avg propensity to cons changes !
Individual export demand

Downward sloping constant-elasticity of foreign demand

\[ X_4(c) = F_4Q(c) \left[ \frac{P_4(c)}{\Phi F_4P(c)} \right]^{\text{EXP}_\text{ELAST}(c)} \]
Government demand function

**Variable**

- $f_{5\text{tot}}$  
  # Overall shift term for government demands #;
- $f_{5\text{tot}2}$  
  # Ratio between $f_{5\text{tot}}$ and real ind tax rev #;
- $(all,c,\text{COM})(all,s,\text{SRC}) f_5(c,s)$  
  # Government demand shift #;
- $w_0\text{tax}_\text{csi}$  
  # Aggregate revenue from all indirect taxes #;
- $p_{5\text{tot}}$  
  # Government price index #;

**Equation**

- $E_{x_5}$  
  # Government demands #
- $(all,c,\text{COM})(all,s,\text{SRC}) x_5(c,s) = f_5(c,s) + f_{5\text{tot}}$;
- $E_{f_{5\text{tot}}}$  
  # Overall government demands shift #
- $f_{5\text{tot}} = w_0\text{tax}_\text{csi} - p_{5\text{tot}} + f_{5\text{tot}2}$;

! note: normally ONE of $f_{5\text{tot}}$ and $f_{5\text{tot}2}$ is exogenous

$f_{5\text{tot}2}$ exogenous ... gov follows real tax rev

$f_{5\text{tot}}$ exogenous gives $x_5(c,s) = f_5(c,s) + f_{5\text{tot}}$ ... gov exog!
All purchaser’s price equations have the same basic form:

\[ P_{Nc} \cdot X_{Nc} = P_{0c} \cdot X_{Nc} \cdot T_c + \sum_{\text{mar}} X_{\text{mar}, c} \cdot P_{\text{mar}} \]

... linearising (and dropping subscripts) ...

\[ [P \cdot X] (p + x) = [P0 \cdot X \cdot T] (p0 + x + t) + \sum_{\text{mar}} [X_{\text{mar}} \cdot P_{\text{mar}}] (x_{\text{mar}} + p_{\text{mar}}) \]

... noting that demand for margins is: \( x_{\text{mar}} = x + a_{\text{mar}} \)

\[ [P \cdot X] p = [P0 \cdot X \cdot T] (p0 + t) + \sum_{\text{mar}} [X_{\text{mar}} \cdot P_{\text{mar}}] (a_{\text{mar}} + p_{\text{mar}}) \]
Causation in short-run closure

\[ \text{GDP} = \text{HH Cons} + \text{Investment} + \text{GOV Cons} + \text{Trade balance} \]

- Real Wage
- Employment
- Tech Change
- Capital Stocks
- Rate of return on capital

Exogenous
Endogenous

Follows factors income
Follows tax revenue
Causation in long-run closure

Real Wage

Employment

Tech Change

Rate of return on capital

Capital Stocks

Rate of return on capital

GDP

HH Cons

Investment

GOV Cons

Trade balance

Endogenous

Exogenous

Follows factors income

Follows tax revenue

Sectoral investment follows capital

Department of Economics, University of Sao Paulo
Interregional CGE Model for Chile

BM-CH Model
Interregional Computable General Equilibrium Model for Chile

The University of Sao Paulo Regional and Urban Economics Lab - NEREUS
November 2018
BM-CH, a bottom-up spatial CGE model of Chile

A multi-sectoral, multi-regional bottom-up CGE model of Morocco’s 15 regions

- each region is modeled as an economy in its own right
- region-specific prices
- region-specific industries
- region-specific consumers

Based on the comparative-static B-MARIA and MMRF models

Database makes allowance for interregional, intra-regional and international trade

- Potential for the representation of regional and central government financial accounts
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- Export demands
- Government demands
- Zero pure profits
- Indirect tax equations
- Market-clearing
- Regional and national macroeconomic variables and price indexes
- Capital accumulation and investment
- Regional population and labor market
Production nest

1. Output
   - Leontief
     - Intermediate Inputs
       - CES
     - Primary Factors
       - CES
       - Other Costs
2. CES
   - Domestic Source
   - Imported Source
   - CES
   - Labor
   - Capital
3. Region r Source
   - Region s Source
Investment demand

Capital good

Leontief

Good 1

up to

Good I

CES

CES

Imported Good 1

Domestic Good 1

Imported Good I

Domestic Good I

CES

from Region 1

from Region 2

up to

from Region 33

KEY

Inputs or Outputs

Functional Form
Household demand

(1) 

Utility

<table>
<thead>
<tr>
<th>LES</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
</tr>
<tr>
<td>Domestic Source</td>
</tr>
<tr>
<td>Imported Source</td>
</tr>
<tr>
<td>CES</td>
</tr>
<tr>
<td>Region r Source</td>
</tr>
<tr>
<td>Region s Source</td>
</tr>
</tbody>
</table>

(2) 

Utility

<table>
<thead>
<tr>
<th>CES</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
</tr>
</tbody>
</table>

(3) 

Utility

<table>
<thead>
<tr>
<th>CES</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
</tr>
</tbody>
</table>

Department of Economics, University of Sao Paulo
Household demand

Each regional household determines optimal consumption bundle by maximizing a Stone-Geary utility function subject to a budget constraint.

A Keynesian-type consumption function determines aggregate regional household expenditure.
Foreign export demand

Export commodities face individual downward-sloping foreign export demand functions

Exports of product $i$ from source $s$ are distinguished from exports of $i$ from source $r$ ($r$ not equal $s$)

$$X4R(i, s) = FEQ(i, s) \times NATFEQ \times \left( \frac{P4R(i, s)}{FEP(i) \times NATFEP} \right)^{EXP\_ELAST(i)}$$
Government demand

Recognise regional governments and central government demands for goods and services for current consumption

Default:

- aggregate regional government demand in region q moves with regional government revenue, with structure of demand exogenous

- aggregate central government demand in region q moves with national government revenue, with structure of demand exogenous
Zero pure profits

Critical assumptions

- no pure profits in the production or distribution of commodities
- price received by the producer is uniform across all customers

Zero pure profits in current production imposed by setting unit prices received by producers equal to unit costs

Zero pure profits in distribution imposed by setting the prices paid by users equal to producer price plus commodity tax plus margins
Indirect taxes

Equations have been added to enable flexible handling of indirect taxes on all flows of goods and services.

Equations allow for variations in tax rates across commodities, their sources and destinations.
Market-clearing

Equations that impose market clearing (demand equals supply) for:

- domestically produced margin and non-margin commodities
- imported commodities
Macro aggregates

Wide range of national and regional macro variables defined...

Two concepts of the real wage rate:

- consumer real wage rate (PLAB/CPI)
- producer real wage rate (PLAB/PGDP)
Investment “dynamics”

Capital, investment and expected rates of return

\[ K_{j,q}(t+1) = (1 - \text{DEP}_{j,q}) \times K_{j,q}(t) + Y_{j,q}(t) \]

Given starting point for capital (t=0) and an explanation of investment, we can trace out time path for capital
Investment “dynamics”

Investment explained by assuming that:

$$\frac{K_{j,q}(t+1)}{K_{j,q}(t)} - 1 = F_{j,q}^t [EROR_{j,q}(t)]$$

Growth in capital related to expected rate of return

- In BM-CH ICGE only assume static expectations, though rational is possible
Rates of return and investment

For static expectations case, the actual rate of return is:

\[ RO_t(j, q) = \frac{P_t(j, q)}{\Pi_t(j, q)} - D(j, q) \]

\[ ro(j, q) = p_t(j, q) - \pi_t(j, q) \]

\[ ro(j, q) = QCOEF(j, q) [p_t(j, q) - \pi_t(j, q)] \]

\textit{QCOEF}: relationship between gross and net rates of return (> 1)
Rates of return and investment

In long-run comparative-static simulations:

- aggregate capital adjusts to maintain $R_{INT} (natr\_tot)$
- capital allocated in line with equation $E_{f\_rate\_xx}$
  - industries with relatively large increases in capital require relatively high rates of return
  - industries with relatively small increases in capital require relatively low rates of return
- industry investment determined by fixed ratios of investment to capital (equation $E\_y$)

Department of Economics, University of Sao Paulo  54
Rates of return and investment

Equalization in the rates of return

\[
\left( \frac{K(j,q)}{K(q)} \right)^{-\beta(j,q)} \quad RO(j,q) = R_{\text{int}}
\]

\[ro(j,q) - r_{\text{int}} = \beta_t(j,q)[k(j,q) - k(q)] + f\_\text{rate}(j,q)\]

*beta*: risk/return ratio

Short-run: *f_rate* endogenous, *k* exogenous

Long-run: *f_rate* exogenous, *k* endogenous
Investment “dynamics”

Growth rate of capital stocks and investment in the short-run:

\[ k_{t+1}(j, q) - k_t(j, q) = 0 \]  % change in capital stocks

\[ y_t(j, q) = 0 \]  % change in investment
Investment “dynamics”

Growth rate of capital stocks and investment in the long-run:

\[
\frac{K_{j,q}(t+1)}{K_{j,q}(t)} = \left(\frac{K_{j,q}(t)}{K_{j,q}(0)}\right)^{1/T}
\]

\[
k_{t+1}(j,q) = \left(1 + \frac{1}{T}\right)k_t(j,q)
\]
Investment in the short run

Fixed capital stocks in the base year values:
- \( \text{curcap}(j,q) \) exogenous (\( =0 \))
- relationship between sectoral rates of return, \( r0(j,q) \), and reference interest rate, \( \text{natr}\_\text{tot} \), is endogenous (\( f\_\text{rate}\_\text{xx}(j,q) \) endogenous)

Percentage change in sectoral investment, \( y(j,q) \) is zero; this can be guaranteed by setting the shift term, \( \text{delf}\_\text{rate}(j,q) \), exogenous and zero

By hypothesis, not only the capital stocks are fixed but also firms’ investment plans
Investment in the short run

\[ E_{r0} \# \text{Definition of rates of return to capital} \#
\]
\[ r0(j,q) = QCOEF(j,q) \ast (plcap(j,q) - pi(j,q)) \];

\[ E_{f\_rate\_xx} \# \text{Capital growth rates related to rates of return} \#
\]
\[ (r0(j,q) - natr\_tot) = BETA\_R(j,q) \ast [curcap(j,q) - kt(q)] + f\_rate\_xx(j,q); \]

\[ E_{curcapT1} \# \text{Capital stock in period T+1} \#
\]
\[ curcap\_t1(j,q) - curcap(j,q) = 0; \]

\[ E_{yT} \# \text{Investment in period T} \#
\]
\[ curcap(j,q) - y(j,q) - 100 \ast delf\_rate(j,q) = 0; \]
Investment in the long run

Capital stocks endogenously determined:

- $\text{curcap}(j,q)$ endogenous
- relationship between sectoral rates of return, $r_0(j,q)$, and reference interest rate, $\text{natr}_\text{tot}$, is given ($f_\text{rate}_\text{xx}(j,q)$ exogenous)

Percentage change in sectoral investment, $y(j,q)$ is endogenous

Firms’ investment plans are carried out, reestablishing returns differentials in the base year

Rate of capital accumulation, but **not the level** of capital stock, remains constant
Investment in the long run

\[ E_{r0} \quad \text{# Definition of rates of return to capital} \quad # \]
\[ r_{0}(j,q) = QCOEF(j,q)*(p1cap(j,q) - pi(j,q)); \]

\[ E_{f \_rate \_xx} \quad \text{# Capital growth rates related to rates of return} \quad # \]
\[ (r_{0}(j,q) - natr_{tot}) = BETA_{R}(j,q)*[curcap(j,q) - kt(q)] + f \_rate \_xx(j,q); \]

\[ E_{curcapT1} \quad \text{# Capital stock in period T+1} \quad # \]
\[ curcap\_t1(j,q) - K\_TERM*curcap(j,q) = 0; \]

\[ E_{yT} \quad \text{# Investment in period T} \quad # \]
\[ VALK_{T1}(j,q)*curcap\_t1(j,q) = VALKT(j,q)*DEP(j)*curcap(j,q) + (INVEST(j,q))*y(j,q) - 100*(VALK\_0(j,q)*(1-DEP(j))) \]

endog. exog.

\( (DEP(j) = 0.96) \)
Critical variables:
- regional population
- regional migration
- regional unemployment
- regional participation rates
- regional wage relativities

Various closures
Regional population and labor market

(1) Fixed
  - wage relativities (determining employment by region), participation and unemployment rates (determining population by region)

(1) Endogenous
  - regional migration

(2) Fixed
  - regional migration, participation rates, wage relativities

(2) Endogenous
  - unemployment rates

(3) Fixed
  - regional migration, participation and unemployment rates

(3) Endogenous
  - wage relativities
Labor market in the short-run

\[ E_{\text{wage\_diff}} \] # Region real-wage diff
#(all,q,REGDEST)

\[ \text{wage\_diff}(q) = p\text{wage}(q) - \text{natxi3} - \text{natrealwage}; \]

\[ E_{\text{del\_labsup}} \] # P-point changes in regional unemployment rates 
#(all,q,REGDEST)

\[ C_{\text{labsup}}(q) \times \text{del\_unr}(q) = C_{\text{EMPLOY}}(q) \times (\text{labsup}(q) - \text{employ}(q)); \]

\[ \text{del\_unr}(q) \] # Percentage-point changes in regional unemployment rate 
#;

---

endog. exog.
Labor market in the long-run

\[
E_{\text{wage\_diff}} \quad \# \text{Region real-wage diff} \\
\quad \# (\text{all, } q, \text{REGDEST})
\]

\[
\text{wage\_diff}(q) = \text{pwage}(q) - \text{natxi3} - \text{natrealwage};
\]

\[
E_{\text{del\_labsup}} \quad \# \text{P-point changes in regional unemployment rates} \\
\quad \# (\text{all, } q, \text{REGDEST})
\]

\[
C_{\text{labsup}}(q) \times \text{del\_unr}(q) = C_{\text{EMPLOY}}(q) \times (\text{labsup}(q) - \text{employ}(q));
\]

\[
\text{del\_unr}(q) \quad \# \text{Percentage-point changes in regional unemployment rate} \\
\]

endog. \quad \text{exog.}

Department of Economics, University of Sao Paulo  \quad 65
Closures

Each equation explains a variable

More variables than equations

Endogenous variables: explained by model

Exogenous variables: set by user

Closure: choice of exogenous variables

Many possible closures

Number of endogenous variables = Number of equations
Length of run, \( T \)

\( T \) is related to our choice of closure

With short-run closure we assume that:

- \( T \) is long enough for price changes to be transmitted throughout the economy, and for price-induced substitution to take place
- \( T \) is not long enough for investment decisions to greatly affect the useful size of sectoral capital stocks [new buildings and equipment take time to produce and install]

\( T \) might be 2 years. So results mean:

- A 10% consumption increase might lead to employment in 2 years time being 1.2% higher than it would be (in 2 years time) if the consumption increase did not occur.
Different closures

Many closures might be used for different purposes

No unique natural or correct closure

Must be at least one exogenous variable measured in local currency units

Normally just one — called the *numéraire*

Often the exchange rate, *natphi*, or *natxi3*, the CPI.

Some quantity variables must be exogenous, such as:
- primary factor endowments
- final demand aggregates
In honor of those that made this gathering possible!
ehaddad@usp.br

www.usp.br/nereus