NEREUS
Núcleo de Economia Regional e Urbana

## Ejercicio Práctico de Aplicación

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## Input-output table



## Input-output model

The input-output table is basically an accounting system - a double entry one similar to that prepared for a business in which sales and purchases or assets and liabilities will be shown but, in this case, for an economy.

The next step is to prepare an economic model so that we can trace the impact of changes in one sector on the rest of the economy.

We do this because the nature of interdependence among sectors varies.

## Input-output model

## Key assumptions

We assume that each sector produces goods and services according to a fixed recipe (formally known as a production function)

$$
a_{i j}=\frac{z_{i j}}{x_{j}} \quad \forall \quad i, j=1,2, \ldots, n
$$

- Fixed technical coefficient
- Constant returns to scale
- Sectors use inputs in fixed proportions


## Production function

$$
x_{j}=f\left(z_{1 j}, \ldots, z_{n j}, W_{j}, M_{j}\right) \quad x_{j}=\min \left(\frac{z_{1 j}}{a_{1 j}}, \ldots, \frac{z_{n j}}{a_{n j}}\right)
$$


(a) Classical Production Function

(b) Leontief Production Function

FIGURE 2-1 Production functions in input space.

## Input-output model

## Key assumptions

Inputs are expressed in monetary terms since it would be difficult to combine tons of iron ore with megawatts of electricity, or hours of labor in some consistent fashion.

This fixed recipe enables us to express the transactions in proportional form, also known as direct coefficients.

## Input-output model

## Key assumptions

The final assumption is that the economy is driven by signals emanating from final demand (consumers, government, exports); this is the exogenous part of the economy, while the interindustry transactions respond to these signals and are therefore endogenous.

## Input-output model

## Basic relations

$$
\sum_{j=1}^{n} z_{i j}+y_{i} \equiv x_{i} \quad \forall i, j=1,2, \ldots, n
$$

$z_{i j}=$ flows of intermediate consumption from sector $i$ to sector $j$
$y_{i}=$ final demand of sector $i$
$x_{i}=$ total supply of sector $i$

## Input-output model

## Basic relations

Replacing $a_{i j}=\frac{z_{i j}}{x_{j}}$ in the previous equation, we have:

$$
\sum_{j=1}^{n} a_{i j} x_{j}+y_{i}=x_{i} \quad \forall i, j=1,2, \ldots, n
$$

$$
\begin{gathered}
\mathbf{A} x+y=x \\
x=(\mathbf{I}-\mathbf{A})^{-1} y \\
\mathbf{B}=(\mathbf{I}-\mathbf{A})^{-1}
\end{gathered}
$$

$\mathbf{A}=$ matrix of technical direct coefficients
$\mathbf{B}=$ Leontief inverse matrix

## Input-output model

## Basic relations

The Leontief inverse matrix B shows the coefficients (economic multipliers) that measure the successive effects on the economy as a result of the initial increase in production of an economic activity branch:

$$
\begin{gathered}
x=(\mathbf{I}-\mathbf{A})^{-1} y \\
{\left[\begin{array}{c}
x_{1} \\
\vdots \\
x_{n}
\end{array}\right]=\left[\begin{array}{ccc}
b_{11} & \cdots & b_{1 n} \\
\vdots & \ddots & \vdots \\
b_{n 1} & \cdots & b_{n n}
\end{array}\right]\left[\begin{array}{c}
y_{1} \\
\vdots \\
y_{n}
\end{array}\right]=\left[\begin{array}{ccc}
b_{11} y_{1} & \cdots & b_{1 n} y_{n} \\
\vdots & \ddots & \vdots \\
b_{n 1} y_{1} & \cdots & b_{n n} y_{n}
\end{array}\right]}
\end{gathered}
$$

The entries reveal the direct and indirect impacts on a sector when final demand in the sector at the top of the column changes by $\$ 1$ (or $\$ 1$ million or $\$ 100$ million). The sum of the elements in a column generate the output multipliers.

## Output multipliers

Calculation of the output multiplier (backward linkages)

$$
o_{j}=\sum_{i=1}^{n} b_{i j}, \quad i, j=1, \ldots, n
$$

## Output multipliers

Decomposition of the output multiplier

Total effect: $\quad E T_{j}=o_{j}$

Indirect effect:
$E I D_{j}=o_{j}-\sum_{i} a_{i j}-1$

Direct effect:

$$
E D_{j}=\sum_{i} a_{i j}
$$

Initial effect: $\quad E I_{j}=1$

## Interregional IO models



## Interregional IO models



## Interregional IO models

## Interregional model

$$
\begin{gathered}
A=\left[\begin{array}{ccc}
A^{L L} & \vdots & A^{L M} \\
\cdots & \cdots & \cdots \\
A^{M L} & \vdots & A^{M M}
\end{array}\right] y=\left[\begin{array}{c}
y^{L} \\
\cdots \\
y^{M}
\end{array}\right] x=\left[\begin{array}{c}
x^{L} \\
\cdots \\
x^{M}
\end{array}\right] \\
\left\{\left[\begin{array}{ccc}
I & \vdots & 0 \\
\cdots & \cdots & \cdots \\
0 & \vdots & I
\end{array}\right]-\left[\begin{array}{ccc}
A^{L L} & \vdots & A^{L M} \\
\cdots & \cdots & \cdots \\
A^{M L} & \vdots & A^{M M}
\end{array}\right]\right\}\left[\begin{array}{c}
x^{L} \\
\cdots \\
x^{M}
\end{array}\right]=\left[\begin{array}{c}
y^{L} \\
\cdots \\
y^{M}
\end{array}\right] \\
(I-A) x=y \longrightarrow x=(I-A)^{-1} y
\end{gathered}
$$

## Multipliers in interregional IO models

$$
A=\left[\begin{array}{cc}
A^{L L} & A^{L M} \\
A^{M L} & A^{M M}
\end{array}\right] \rightarrow B=(I-A)^{-1}=\left[\begin{array}{cc}
B^{L L} & B^{L M} \\
B^{M L} & B^{M M}
\end{array}\right]
$$

Intraregional effects: $\quad B^{L L}, B^{M M} \rightarrow O_{j}^{L L}, O_{j}^{M M}$

Interregional effects: $\quad B^{M L}, B^{L M} \rightarrow O_{j}^{M L}, O_{j}^{L M}$

National effects: $\quad O_{j}^{L}=O_{j}^{L L}+O_{j}^{M L}$ e $O_{j}^{M}=O_{j}^{M M}+O_{j}^{L M}$

Sectoral effects: $\quad O_{i j}^{L}=\alpha_{i j}^{L L}+\alpha_{i j}^{M L}$ e $O_{i j}^{M}=\alpha_{i j}^{M M}+\alpha_{i j}^{L M}$

## Multipliers in interregional IO models

Multipliers vary not only across sectors but also across regions.

A small regional economy, with a modest representation of industry, may not be able to provide all the necessary inputs required by local industry. Thus, there will be considerable importation of inputs (sometimes referred to as leakages).

In general, the larger the value of the imports, the lower the value of the multiplier.

We would expect multipliers to decrease as we move from the country as a whole to a macro-region, an individual province, a metropolitan region and finally to a municipality.

## Mexican interregional input-output system, 2013



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The University of São Paulo
Regional and Urban Economics Lab
Interregional Input-Output Tables for Mexico, 2013
Reference: Haddad, E. A., Araújo, I. F., Ibarrarán, M. E., Boyd, R., Elizondo, A., Liedo, P., Belausteguigoitia, J. C., and Menchero, M. (2019). Interregional Input-Output System for Mexico, 2013, TD NEREUS 07-2019, The University of São Paulo Regional and Urban Economics Lab (NEREUS).
http://www.scielo.org.mx/scielo.php?script=sci arttext\&pid=S244866552020000300007
https://www.researchgate.net/publication/344121142 Interstate inputoutput model for Mexico 2013

## List of sectors

| Id | Cod. <br> SUT* | Sectors |
| :---: | :---: | :---: |
| S1 | 111 | Agriculture |
| S2 | 112 | Animal production |
| S3 | 113 | Forestry and logging |
| S4 | 114 | Fishing and aquaculture |
| S5 | 115 | Agriculture, farming, forestry and fishing support service activities |
| S6 | 211 | Extraction of crude petroleum and natural gas |
| S7 | 212-213 | Mining and support service activities |
| S8 | 221 | Electric power generation, transmission and distribution |
| S9 | 222 | Water and gas supply by pipelines to the final consumer |
| S10 | 236-238 | Construction |
| S11 | 311 | Manufacture of food products |
| S12 | 312 | Manufacture of beverages and tobacco products |
| S13 | 313-314 | Manufacture of textiles |
| S14 | 315-316 | Manufacture of wearing apparel |
| S15 | 321 | Manufacture of wood and of products of wood and cork, except furniture |
| S16 | 322-323 | Manufacture of paper and paper products; Printing and reproduction of recorded media |
| S17 | 324-326 | Manufacture of coke and refined petroleum products; Manufacture of chemicals and chemical products; Manufacture of rubber and plastics products |
| S18 | 327 | Manufacture of other non-metallic mineral products |
| S19 | 331-332 | Manufacture of basic metals; Manufacture of fabricated metal products, except machinery and equipment |
| S20 | 333-336 | Manufacture of machinery and equipment n.e.c.; Manufacture of computer, electronic and optical products; Manufacture of electrical equipment; Manufacture of motor vehicles, trailers and semi-trailers; Manufacture of other transport equipment |
| S21 | 337 | Manufacture of furniture |
| S22 | 339 | Other manufacturing |
| S23 | 431 | Wholesale trade |
| S24 | 461 | Retail trade |
| S25 | 481-493 | Transportation and storage |
| S26 | 511-519 | Information and communication |
| S27 | 521-524 | Financial and insurance activities |
| S28 | 531-533 | Real estate activities |
| S29 | 541 | Professional, scientific and technical activities |
| S30 | 551 | Activities of head offices; management consultancy activities |
| S31 | 561-562 | Administrative and support service activities |
| S32 | 611 | Education |
| S33 | 621-624 | Human health and social work activities |
| S34 | 711-713 | Arts, entertainment and recreation |
| S35 | 721-722 | Accommodation and food service activities |
| S36 | 811-814 | Other service activities |
| S37 | 931 | Public administration and defense; compulsory social security; Activities of extraterritorial organizations and bodies |

## Aggregate trade flows in Mexico

## Annex 4.A. Interregional Trade in Mexico, 2013 (in MXN billions)

|  | DESTINATION |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | R1 | R2 | R3 | R4 | R5 | R6 | R7 | R8 | R9 | R10 | R11 | R12 | R13 | R14 | R15 | R16 | R17 | R18 | R19 | R20 | R21 | R22 | R23 | R24 | R25 | R26 | R27 | R28 | R29 | R30 | R31 | R32 | ROW |  |
| R1 | 159 | 2 | 0 | 6 | 4 | 1 | 2 | 4 | 20 | 1 | 7 | 1 | 1 | 10 | 6 | 2 | 1 | 1 | 8 | 1 | 2 | 3 | 1 | 4 | 1 | 2 | 4 | 3 | 0 | 4 | 1 | 5 | 91 | 355 |
| R2 | 2 | 456 | 3 | 29 | 11 | 1 | 6 | 12 | 27 | 2 | 3 | 1 | 1 | 7 | 7 | 1 | 1 | 1 | 13 | 1 | 3 | 2 | 2 | 2 | 3 | 16 | 14 | 4 | 0 | 9 | 2 | 3 | 256 | 899 |
| R3 | 1 | 8 | 84 | 11 | 5 | 0 | 2 | 4 | 13 | 1 | 2 | 0 | 0 | 2 | 3 | 0 | 0 | 0 | 7 | 0 | 1 | 1 | 0 | 1 | 1 | 4 | 5 | 2 | 0 | 3 | 0 | 1 | 8 | 170 |
| R4 | 1 | 5 | 0 | 164 | 8 | 0 | 15 | 4 | 45 | 1 | 26 | 0 | 13 | 23 | 47 | 3 | 6 | 1 | 32 | 13 | 6 | 11 | 1 | 6 | 1 | 4 | 44 | 41 | 2 | 42 | 2 | 1 | 347 | 915 |
| R5 | 5 | 13 | 2 | 20 | 478 | 1 | 5 | 28 | 39 | 5 | 9 | 2 | 2 | 15 | 15 | 4 | 2 | 1 | 93 | 3 | 7 | 5 | 3 | 6 | 5 | 12 | 12 | 17 | 1 | 13 | 3 | 5 | 448 | 1279 |
| R6 | 1 | 2 | 0 | 5 | 3 | 73 | 1 | 2 | 12 | 0 | 2 | 0 | 0 | 6 | 3 | 1 | 0 | 0 | 4 | 0 | 1 | 1 | 0 | 1 | 0 | 2 | 3 | 1 | 0 | 2 | 0 | 1 | 8 | 139 |
| R7 | 2 | 8 | 1 | 16 | 7 | 1 | 233 | 7 | 39 | 2 | 5 | 2 | 2 | 7 | 11 | 2 | 1 | 1 | 13 | 2 | 5 | 3 | 3 | 2 | 2 | 6 | 20 | 5 | 1 | 13 | 4 | 1 | 33 | 462 |
| R8 | 3 | 13 | 1 | 18 | 21 | 1 | 5 | 416 | 32 | 5 | 6 | 1 | 1 | 11 | 11 | 3 | 2 | 1 | 29 | 2 | 5 | 3 | 2 | 4 | 4 | 14 | 11 | 8 | 1 | 10 | 2 | 4 | 330 | 979 |
| R9 | 22 | 46 | 6 | 59 | 58 | 6 | 21 | 59 | 2566 | 12 | 68 | 18 | 44 | 69 | 256 | 25 | 28 | 7 | 73 | 21 | 72 | 49 | 8 | 34 | 15 | 41 | 60 | 51 | 13 | 92 | 10 | 14 | 166 | 4088 |
| R10 | 2 | 5 | 1 | 8 | 12 | 0 | 2 | 11 | 20 | 148 | 4 | 1 | 1 | 7 | 7 | 2 | 1 | 1 | 18 | 1 | 2 | 2 | 1 | 3 | 3 | 5 | 5 | 5 | 0 | 5 | 1 | 3 | 31 | 317 |
| R11 | 11 | 12 | 2 | 18 | 17 | 2 | 5 | 16 | 99 | 4 | 556 | 3 | 4 | 32 | 28 | 9 | 3 | 2 | 28 | 3 | 10 | 16 | 3 | 12 | 5 | 10 | 13 | 13 | 1 | 16 | 3 | 7 | 187 | 1151 |
| R12 | 2 | 4 | 0 | 11 | 6 | 0 | 2 | 6 | 58 | 1 | 4 | 138 | 1 | 5 | 11 | 1 | 3 | 0 | 10 | 1 | 5 | 3 | 1 | 2 | 1 | 3 | 8 | 3 | 1 | 7 | 1 | 1 | 19 | 318 |
| R13 | 2 | 4 | 1 | 7 | 5 | 1 | 2 | 5 | 101 | 1 | 6 | 2 | 179 | 7 | 18 | 2 | 2 | 1 | 8 | 1 | 9 | 5 | 1 | 3 | 1 | 3 | 6 | 4 | 2 | 10 | 1 | 1 | 33 | 435 |
| R14 | 18 | 22 | 3 | 36 | 30 | 6 | 9 | 28 | 145 | 7 | 38 | 4 | 6 | 897 | 44 | 18 | 5 | 6 | 50 | 4 | 15 | 17 | 5 | 17 | 10 | 19 | 24 | 23 | 2 | 27 | 5 | 11 | 306 | 1855 |
| R15 | 9 | 17 | 3 | 26 | 24 | 3 | 8 | 22 | 659 | 5 | 27 | 8 | 13 | 34 | 1052 | 12 | 13 | 3 | 43 | 7 | 31 | 25 | 5 | 13 | 6 | 15 | 23 | 19 | 5 | 35 | 5 | 5 | 300 | 2475 |
| Z R16 | 5 | 8 | 1 | 10 | 12 | 1 | 2 | 10 | 63 | 2 | 14 | 1 | 3 | 30 | 21 | 246 | 2 | 1 | 18 | 2 | 6 | 8 | 1 | 5 | 2 | 7 | 8 | 7 | 1 | 10 | 2 | 2 | 51 | 562 |
| - R17 | 1 | 2 | 0 | 5 | 3 | 0 | 1 | 3 | 51 | 1 | 3 | 2 | 1 | 4 | 11 | 1 | 157 | 0 | 5 | 1 | 5 | 2 | 1 | 1 | 1 | 2 | 4 | 2 | 1 | 5 | 1 | 1 | 62 | 338 |
| \% R18 | 1 | 2 | 0 | 4 | 4 | 0 | 1 | 3 | 15 | 1 | 2 | 0 | 0 | 6 | 3 | 1 | 0 | 78 | 6 | 0 | 1 | 1 | 0 | 1 | 1 | 2 | 3 | 2 | 0 | 2 | 0 | 1 | 7 | 154 |
| R19 | 11 | 29 | 4 | 44 | 118 | 3 | 11 | 52 | 89 | 9 | 20 | 4 | 4 | 30 | 27 | 7 | 4 | 3 | 1148 | 5 | 13 | 10 | 5 | 15 | 10 | 25 | 28 | 64 | 2 | 27 | 6 | 12 | 388 | 2226 |
| R20 | 2 | 6 | 1 | 18 | 7 | 1 | 5 | 6 | 51 | 1 | 5 | 1 | 2 | 6 | 11 | 2 | 2 | 1 | 12 | 189 | 8 | 3 | 2 | 2 | 2 | 5 | 12 | 4 | 1 | 13 | 2 | 2 | 27 | 409 |
| R21 | 4 | 7 | 1 | 21 | 9 | 1 | 7 | 9 | 149 | 2 | 11 | 3 | 8 | 14 | 36 | 4 | 7 | 1 | 17 | 6 | 428 | 8 | 3 | 5 | 2 | 6 | 18 | 7 | 7 | 36 | 3 | 2 | 164 | 1006 |
| R22 | 4 | 5 | 1 | 11 | 8 | 1 | 3 | 7 | 74 | 2 | 16 | 2 | 3 | 13 | 22 | 4 | 2 | 1 | 13 | 2 | 7 | 289 | 1 | 7 | 2 | 4 | 8 | 6 | 1 | 10 | 2 | 3 | 119 | 651 |
| R23 | 1 | 6 | 0 | 33 | 8 | 0 | 2 | 8 | 35 | 1 | 4 | 0 | 1 | 3 | 6 | 1 | 1 | 0 | 14 | 1 | 3 | 2 | 144 | 2 | 1 | 5 | 12 | 6 | 0 | 6 | 3 | 1 | 10 | 320 |
| R24 | 7 | 5 | 1 | 10 | 11 | 1 | 3 | 8 | 46 | 2 | 13 | 1 | 2 | 16 | 14 | 3 | 2 | 1 | 22 | 2 | 5 | 8 | 1 | 272 | 2 | 4 | 7 | 10 | 1 | 9 | 1 | 6 | 113 | 611 |
| R25 | 3 | 14 | 1 | 17 | 18 | 1 | 3 | 15 | 36 | 4 | 5 | 1 | 1 | 13 | 9 | 2 | 1 | 1 | 23 | 1 | 4 | 3 | 1 | 3 | 241 | 15 | 10 | 7 | 1 | 8 | 2 | 3 | 34 | 500 |
| R26 | 3 | 33 | 3 | 33 | 19 | 2 | 8 | 23 | 42 | 3 | 8 | 2 | 2 | 17 | 16 | 5 | 3 | 2 | 27 | 3 | 7 | 4 | 3 | 4 | 9 | 458 | 18 | 7 | 1 | 18 | 4 | 4 | 244 | 1035 |
| R27 | 3 | 13 | 2 | 33 | 11 | 1 | 18 | 11 | 56 | 3 | 18 | 3 | 9 | 18 | 34 | 5 | 4 | 1 | 25 | 9 | 10 | 8 | 6 | 6 | 5 | 11 | 303 | 23 | 2 | 40 | 9 | 2 | 177 | 878 |
| R28 | 5 | 13 | 2 | 23 | 26 | 2 | 5 | 18 | 51 | 4 | 11 | 2 | 3 | 15 | 16 | 4 | 2 | 1 | 66 | 2 | 6 | 6 | 2 | 9 | 5 | 11 | 13 | 404 | 1 | 14 | 3 | 6 | 230 | 980 |
| R29 | 1 | 1 | 0 | 3 | 2 | 0 | 1 | 2 | 26 | 0 | 2 | 0 | 1 | 2 | 6 | 1 | 1 | 0 | 3 | 1 | 9 | 2 | 0 | 1 | 0 | 1 | 2 | 1 | 69 | 5 | 1 | 0 | 20 | 164 |
| R30 | 6 | 17 | 3 | 30 | 18 | 2 | 11 | 18 | 149 | 4 | 18 | 6 | 10 | 24 | 44 | 7 | 6 | 2 | 33 | 8 | 31 | 11 | 6 | 8 | 6 | 15 | 31 | 17 | 5 | 706 | 7 | 4 | 142 | 1404 |
| R31 | 1 | 4 | 0 | 35 | 4 | 0 | 3 | 4 | 18 | 1 | 2 | 1 | 1 | 4 | 5 | 1 | 1 | 0 | 7 | 1 | 2 | 1 | 5 | 1 | 1 | 3 | 15 | 3 | 0 | 6 | 186 | 1 | 22 | 341 |
| R32 | 4 | 2 | 0 | 5 | 6 | 0 | 1 | 4 | 16 | 1 | 5 | 0 | 1 | 6 | 5 | 2 | 1 | 0 | 12 | 0 | 2 | 2 | 0 | 4 | 1 | 2 | 3 | 3 | 0 | 5 | 1 | 100 | 33 | 228 |
| ROW | 96 | 215 | 16 | 139 | 369 | 15 | 73 | 269 | 603 | 37 | 229 | 26 | 70 | 330 | 450 | 60 | 66 | 15 | 492 | 51 | 233 | 146 | 26 | 133 | 48 | 247 | 153 | 226 | 29 | 230 | 38 | 37 | 0 | 5167 |
| TOTAL | 398 | 1000 | 143 | 907 | 1341 | 128 | 477 | 1092 | 5445 | 274 | 1148 | 235 | 391 | 1678 | 2256 | 439 | 328 | 133 | 2372 | 343 | 955 | 661 | 245 | 590 | 394 | 979 | 902 | 999 | 150 | 1440 | 311 | 250 | 4407 | 32809 |

## Interregional linkages

The conventional input-output model is given by the system of matrix equations:
$x=(I-A)^{-1} f=B f$
where $x$ and $f$ are respectively the vectors of gross output and final demand; $A$ consists of input coefficients $a_{i j}$ defined as the amount of product $i$ required per unit of product $j$ (in monetary terms), for $i, j=1, \ldots, \mathrm{n}$; and $B$ is known as the Leontief inverse.

## Interregional input-output model

In an interregional context, with $R$ different regions, we have:

$$
x=\left[\begin{array}{c}
x^{1} \\
\vdots \\
x^{R}
\end{array}\right] ; A=\left[\begin{array}{ccc}
A^{11} & \cdots & A^{1 R} \\
\vdots & \ddots & \vdots \\
A^{R 1} & \cdots & A^{R R}
\end{array}\right] ; f=\left[\begin{array}{c}
f^{1} \\
\vdots \\
f^{R}
\end{array}\right] ; \text { and } B=\left[\begin{array}{ccc}
B^{11} & \cdots & B^{1 R} \\
\vdots & \ddots & \vdots \\
B^{R 1} & \cdots & B^{R R}
\end{array}\right]
$$

and

$$
\begin{aligned}
& x^{1}=B^{11} f^{1}+\cdots+B^{1 R} f^{R} \\
& \vdots \\
& x^{R}=B^{R 1} f^{1}+\cdots+B^{R R} f^{R}
\end{aligned}
$$

## Interregional input-output model

Let us also consider different components of $f$, which include demands originating in the specific regions, $v^{r s}, s=1, \ldots, R$, and abroad, $e$. We obtain information of final demand from origin $s$ in the IIOM-MEX, allowing us to treat $v$ as a matrix which provides the monetary values of final demand expenditures from the domestic regions in Mexico and from the foreign region.

$$
v=\left[\begin{array}{ccc}
v^{11} & \cdots & v^{1 R} \\
\vdots & \ddots & \vdots \\
v^{R 1} & \cdots & v^{R R}
\end{array}\right] ; e=\left[\begin{array}{c}
e^{1} \\
\vdots \\
e^{R}
\end{array}\right]
$$

Thus:

$$
\begin{gathered}
x^{1}=B^{11}\left(v^{11}+\cdots+v^{R 1}+e^{1}\right)+\cdots+B^{1 R}\left(v^{1 R}+\cdots+v^{R R}+e^{R}\right) \\
\vdots \\
x^{R}=B^{R 1}\left(v^{11}+\cdots+v^{R 1}+e^{1}\right)+\cdots+B^{R R}\left(v^{1 R}+\cdots+v^{R R}+e^{R}\right)
\end{gathered}
$$

# Spatial propagation of final demand shocks (...) 

Table 2. Components of Decomposition of Regional Output Based on the Sources of Final Demand: Mexico, 2013 (in \%)

|  | ORIGIN OF FINAL DEMAND |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | R1 | R2 | R3 | R4 | R5 | R6 | R7 | R8 | R9 | R10 | R11 | R12 | R13 | R14 | R15 | R16 | R17 | R18 | R19 | R20 | R21 | R22 | R23 | R24 | R25 | R26 | R27 | R28 | R29 | R30 | R31 | R32 | ROW |
| R1 | 34.43 | 0.65 | 0.09 | 1.86 | 0.94 | 0.14 | 0.45 | 0.97 | 6.43 | 0.22 | 1.31 | 0.12 | 0.17 | 1.71 | 1.18 | 0.34 | 0.15 | 0.13 | 1.98 | 0.17 | 0.45 | 0.53 | 0.17 | 0.70 | 0.22 | 0.40 | 1.18 | 0.80 | 0.06 | 1.09 | 0.16 | 1.06 | 39.75 |
| R2 | 0.17 | 39.09 | 0.22 | . 57 | 96 | 06 | . 67 | 1.12 | 3.43 | 12 | . 31 | 0.06 | 07 | 0.48 | 0.58 | . 12 | .08 | 0.06 | 1.26 | 0.09 | 0.32 | 0.17 | 0.14 | 0.18 | 0.21 | 1.12 | 1.65 | 0.42 | 0.03 | 0.93 | 0.13 | 0.33 | 41.84 |
| R3 | 0.46 | 78 | 39.70 | 7.88 | 70 | . 08 | . 38 | 2.39 | 9.69 | 38 | 0.92 | 0.10 | 22 | . 04 | . 63 | 0.22 | 0.17 | 0.09 | 3.96 | 0.16 | 0.83 | 0.50 | 0.17 | 0.50 | 0.31 | 1.81 | 3.72 | 1.23 | 0.09 | 2.13 | 0.24 | 0.65 | 9.87 |
| R4 | 0.45 | 1.04 | 18 | 18.52 | 1.25 | 18 | 84 | 1.11 | 8.28 | 26 | 01 | 0.24 | 25 | 1.46 | . 93 | 0.43 | 0.30 | 0.16 | 2.42 | 0.26 | 0.8 | 0.58 | 0.40 | 0.53 | 0.45 | 0.87 | 2.47 | 0.70 | 0.12 | 1.74 | 0.48 | 0.43 | 83 |
| R5 | 0.23 | 0.75 | 0.11 | 1.75 | 23.40 | 0.08 | 40 | 29 | 3.54 | 23 | 0.45 | 09 | 09 | 0.66 | 0.71 | 0.17 | 0.11 | 0.09 | 4.26 | 0.13 | 0.30 | 0.23 | 0.17 | 0.26 | 0.27 | 0.48 | . 01 | 0.85 | 0.04 | 0.80 | 0.15 | 0.30 | 56.59 |
| R6 | 0.77 | 46 | 0.10 | 30 | 1.70 | 44.73 | 0.85 | . 82 | 10.76 | 24 | 1.32 | 0.14 | 0.24 | 3.29 | . 96 | 0.45 | 0.24 | 0.19 | 2.93 | 0.19 | 0.79 | 0.67 | 0.20 | 0.65 | 0.30 | 1.07 | 2.49 | 1.06 | 0.10 | 1.73 | 0.26 | 0.73 | 29 |
| R7 | 0.45 | 80 | 0.25 | , 03 | . 52 | 16 | 40.04 | 1.69 | 11.79 | . 32 | 1.03 | 30 | . 33 | 1.40 | 2.00 | 0.41 | 0.30 | 0.16 | 3.11 | 0.37 | . 02 | 0.67 | 0.71 | 0.49 | 0.47 | 13 | . 04 | . 08 | 0.14 | 2.17 | 0.79 | 0.28 | 15.54 |
| R8 | 0.23 | 10 | 0.10 | 97 | 33 | . 07 | 0.43 | 31.77 | 3.68 | . 22 | 0.45 | 0.08 | 0.09 | 0.65 | 0.70 | 0.16 | 0.10 | 0.08 | 2.18 | 0.12 | 0.31 | 0.23 | 0.15 | 0.24 | 0.25 | 0.80 | 09 | 0.60 | 0.04 | 0.82 | 0.15 | 0.33 | 45 |
| R9 | 0.56 | 28 | 0.14 | . 75 | 1.34 | 0.14 | 0.56 | 50 | 58.06 | 0.29 | 1.54 | 0.33 | 0.84 | 1.53 | 4.97 | 0.51 | 0.58 | 0.16 | 2.06 | 0.44 | 1.48 | 1.07 | 0.22 | 0.73 | 0.33 | 0.88 | 1.58 | 1.21 | 0.26 | 2.17 | 0.29 | 0.33 | 0.86 |
|  | 0.71 | 86 | 0.23 | 3.44 | 3.10 | 0.15 | 0.76 | 3.33 | 9.65 | 34.10 | 1.28 | 0.18 | 0.26 | 1.85 | . 91 | 0.38 | 0.26 | 0.21 | 5.54 | 0.24 | 0.79 | 0.64 | 0.31 | 0.73 | 0.68 | . 27 | 2.06 | 1.59 | 0.10 | . 70 | 0.30 | 0.99 | 9.40 |
| R11 | 0.80 | . 04 | 0.15 | 2.12 | 19 | 0.18 | 0.52 | 1.32 | 10.96 | 0.27 | 35.69 | 0.20 | . 29 | 2.11 | . 99 | 0.60 | 0.24 | 0.17 | 2.42 | 0.22 | 0.76 | 1.10 | 0.24 | 0.82 | 0.36 | 0.71 | 1.42 | 1.08 | 0.10 | 1.41 | 0.24 | 0.58 | 28.70 |
| R12 | 0.56 | 49 | 0.09 | 4.22 | 1.78 | 0.09 | 0.83 | 1.85 | 20.99 | 0.21 | 17 | 35.14 | 0.40 | . 46 | . 97 | 0.30 | 0.69 | 0.10 | 3.19 | 0.26 | 1.34 | 0.84 | 0.21 | 0.57 | 0.20 | 0.96 | 2.89 | 1.15 | 0.16 | 2.26 | 0.33 | 0.42 | 10.90 |
| R13 | 0.45 | 0.98 | 0.14 | 2.56 | . 03 | 0.15 | 0.67 | 13 | 26.85 | 0.23 | 17 | 0.29 | 29.01 | 1.44 | 34 | 0.45 | 0.41 | 0.13 | 2.03 | 0.31 | 1.66 | 0.90 | 0.28 | 0.53 | 0.33 | 0.68 | . 8 | 0.82 | 0.25 | 2.26 | 0.31 | 0.37 | 17.01 |
| R14 | 0.87 | 1.32 | 0.17 | 2.45 | 1.45 | 0.29 | 0.56 | 1.62 | 10.38 | 0.31 | 81 | . 18 | 0.29 | 35.19 | 2.06 | 0.80 | 0.25 | 0.29 | 2.92 | 0.23 | 0.77 | 0.84 | 0.26 | 0.78 | 0.44 | 0.89 | . 60 | 1.32 | 0.10 | 1.50 | 0.26 | 0.59 | 7.21 |
| R15 | 0.36 | 0.80 | 0.11 | 1.53 | 0.89 | 0.12 | 0.41 | 0.94 | 28 | 0.18 | . 02 | 0.26 | 0.43 | 1.20 | 32.08 | 0.41 | . 44 | 0.11 | 1.85 | 0.26 | 0.98 | 0.85 | 0.22 | 0.44 | 0.25 | 0.54 | 1.20 | 0.79 | 0.15 | 1.46 | 0.23 | 0.23 | 21.13 |
| R16 | 0.83 | 1.42 | 0.14 | 2.69 | 1.71 | 0.19 | 0.55 | 1.78 | 69 | 0.26 | 2.02 | 0.20 | 0.37 | 3.53 | 2.75 | 33.48 | 0.32 | . 21 | 3.36 | 0.25 | 0.92 | 1.16 | 0.25 | 0.72 | 0.33 | 0.95 | 1.85 | 1.32 | 0.13 | 1.70 | 0.29 | 0.47 | 19.13 |
| < R17 | 0.30 | 0.70 | 0.09 | 2.13 | 0.80 | 0.09 | 0.52 | 0.79 | 16.84 | 0.15 | 0.71 | 0.34 | 0.29 | 0.88 | 2.44 | . 26 | 35.79 | 0.08 | 1.49 | 0.21 | 1.11 | 0.53 | 0.17 | 0.34 | 0.20 | 0.48 | 1.47 | 0.58 | 0.13 | 1.53 | 0.19 | 0.26 | 28.10 |
| R18 | 0.78 | 1.66 | 0.13 | .74 | 2.41 | 0.17 | 0.75 | 2.28 | 12.43 | 0.49 | 1.61 | 0.14 | 0.30 | 3.10 | 2.1 | 0.47 | 0.22 | 42.6 | 4.18 | 0.18 | 0.8 | 0.87 | 0.19 | 0.70 | 0.44 | 1.31 | 2.17 | 1.42 | 0.10 | 1.72 | 0.2 | 0.61 | 9.54 |
| 促 | 0.38 | 1.15 | 0.15 | 2.59 | 3.33 | 0.12 | 0.59 | 1.8 | 5.30 | 0.32 | 0.70 | 0.1 | 0.13 | 1.00 | 0.99 | 0.26 | 0.15 | 0.11 | 38.21 | 0.18 | 0.46 | 0.35 | 0.2 | 0.45 | 0.36 | 0.75 | 1.5 | 1.93 | 0.05 | 1.2 | 0.2 | 0.54 | 28 |
| $\sim$ | 0.49 | 50 | 0.18 | 5.49 | 1.66 | 0.14 | 1.30 | 1.59 | 15.94 | 0.27 | 1.11 | 0.28 | 0.38 | 1.35 | 2.39 | 0.39 | 0.36 | 0.13 | 3.09 | 34.82 | 1.76 | 0.77 | 0.38 | 0.52 | 0.38 | 0.97 | 3.56 | 1.06 | 0.19 | 3.07 | 0.44 | 0.44 | 13.59 |
| R21 | 0.32 | 0.74 | 0.12 | 2.70 | 0.78 | 0.12 | 0.78 | 0.85 | 16.40 | 0.16 | 0.90 | 0.27 | 0.48 | 1.10 | 2.41 | 0.36 | 0.46 | 0.12 | 1.70 | 0.46 | 30.69 | 0.63 | 0.29 | 0.38 | 0.24 | 0.48 | 2.1 | 0.68 | 0.39 | 3.20 | 0.30 | 0.27 | 29.06 |
| R22 | 0.53 | 0.77 | 0.12 | 2.25 | 1.00 | 0.14 | 0.58 | 1.01 | 13.30 | 0.21 | 1.77 | 0.20 | 0.33 | 1.5 | 2.44 | 0.49 | 0.27 | 0.13 | 1.95 | 0.2 | 0.82 | 31.28 | 0.22 | 0.72 | 0.28 | 0.54 | 1.54 | 0.89 | 0.11 | 1.59 | 0.24 | 0.44 | 32.10 |
| R23 | 0.51 | 1.87 | 0.08 | 11.15 | 2.37 | 0.07 | 0.77 | 2.60 | 12.97 | 0.41 | 1.31 | 0.12 | 0.40 | 1.00 | 2.07 | 0.24 | 0.19 | 0.07 | 4.91 | 0.2 | 0.98 | 0.80 | 35.59 | 0.61 | 0.21 | 1.34 | 3.88 | 1.85 | 0.14 | 2.16 | 1.1 | 0.23 | 7.7 |
| R24 | 0.83 | 0.84 | 0.13 | 2.36 | 1.36 | 0.16 | 0.58 | 1.21 | 17 | 0.27 | 1.59 | 0.17 | 0.23 | 1.74 | 1.71 | 0.41 | 0.22 | 0.15 | 3.14 | 0.22 | 0.6 | 0.88 | 0.23 | 31.63 | 0.30 | 0.54 | 1.52 | 1.3 | 0.08 | 1.4 | 0.2 | 0.84 | 33.83 |
| R25 | 0.58 | 3.07 | 0.27 | 4.29 | 3.08 | 0.15 | 0.81 | 3.10 | 9.90 | 0.54 | 1.12 | 0.16 | 0.25 | 2.02 | 1.75 | 0.36 | 0.23 | 0.23 | 4.91 | 0.22 | 0.77 | 0.64 | 0.30 | 0.58 | 38.34 | 2.52 | 2.43 | 1.50 | 0.10 | 1.72 | 0.32 | 0.62 | 13.10 |
| R26 | 0.26 | 2.48 | 0.27 | 3.66 | 1.09 | 0.14 | 0.79 | 1.69 | 5.20 | 0.23 | 0.60 | 0.14 | 0.13 | 1.06 | 1.07 | 0.28 | 0.16 | 0.16 | 1.98 | 0.20 | 0.48 | 0.30 | 0.30 | 0.28 | 0.66 | 29.26 | 1.93 | 0.63 | 0.06 | 1.43 | 0.27 | 0.38 | 2.45 |
| R27 | 0.48 | 1.64 | 0.27 | 4.85 | 1.38 | 0.21 | 1.19 | 1.53 | 10.06 | 0.37 | 1.07 | 0.35 | 0.31 | 1.58 | 2.09 | 0.51 | 0.32 | 0.18 | 2.57 | 0.33 | 1.10 | 0.63 | 0.70 | 0.56 | 0.62 | 1.18 | 26.21 | 0.83 | 0.15 | 2.21 | 0.99 | 0.40 | 33.15 |
| R28 | 0.44 | 1.25 | 0.16 | 2.78 | 2.08 | 0.14 | 0.59 | 1.59 | 6.83 | 0.31 | 0.82 | 0.17 | 0.16 | 1.18 | 1.23 | 0.32 | 0.16 | 0.13 | 5.06 | 0.16 | 0.57 | 0.44 | 0.23 | 0.61 | 0.40 | 0.82 | 1.55 | 29.12 | 0.07 | 1.28 | 0.26 | 0.56 | 38.52 |
| R29 | 0.37 | 0.88 | 0.12 | 2.38 | 0.90 | 0.12 | 0.68 | 1.03 | 18.46 | 0.18 | 0.98 | 0.26 | 0.59 | 1.21 | 2.63 | 0.36 | 0.41 | 0.12 | 1.95 | 0.42 | 3.18 | 0.74 | 0.29 | 0.42 | 0.25 | 0.59 | 1.95 | 0.78 | 30.69 | 2.78 | 0.33 | 0.28 | 23.65 |
| R30 | 0.44 | 1.35 | 0.19 | 2.80 | 1.20 | 0.15 | 0.78 | 1.40 | 13.97 | 0.27 | 1.07 | 0.32 | 0.43 | 1.45 | 2.39 | 0.43 | 0.37 | 0.15 | 2.48 | 0.40 | 1.65 | 0.71 | 0.43 | 0.51 | 0.41 | 0.88 | 2.36 | 1.00 | 0.22 | 39.29 | 0.48 | 0.32 | 19.73 |
| R31 | 0.30 | 1.31 | 0.16 | 12.47 | 1.22 | 0.09 | 1.13 | 1.30 | 7.73 | 0.19 | 0.74 | 0.17 | 0.23 | 0.98 | 1.38 | 0.25 | 0.21 | 0.10 | 2.45 | 0.29 | 0.71 | 0.45 | 1.46 | 0.36 | 0.26 | 0.73 | 5.17 | 0.99 | 0.10 | 1.8 | 44.08 | 0.24 | 10.93 |
| R32 | 1.32 | 1.00 | 0.12 | 2.54 | 1.74 | 0.14 | 0.60 | 1.62 | 8.78 | 0.38 | 1.45 | 0.16 | 0.23 | 1.79 | 1.72 | 0.38 | 0.24 | 0.14 | 3.86 | 0.19 | 0.73 | 0.72 | 0.22 | 1.03 | 0.29 | 0.66 | 1.55 | 1.16 | 0.09 | 1.52 | 0.23 | 36.23 | 27.17 |
| TOTAL | 0.92 | 2.49 | 0.40 | 3.35 | 2.53 | 0.37 | 1.30 | 2.54 | 18.55 | 0.65 | 2.54 | 0.62 | 0.80 | 3.63 | 4.91 | 1.08 | 0.74 | 0.38 | 5.50 | 0.78 | 2.00 | 1.41 | 0.70 | 1.23 | 1.04 | 1.88 | 2.58 | 2.06 | 0.32 | 3.58 | 0.85 | 0.71 | 27.55 |

Source: Calculations by the authors.

## Spatial propagation of final demand shocks (...)

Figure 3. Identification of Regions Relatively More Affected by a Specific Regional Demand, by Origin of Final Demand (cont.)

Legend

| $\square$ | Origin of final demand |
| :--- | :--- |
| $\square$ | $0.0 \%-0.5 \%$ |
| $\square$ | $0.5 \%-1.0 \%$ |
|  | $1.0 \%-2.0 \%$ |
|  | $2.0 \%-3.0 \%$ |
| $\square$ | $>3.0 \%$ |

## Spatial propagation of final demand shocks (...)



## Spatial propagation of final demand shocks



## Many potential applications!

Input-output applications

- Mexican regions (how do they relate?), structural decomposition analysis (historical estimation, updating), main drivers of sectoral and regional growth, impact of interregional government transfers, impact analysis...

Interregional CGE applications

- Economic impacts of drought, regional impacts of climate change (agriculture), specific transportation projects (accessibility), simulate TFP-enhancing policies (sectors and regions), other usual CGE applications, ...


## Exercise - Interregional IO (decomposition of multiplier effects)

Data: Mexican 32-region interregional input-output table, 2013

## 1. Agriculture (S1) in Sinaloa (R25)

2. Mexican exports and KIBS

Excel files (...)

|  |
| :---: |
| Interstate input-output model for Mexico, 2013 ${ }^{1}$ |
| Modelo interestatal de insumo-producto para México, 2013 |
| (Received: 03/March/2020; accepted: 09/July/2020; published: 04/September/2020) |
|  |

## Interregional CGE Model for Mexico

## BM-MX Model

## Interregional Computable General Equilibrium Model for Mexico



The University of Sao Paulo Regional and Urban Economics Lab - NEREUS
May 2019

## BMMX ICGE, a bottom-up spatial CGE model of Mexico

A multi-sectoral, multi-regional bottom-up CGE model of Mexico's 32 regions

- each region is modeled as an economy in its own right
- region-specific prices
- region-specific industries
- region-specific consumers

Based on the comparative-static B-MARIA and MMRF models

Database makes allowance for interregional, intra-regional and international trade

- Potential for the representation of regional and Federal government financial accounts


## Stylized flows



## Core database

|  |  | ABSORPTION MATRIX |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 |
|  |  | Producers | Investors | Household | Export | Regional Govt. | Federal Govt. |
|  | Size | J x Q | J x Q | Q | 1 | Q | Q |
| Basic Flows | Ix S | BAS1 | BAS2 | BAS3 | BAS4 | BAS5 | BAS6 |
| Margins | Ix S x R | MAR1 | MAR2 | MAR3 | MAR4 | MAR5 | MAR6 |
| Taxes | Ix S | TAX1 | TAX2 | TAX3 | TAX4 | TAX5 | TAX6 |
| Labor | 1 | LABR | $I=$ number of commodities <br> $J=$ number of industries <br> $R=$ number of commodities used as margins <br> $Q=$ number of regions <br> $S=Q$ domestic regions +1 foreign import |  |  |  |  |
| Capital | 1 | CPTL |  |  |  |  |  |
| Other | 1 | OCTS |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

## Features of database

Commodity flows are valued at "basic prices" (BAS):

- do not include user-specific taxes or margins

For each user of each imported good and each domestic good, there are numbers showing:

- tax levied on that usage (TAX)
- usage of margins - transport (MAR)*

Single-production:

- each commodity may be produced by one industry
- each industry may produce one commodity

For each industry the total cost of production is equal to the total value of output

For each commodity the total value of sales is equal to the total value of output

## Features of database (cont.)

Domestic producers

- J industries in Q regions

Investors

- J industries in Q regions

Households
" one representative household for each of the Q regions

Each of the I commodity types can be obtained from the region, from other regions, or imported from overseas

## Features of database (cont.)

Aggregate foreign purchaser of exports

Other demand category corresponding to the Q regional governments

Other demand category corresponding to the central government in the Q regions

Commodity taxes and margins explicitly recognised

## Building blocks

$\checkmark$ Producer's demands for inputs
$\checkmark$ Investor demands
$\checkmark$ Household demands
$\checkmark$ Export demands
$\checkmark$ Government demands
$\checkmark$ Margins demands
$\checkmark$ Zero pure profits
$\checkmark$ Indirect tax equations
$\checkmark$ Market-clearing
$\checkmark$ Regional and national macroeconomic variables and price indexes
$\checkmark$ Capital accumulation and investment
$\checkmark$ Regional population and labor market

## Production nest

(1)


## Investment demand



## Household demand

(1)
(2)
(3)


## Foreign export demand



## Government demand

Recognise regional governments and Federal government demands for goods and services for current consumption (not properly calibrated yet!)

Neither modelled explicitly

Default:

- aggregate regional government demand in region q moves with regional government revenue, with structure of demand exogenous
- aggregate Federal government demand in region q moves with national government revenue, with structure of demand exogenous


## Demand for margin (transportation) services*

Margins commodities (identified in the set MARGCOM) provide freight services

- these commodities are consumed directly and used indirectly to facilitate the movements of products
- latter type of use is margins demand

Margins demand for margin commodity i is assumed to be proportional to the volume of the underlying flow

- e.g., margins use of transportation services in taking agricultural products to manufacturing is modelled as proportional to the volume of agricultural product used in manufacturing


## Zero pure profits

Critical assumptions

- no pure profits in the production or distribution of commodities
- price received by the producer is uniform across all customers

Zero pure profits in current production imposed by setting unit prices received by producers equal to unit costs

Zero pure profits in distribution imposed by setting the prices paid by users equal to producer price plus commodity tax plus margins

## Indirect taxes

Equations have been added to enable flexible handling of indirect taxes on all flows of goods and services

Equations allow for variations in tax rates across commodities, their sources and destinations

## Market-clearing

Equations that impose market clearing (demand equals supply) for:

- domestically produced margin and non-margin commodities
- imported commodities


## Macro aggregates

Wide range of national and regional macro variables defined...

Two concepts of the real wage rate

- consumer real wage rate (PLAB/CPI)
- producer real wage rate (PLAB/PGDP)


## Investment "dynamics"

Capital, investment and expected rates of return

$$
K_{j, q}(t+1)=\left(1-D E P_{j, q}\right) \times K_{j, q}(t)+Y_{j, q}(t)
$$

Given starting point for capital ( $\mathrm{t}=0$ ) and an explanation of investment, we can trace out time path for capital

## Investment "dynamics"

Investment explained by assuming that:
$\frac{K_{j, q}(t+1)}{K_{j, q}(t)}-1=F_{j, q}^{t}\left[E R O R_{j, q}(t)\right]$

Growth in capital related to expected rate of return

- In BMMX ICGE only assume static expectations, though rational is possible


## Rates of return and investment

For static expectations case, the actual rate of return is:

$$
\begin{aligned}
& R O_{t}(j, q)=\frac{P_{t}(j, q)}{\Pi_{t}(j, q)}-D(j, q) \\
& \operatorname{ro}(j, q)=p_{t}(j, q)-\pi_{t}(j, q) \\
& \operatorname{ro}(j, q)=Q \operatorname{COEF}(j, q)\left[p_{t}(j, q)-\pi_{t}(j, q)\right]
\end{aligned}
$$

QCOEF: relação entre taxa bruta e taxa líquida de retorno (> 1 )

## Rates of return and investment

In long-run comparative-static simulations:

- aggregate capital adjusts to maintain $\mathrm{R}_{\mathrm{INT}}$ (natr_tot)
- capital allocated in line with equation E_f_rate_xx
- industries with relatively large increases in capital require relatively high rates of return
- industries with relatively small increases in capital require relatively low rates of return
- industry investment determined by fixed ratios of investment to capital (equation E_y)


## Rates of return and investment

Equalization in the rates of return

$$
\begin{aligned}
& \left(\frac{K(j, q)}{K(q)}\right)^{-\beta(j, q)} R O(j, q)=R_{\mathrm{int}} \\
& r o(j, q)-r_{\mathrm{int}}=\beta_{t}(j, q)[k(j, q)-k(q)]+f_{-} r a t e(j, q)
\end{aligned}
$$

beta: risk/return ratio

Short-run: f_rate endogenous, $k$ exogenous
Long-run: f_rate exogenous, $k$ endogenous

## Investment "dynamics"

Growth rate of capital stocks and investment in the short-run:

$$
\begin{array}{ll}
k_{t+1}(j, q)-k_{t}(j, q)=0 & \text { \% change in capital stocks } \\
y_{t}(j, q)=0 & \text { \% change in investment }
\end{array}
$$

## Investment "dynamics"

Growth rate of capital stocks and investment in the long-run:

$$
\begin{aligned}
& \frac{K_{j, q}(t+1)}{K_{j, q}(t)}=\left(\frac{K_{j, q}(t)}{K_{j, q}(0)}\right)^{1 / T} \\
& k_{t+1}(j, q)=\left(1+\frac{1}{T}\right) k_{t}(j, q)
\end{aligned}
$$

## Investment in the short run

Fixed capital stocks in the base year values:

- curcap $(j, q)$ exogenous (=0)
- relationship between sectoral rates of return, $r O(j, q)$, and reference interest rate, natr_tot, is endogenous (f_rate_xx(j,q) endogenous)

Percentage change in sectoral investment, $y(j, q)$ is zero; this can be guaranteed by setting the shift term, delf_rate $(j, q)$, exogenous and zero

By hypothesis, not only the capital stocks are fixed but also firms' investment plans

## Investment in the long run

Capital stocks endogenously determined:

- curcap( $j, q$ ) endogenous
- relationship between sectoral rates of return,, rO(j,q), and reference interest rate, natr_tot, is given (f_rate_xx(j,q) exogenous)

Percentage change in sectoral investment, $y(j, q)$ is endogenous

Firms' investment plans are carried out, reestablishing returns differentials in the base year

Rate of capital accumulation, but not the level of capital stock, remains constant

## Regional population and labor market

Critical variables:

- regional population
- regional migration
- regional unemployment
- regional participation rates
- regional wage relativities

Various closures

## Regional population and labor market

(1) Fixed
" wage relativities (determining employment by region), participation and unemployment rates (determining population by region)
(1) Endogenous

- regional migration
(2) Fixed
- regional migration, participation rates, wage relativities
(2) Endogenous
" unemployment rates
(3) Fixed
" regional migration, participation and unemployment rates
(3) Endogenous
- wage relativities


## Closures

Each equation explains a variable
More variables than equations
Endogenous variables: explained by model
Exogenous variables: set by user
Closure: choice of exogenous variables
Many possible closures
Number of endogenous variables $=$ Number of equations

## Exercise - CGE model (simulating the short-run effects of a drought in Northern Mexico)

1. Most years, one or more regions of Mexico suffer from low rainfall. As a continuing result of climate change, during 2020 an unusually severe drought in Sinaloa is expected to affect farmers all over the region.
Agricultural outputs will fall sharply, with an expected loss of $10 \%$ (what if climate variability would also affect cattle raising output?). We will simulate the effects of such a drought using the BMMX ICGE model.

Guide for the BMMX Interregional CGE Model for Mexico Using Customized RunGEM

GEMPACK...

## Production nest

(1)


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