Chapter 1

COMPUTABLE GENERAL EQUILIBRIUM MODELLING FOR POLICY ANALYSIS AND FORECASTING

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Contents

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1. Introduction

Over the last 35 years, computable general equilibrium (CGE) models have been used in the analysis of an enormous variety of questions. These have included

the effects on
- macro variables, including measures of nation-wide or even global economic welfare;
- industry variables;
- regional (both sub-national and super-national) variables;
- labour market variables;
- distributional variables; and
- environmental variables

of changes in
- taxes, public consumption and social security payments;
- tariffs and other interferences in international trade;
- environmental policies;
- technology;
- international commodity prices and interest rates;
- wage setting arrangements and union behavior; and
- known levels and exploitability of mineral deposits (the Dutch disease).

While most of these questions have been analyzed in single-country, single-period models, there are now numerous CGE models which are either multi-regional or multi-period (dynamic) or both. By going multi-regional, CGE modelling has thrown light on both intra-country and inter-country regional questions. In the first category are issues (important in federations) concerning the effects of tax and expenditure activities of provincial governments. In the second category are issues such as the effects of the formation of trading blocks and the effects of different approaches to reducing world output of greenhouse gases. By going dynamic, CGE modelling has the potential to broaden and deepen its answers to all questions with which it has been confronted. It has also entered the forecasting arena. CGE models are now used to generate forecasts of the prospects of different industries, labour force groups and regions These forecasts feed into investment decisions by private and public sector organizations affecting stocks of physical and human capital.

The main objective of this chapter is to show how CGE models can be constructed and applied. We try to achieve this objective by describing the construction and application of an illustrative model. Although the model is small, it illustrates the key aspects of CGE modelling, including

- input–output data,
- elasticity parameters,
- theoretical specification,

- solution algorithm and
- result interpretation.

The illustrative model can be used in two ways: as a single-period model suitable for comparative-static analyses; and as a model for multi-period forecasting.

The chapter is organized as follows. In the remainder of this section (Subsections 1.1 and 1.2) we define CGE modelling and provide a brief history of its development. Then in Section 2 we discuss the computation of solutions for CGE models. The illustrative model is in Section 3. Section 4 is an overview of what we see as the field's achievements, failures and potential.

Readers can choose their own path through the chapter. After completing this section, some readers may like to skip the mathematics in Section 2 and move straight to the illustrative model in Section 3. Others might like to start with Section 4 and then work back through the more technical material in Sections 2 and 3.

1.1. Definition

The distinguishing characteristics of computable general equilibrium (CGE) models are as follows.

(i) They include explicit specifications of the behavior of several economic actors (i.e. they are *general*). Typically they represent households as utility maximizers and firms as profit maximizers or cost minimizers. Through the use of such optimizing assumptions they emphasize the role of commodity and factor prices in influencing consumption and production decisions by households and firms. They may also include optimizing specifications to describe the behavior of governments, trade unions, capital creators, importers and exporters.

(ii) They describe how demand and supply decisions made by different economic actors determine the prices of at least some commodities and factors. For each commodity and factor they include equations ensuring that prices adjust so that demands added across all actors do not exceed total supplies. That is, they employ market *equilibrium* assumptions.

(iii) They produce numerical results (i.e. they are *computable*). The coefficients and parameters in their equations are evaluated by reference to a numerical database. The central core of the database of a CGE model is usually a set of input–output accounts showing for a given year the flows of commodities and factors between industries, households, governments, importers and exporters. The input–output data are normally supplemented by numerical estimates of various elasticity parameters. These may include substitution elasticities between different inputs to production processes, estimates of price and income elasticities of demand by households for different commodities, and foreign elasticities of demand for exported products.

An alternative name for CGE models is applied general equilibrium (AGE) models. This name emphasizes the idea that in CGE modelling the database and numerical results are intended to be more than merely illustrative. CGE models use data for actual countries or regions and produce numerical results relating to specific real-world situations.

1.2. *Brief history*

On our definition, the first CGE model was that of Johansen (1960).[1] His model was *general* in that it contained 20 cost-minimizing industries and a utility-maximizing household sector. For these optimizing actors, prices played an important role in determining their consumption and production decisions. His model employed market *equilibrium* assumptions in the determination of prices. Finally, it was *computable* (and applied). It produced a numerical, multi-sectoral description of growth in Norway using Norwegian input–output data and estimates of household price and income elasticities derived using Frisch's (1959) additive utility method.

Following Johansen's contribution, there was a surprisingly long pause in the development of CGE modelling with no further significant progress until the 1970s. The 1960s were a period in which leading general-equilibrium economists developed and refined theoretical propositions on the existence, uniqueness, optimality and stability of solutions to general equilibrium models.[2] Rather than being computable (numerical), their models were expressed in general, algebraic terms.

The most direct link between this theoretical work and CGE modelling was made by Scarf (1967a, 1967b and 1973). Drawing on the mathematics of the theoretical existence theorems, Scarf designed an algorithm for computing solutions to numerically specified general equilibrium models. This algorithm had finite convergence properties, i.e. for a wide class of general equilibrium models, the algorithm was certain to produce a solution in a finite number of steps.

Scarf was of great importance in stimulating interest in CGE modelling in North America. In the early 1970s, his students John Shoven and John Whalley became leading contributors to the field [see, for example, Shoven and Whalley (1972, 1973, 1974)]. In 1991, when Scarf was awarded a distinguished fellowship of the American Economic Association, the citation, read in part:

> Scarf's path-breaking technique for the computation of equilibrium prices has resulted in a new subdiscipline of economics: the study of applied general equilibrium models... Scarf was the catalyst behind the creation of this subfield of the profession

[1]On a broader definition, CGE modelling starts with Leontief's (1936, 1941) input–output models of the 1930s and includes the economy-wide mathematical programming models of Sandee (1960), Manne (1963) and others developed in the 1950s and 60s. We regard these contributions as vital forerunners of CGE models. On our definition, input–output and programming models are excluded from the CGE class because they have insufficient specification of the behavior of individual actors and the role of prices. Scarf (1994) takes a similar view of the origins of the field.

[2]See Arrow and Hahn (1971).

and in the transformation of the general equilibrium model from a purely theoretical construct to a useful tool for policy analysis. (American Economic Review, 82(4), September 1992.)

In our view, this misrepresents Scarf's contribution. Johansen had already solved a relatively large CGE model by a simple, computationally efficient method[3] well before the Scarf algorithm was invented. Scarf's technique was never the most effective method for doing CGE computations. Even those CGE modellers who embraced the Scarf technique in the 1970s had by the 1980s largely abandoned it. When dealing with models capable of giving practical answers to policy and forecasting questions, they switched to older methods such as Newton–Raphson and Euler algorithms. For this reason, and also because it has been reviewed extensively elsewhere including in other volumes of the Handbooks in Economics,[4] Scarf's approach receives only passing mentions in the remainder of this chapter.

While the 1960s were not an active period in CGE modelling, they were a key decade in the development of large-scale, economy-wide econometric models (e.g. the Wharton, DRI, MPS, St Louis, Michigan and Brookings models).[5] Relative to CGE models, the economy-wide econometric models paid less attention to economic theory and more attention to time-series data. In CGE models, the specifications of demand and supply functions are completely consistent with underlying theories of optimizing behavior by economic actors. In economy-wide econometric models, the role of optimizing theories of the behavior of individual actors is usually restricted to that of suggesting variables to be tried in regression equations.

In the 1960s, the underlying philosophy of the econometric approach of "letting the data speak" seemed attractive to applied economists. This may be part of the explanation of the pause in the development of the CGE approach. In the 1970s there were two factors, apart from Scarf's bridge with the theoretical literature, which stimulated interest in the CGE approach.

First, there were major shocks to the world economy including a sudden escalation in energy prices, a sharp change in the international monetary system and rapid growth in real wage rates. Without tight theoretical specifications, the econometric models could not provide useful simulations of the effects of shocks such as these which carried economies away from established trends.

CGE models are often vulnerable to the criticism that their behavioral specifications (e.g. utility maximization and cost minimization) are imposed without empirical validation.[6] However, with these specifications in place, CGE models can offer in-

[3]Section 2 contains a description of Johansen's method.

[4]See, for example, Scarf (1982) and Kehoe (1991).

[5]For an historical perspective on these models, see the papers in Kmenta and Ramsey (1981).

[6]Among CGE modellers, Dale Jorgenson and his colleagues are the least vulnerable to this criticism. Starting in 1974, Jorgenson has emphasized the need for econometric estimation of all parameters [see, for example, Hudson and Jorgenson (1974), Jorgenson (1984) and Jorgenson and Wilcoxen (1994)]. To support his many CGE applications to energy and environmental issues in the U.S., he has made econometric estimates for cost functions, indirect utility functions and trade parameters at a detailed level.

sights into the likely effects of shocks for which there is no historical experience. For example, up to 1973, there was no modern experience of a sharp change in oil prices. Consequently, in regression equations based on pre-1973 time-series data, the price of oil has an insignificant or zero coefficient. This meant that models relying heavily on time-series analysis implied that movements in oil prices would not be an important determinant of economic activity. In detailed CGE models, inputs of oil appear as variables in production functions. Then through cost-minimizing calculations, increases in the price of oil act on economic activity in CGE simulations in the same way as increases in the prices of other inputs. In the 1970s, interest in CGE modelling increased as applied economists recognized the power of optimizing assumptions in translating broad experience (e.g. experience of cost increases) into plausible predictions of the effects of particular shocks for which we may have no experience (e.g. the effects of an increase in oil prices).

The second factor driving the growth of CGE modelling over the last 20 years has been its increasing ability to handle detail. The key ingredients have been improved data bases (e.g. the availability of unit records from Censuses) and improved computer programs (e.g. the availability of programs such as GEMPACK, GAMS, HERCULES and CASGEN).[7] In our consulting work in Australia, we can now use CGE models to satisfy demands for analyses disaggregated into effects on 120 industries, 56 regions, 280 occupations, and several hundred family types. At this level of detail, no other technique has as much to offer as CGE modelling.[8] As CGE modellers have learnt to handle more detail, CGE results have become of interest to public and private sector organizations concerned with, among other things: industries; regions; employment; education and training; income distribution; social welfare and the environment.

CGE modelling is now an established field of applied economics. Several detailed surveys of CGE modelling have appeared in leading journals and in books from prominent publishers [e.g. Shoven and Whalley (1984), Pereira and Shoven (1988), Robinson (1989, 1991), Bandara (1991) and Bergman (1990)]. There are regular international meetings of CGE modellers, often followed by the production of a conference volumes [e.g. Kelley, Sanderson and Williamson (1983), Scarf and Shoven (1984), Piggott and Whalley (1985 and 1991), Srinivasan and Whalley (1986), Bergman, Jorgenson and Zalai (1990), Bergman and Jorgenson (1990), Don, van de Klundert and van Sinderen (1991), Devarajan and Robinson (1993) and Mercenier and Srinivasan (1994)]. Numerous monographs have been published giving detailed descriptions of the construction and application of CGE models [e.g. Johansen (1960), Adelman and Robinson (1978), Keller (1980), Dixon, Parmenter, Sutton and Vincent (1982), Harris

[7]Descriptions of general-purpose software for solving CGE models include Pearson (1988), Codsi and Pearson (1988), Bisschop and Meeraus (1982), Brooke, Kendrick and Meeraus (1988), Drud, Kendrick and Meeraus (1986), Meeraus (1983), and Rutherford (1985a and b). The existence of this software means that economists interested in building and applying CGE models no longer need either a high level of skill in programming or a sophisticated understanding of algorithms for solving systems of equations.

[8]The main alternative is input–output analysis and its extensions. We return to this in Section 4.

with Cox (1983), Ballard, Fullerton, Shoven and Whalley (1985), Whalley (1985), McKibbin and Sachs (1991) and Horridge, Parmenter and Pearson (1993)]. At least three CGE textbooks are now available for graduate students and advanced undergraduates [Dervis, de Melo and Robinson (1982), Shoven and Whalley (1992) and Dixon, Parmenter, Powell and Wilcoxen (1992)[9]] and graduate students all over the world are engaged in writing CGE theses.

Is the field past its peak? Is it in danger of going stale? We don't think so. We think that CGE modelling will generate high-profile academic careers for many years to come. More importantly, it is likely to be increasingly influential in policy making and in business.

For applied economists with a strong interest in theory, the CGE field offers the challenge of incorporating into the models ideas from modern macro- and microeconomics. Of the ideas emerging from macroeconomics, rational expectations and the differences between the effects of anticipated and unanticipated shocks have received considerable attention in CGE modelling [e.g. Ballard and Goulder (1985), Bovenberg and Goulder (1991), Mercenier and Sampaio de Souza (1994), Jorgenson and Wilcoxen (1994) and Dixon, Parmenter, Powell and Wilcoxen (1992, Chapter 5)]. We expect that CGE models will soon appear incorporating other ideas from modern macro such as technical change related to accumulation of human capital, and hysteresis in labour markets and international trade. Drawing from ideas in modern micro theory, CGE models are being constructed which include product differentiation at the firm level, economies of scale, free and costly entry and exit, price discrimination and game-theoretic behavior [see, for example, Harris with Cox (1983), Harris (1984), Cox and Harris (1985, 1986), Norman (1990) and Mercenier (1994a and b)].

For applied economists with a strong empirical/statistical interest, the challenges offered by CGE modelling are unbounded. They include: compilation of timely input–output and other data with economically meaningful industrial, regional, occupational, environmental and social classifications; the measurement of outputs for difficult industries such as banking; the measurement of capital inputs; the estimation of elasticity parameters; the estimation of trends in tastes and technology; the selection of important issues for analysis; and the representation of results in a clear and persuasive manner.

2. Solving a CGE model

There are two main approaches to solving CGE models: non-linear programming and derivative methods. In Subsection 2.1 we describe briefly the programming approach. Then in Subsection 2.2 we give a fuller account of a particular derivative method (the Johansen/Euler method). This latter method is the one that we use to

[9]This last textbook is accompanied by a set of teaching diskettes prepared by Pearson (1992).

solve the illustrative model in Section 3. Subsection 2.3 is a discussion of how to solve multi-period or intertemporal models. A special problem with the use of derivative techniques in solving these models is the need to construct an initial solution.

2.1. The programming approach[10]

The programming approach relies on the idea that the solution to a CGE model can often be deduced from the solution to an optimization problem. Consider, for example a two consumer, pure-exchange (no production) model. A solution for this model is a list of non-negative vectors,

$$\Xi = \{P, C(1), C(2)\}$$

satisfying the following conditions:

$$C(i) \text{ maximizes } U_i(C(i)) \quad \text{subject to } P'(C(i) - Z(i)) = 0, \quad i = 1, 2, \tag{2.1}$$

$$\sum_i C(i) = \sum_i Z(i) \tag{2.2}$$

and

$$P'\underline{1} = 1, \tag{2.3}$$

where

$C(i)$ is the consumption vector for consumer i,
P is the vector of commodity prices,
$Z(i)$ is the exogenously given endowment vector of consumer i;

and

U_i is consumer i's utility function which we assume is strictly concave.

Condition (2.1) means that consumers maximize their utility functions, U_i, subject to their budget constraints. Condition (2.2) ensures that demand equals supply for each good. (For convenience, we assume that there are no goods in excess supply at zero price.) Condition (2.3) sets the overall level of prices.

[10]This approach to CGE computation was developed by several authors including Dixon (1975, 1978a) and Ginsburgh and Waelbroeck (1981). For a recent contribution, see Rutherford (1992). The theoretical underpinnings were given by Negishi (1960).

One approach to finding Ξ is to solve a sequence of non-linear programming problems of the form:

choose $C(1), C(2)$ to maximize

$$W_1 U_1(C(1)) + W_2 U_2(C(2)) \tag{2.4}$$

subject to

$$\sum_i C(i) = \sum_i Z(i), \tag{2.5}$$

where W_1 and W_2 are a pair of non-negative numbers adding to one.

At a solution $(\overline{C}(1),\ \overline{C}(2))$ to problem (2.4)–(2.5), there will exist a vector of Lagrangian multipliers, Π, such that

$$W_i \nabla U_i\big(\overline{C}(i)\big) = \Pi, \quad i = 1, 2. \tag{2.6}$$

Hence, for both $i = 1$ and 2,

$$\overline{C}(i) \text{ maximizes } U_i(C(i)) \quad \text{subject to } \Pi'\big(C(i) - \overline{C}(i)\big) = 0. \tag{2.7}$$

It is also true that

$$\sum_i \overline{C}(i) = \sum_i Z(i). \tag{2.8}$$

If it happens that

$$\Pi'\big(\overline{C}(i)\big) = \Pi'(Z(i)), \tag{2.9}$$

then we have found a solution, Ξ, to our CGE model as follows:

$$P = \big(1/(\Pi'\underline{1})\big)\Pi,$$

$$C(i) = \overline{C}(i), \quad i = 1, 2.$$

If (2.9) is not satisfied, then we vary the weights, W_i, resolving problem (2.4)–(2.5) until it is satisfied.

Non-linear programming methods can be extended well beyond the pure-exchange case. They have been applied successfully in solving CGE models which include production, investment, capital accumulation, taxes and trade.[11] Nevertheless, we have found derivative methods to be more convenient and flexible. This is also the current experience of most other people in the CGE field.

2.2. *The derivative approach: The Johansen/Euler method*

We consider a model for which a vector V of length n is a solution (or equilibrium) if it satisfies a system of equations

$$F(V) = 0, \tag{2.10}$$

where F is a vector function of length m. The components of the vector V represent: demands for and supplies of commodities and factors; prices; taxes and subsidies; surpluses and deficits; technological coefficients and other economic variables. The equation system (2.10) imposes conditions such as: demands equal supplies; prices equal costs; and demands depend on relative prices and expenditure levels.

We assume that F is differentiable and that the number of variables, n, exceeds the number of equations, m. Exogenously given values are assigned to $n - m$ variables.

Finally, we assume that an initial solution, V^{I}, is known. That is, we have a vector V^{I} such that

$$F(V^{\mathrm{I}}) = 0. \tag{2.11}$$

For a one-period model, finding an initial solution is usually trivial. As we will see in Section 3, V^{I} can normally be read from the model's input–output database. The problem is a little harder for some multi-period models where the initial solution, V^{I}, has to be constructed. As explained in Subsection 2.3, in most cases, this can be done quite easily.

In computing solutions to a CGE model, we should take advantage of our knowledge of V^{I}. By not using an initial solution, non-linear programming methods[12] and combinatorial approaches such as the Scarf algorithm neglect valuable information. Given an initial solution, we can generate new solutions for our model by elementary derivative techniques, e.g. variants of Newton's and Euler's methods. We have found variants of Euler's method to be particularly easy to use and effective in large-scale CGE computations.[13]

[11]For a recent application of the non-linear programming method in a model including these features, see Dixon (1991).

[12]Non-linear programming packages normally store the latest solution to a problem and then proceed from there when a new solution is required after a change in parameter values. However, with any alteration in the structure of the problem to be solved, all information on old solutions is usually lost.

[13]This is the approach underlying the GEMPACK programs for solving CGE models, see Pearson (1988) and Codsi and Pearson (1988).

To describe the Euler method in the context of CGE modelling we start by rewriting (2.10) as

$$F(V_1, V_2) = 0, \tag{2.12}$$

where V_1 is the vector of length m of endogenous variables and V_2 is the vector length $n - m$ of exogenous variables. Then we totally differentiate (2.12), recognizing that if we are to continue to have a solution to our model, then deviations, $\mathrm{d}V_1$ and $\mathrm{d}V_2$, from V^{I} must satisfy, to a linear approximation,

$$F_1(V^{\mathrm{I}})\,\mathrm{d}V_1 + F_2(V^{\mathrm{I}})\,\mathrm{d}V_2 = 0, \tag{2.13}$$

where F_1 and F_2 are matrices of partial derivatives of F evaluated at V^{I}.

To make a one-step Euler or Johansen[14] approximation, we compute

$$\mathrm{d}V_1 = B(V^{\mathrm{I}})\,\mathrm{d}V_2 \tag{2.14}$$

where

$$B(V^{\mathrm{I}}) = -F_1^{-1}(V^{\mathrm{I}})F_2(V^{\mathrm{I}}). \tag{2.15}$$

Provided we can evaluate $B(V^{\mathrm{I}})$, then Eq. (2.14) can tell us how, in the region of V^{I}, the endogenous variables (V_1) are affected by movements in the exogenous variables (V_2). For example, using (2.14), we might compute the effects on output and employment in the footwear industry (components of V_1) of changes in various taxes and tariffs (components of V_2).

Four issues need to be discussed concerning the Johansen/Euler computation, (2.14)–(2.15).

Is $F_1(V^{\mathrm{I}})$ invertible? First, is it legitimate to assume that $F_1(V^{\mathrm{I}})$ is invertible? From the implicit functions theorem,[15] we know that the existence of $F_1^{-1}(V^{\mathrm{I}})$ is a necessary and sufficient condition for the existence of a unique m-vector function G satisfying

$$F(G(V_2), V_2) = 0 \tag{2.16}$$

for all V_2 in a neighbourhood of V_2^{I}. Consequently, if $F_1(V^{\mathrm{I}})$ is singular, then our model either contains no answer to the question of how V_1 is affected by variations in V_2 in the region of V_2^{I}, or it contains multiple answers. In either case, failure to be able to apply (2.14)–(2.15) would not be the fundamental problem. Rather, our problem

[14]Equations (2.14)–(2.15) describe the computations made by Johansen (1960).

[15]See, for example, Apostol (1957, pp. 146–148).

would be either that there were contradictions within our model or that the model was under-specified to answer the question at hand. We can conclude that the assumption that $F_1^{-1}(V^{\mathrm{I}})$ exists does not limit the applicability of our computing method: the assumption will be valid if we are attempting to compute the answer to a question to which our model has a unique solution.

Approximation errors. The second issue concerns approximation errors. The matrix $B(V^{\mathrm{I}})$ shows the partial derivatives, evaluated at V^{I}, of the endogenous variables (V_1) with respect to the exogenous variables (V_2). That is, $B(V^{\mathrm{I}})$ is the Jacobian matrix of the solution function, G. Thus, for a given movement in the exogenous variables, the valuation of $\mathrm{d}V_1$ via (2.14) provides only a first order approximation to the effects on the endogenous variables implied by the model (2.12). Where $(\mathrm{d}V_1)_{\text{true}}$ is the exact vector of effects implied by the model and $(\mathrm{d}V_1)_{(\cdot 1)}$ is the set of effects computed via (2.14), we have

$$\begin{aligned}(\mathrm{d}V_1)_{\text{true}} &= G\big(V_2^{\mathrm{I}} + \mathrm{d}V_2\big) - G\big(V_2^{\mathrm{I}}\big)\\ &= B(V^{\mathrm{I}})\,\mathrm{d}V_2 + \mathrm{HOT}\\ &= (\mathrm{d}V_1)_{(\cdot 1)} + \mathrm{HOT}.\end{aligned} \tag{2.17}$$

HOT is the vector of values of higher order terms in a Taylor's series.

If $\mathrm{d}V_2$ is small, then $(\mathrm{d}V_1)_{(\cdot 1)}$ will be a good approximation to $(\mathrm{d}V_1)_{\text{true}}$, i.e., the components of HOT will be small. But what do we do if $\mathrm{d}V_2$ is not small? In any case, how can we tell whether $\mathrm{d}V_2$ is small enough for $(\mathrm{d}V_1)_{(\cdot 1)}$ to be a satisfactory approximation to $(\mathrm{d}V_1)_{\text{true}}$?

One way to answer these questions is to make a multi-step Johansen/Euler computation. For example, we can compute the effects of the change, $\mathrm{d}V_2$, in the exogenous variables in two steps rather than one. In the first step, we compute

$$(\mathrm{d}V_1)_{(1,2)} = B_{(1,2)}\big(\tfrac{1}{2}\,\mathrm{d}V_2\big). \tag{2.18}$$

The subscripts (1,2) refer to the first step of a two-step computation. $(\mathrm{d}V_1)_{(1,2)}$ is our approximation to the effect on the endogenous variables of a vector of shocks of half the size of those in which we are ultimately interested. $B_{(1,2)}$ is the B matrix used in the first step of the two-step computation. It is the same as $B(V^{\mathrm{I}})$ defined in (2.15).

Having computed $(\mathrm{d}V_1)_{(1,2)}$, we re-evaluate the B matrix as

$$B_{(2,2)} = -F_1^{-1}(V_{(1,2)})F_2(V_{(1,2)}) \tag{2.19}$$

where $V_{(1,2)}$ is the vector of values of the variables at the end of the first step of the two-step computation. That is

$$V_{(1,2)} = \big[V_1^{\mathrm{I}} + (\mathrm{d}V_1)_{(1,2)},\ V_2^{\mathrm{I}} + \tfrac{1}{2}(\mathrm{d}V_2)\big]. \tag{2.20}$$

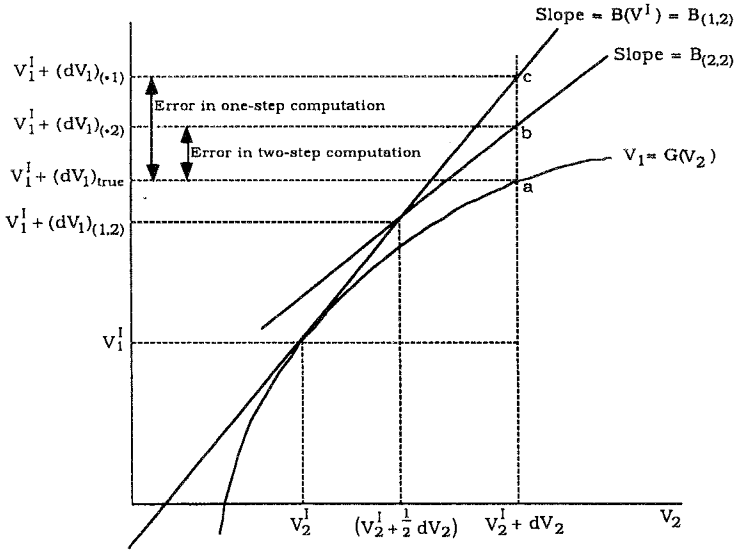

Figure 1.1. Johansen/Euler solutions.

Now we can compute

$$(\mathrm{d}V_1)_{(2,2)} = B_{(2,2)}(\tfrac{1}{2}\,\mathrm{d}V_2), \tag{2.21}$$

i.e., we compute the effect on the endogenous variables of the remaining half of the shocks to the exogenous variables.

Finally, we compute our two-step approximation to $(\mathrm{d}V_1)_{\text{true}}$ as

$$(\mathrm{d}V_1)_{(\cdot 2)} = (\mathrm{d}V_1)_{(1,2)} + (\mathrm{d}V_1)_{(2,2)}. \tag{2.22}$$

In many (perhaps most) general equilibrium models the solution functions, G, are well approximated by quadratic functions over variable ranges relevant to simulations. In these cases, we find that the two-step computation of $\mathrm{d}V_1$ involves about half the error of the one-step computation. That is we find that

$$(\mathrm{d}V_1)_{(\cdot 2)} - (\mathrm{d}V_1)_{\text{true}} \simeq \tfrac{1}{2}\left[(\mathrm{d}V_1)_{(\cdot 1)} - (\mathrm{d}V_1)_{\text{true}}\right]. \tag{2.23}$$

This is not an appropriate place to offer a rigorous justification of (2.23).[16] Instead we offer a diagram and some suggestive algebra.

The diagram (Fig. 1.1) illustrates a 2 variable case in which we are concerned with the effects on the endogenous variable (V_1) of moving the exogenous variable (V_2)

[16]For a complete discussion of results such as (2.23), see Dahlquist, Bjorck and Anderson (1974, pp. 269–273).

from V_2^{I} to $V_2^{\mathrm{I}} + \mathrm{d}V_2$. We assume that the form of G is unknown but that we do know the initial solution $(V_1^{\mathrm{I}}, V_2^{\mathrm{I}})$ and also how to evaluate derivatives of G, e.g. via (2.14)–(2.15). When we use a one-step Johansen/Euler calculation to compute the effect on V_1 of moving V_2 from V_2^{I} to $V_2^{\mathrm{I}} + \mathrm{d}V_2$, we obtain the answer $(\mathrm{d}V_1)_{(\cdot 1)}$, having an error of ac. When we use a two-step computation, the error is reduced to ab. Notice that with the G function drawn approximately quadratic as in our diagram, the two-step error, ab, is approximately half the one-step error, ac.

Now we turn to the suggestive algebra. We return to the m-equation-n-variable case and we assume that each of the m endogenous variables is a quadratic function of the $n - m$ exogenous variables. For the jth endogenous variable, we have

$$V_1(j) = G_j(V_2) = a_j + b_j' V_2 + \tfrac{1}{2} V_2' Q_j V_2, \tag{2.24}$$

where a_j, b_j and Q_j are parameters with a_j being a scalar, b_j being a vector of length $n - m$ and Q_j being a symmetric matrix of size $(n - m) \times (n - m)$.

The vector of first-order partial derivatives of G_j, i.e. the jth row of the Jacobian matrix of G, is given by

$$B_j(V_2) = b_j' + V_2' Q_j. \tag{2.25}$$

We assume that we can evaluate B_j correctly at each step of a Johansen/Euler procedure.[17] Then we find that the one- and two-step errors in the evaluation of the effect on the jth endogenous variable of a movement in the exogenous variables from V_2^{I} to $V_2^{\mathrm{I}} + \mathrm{d}V_2$ are as follows:

$$\begin{aligned} \text{Error (one-step)} &= (\mathrm{d}V_1(j))_{(\cdot 1)} - (\mathrm{d}V_1(j))_{\mathrm{true}}, \\ &= B_j(V_2^{\mathrm{I}})\, \mathrm{d}V_2 - \left[G_j(V_2^{\mathrm{I}} + \mathrm{d}V_2) - G_j(V_2^{\mathrm{I}})\right], \\ &= -\tfrac{1}{2}(\mathrm{d}V_2)' Q_j (\mathrm{d}V_2); \end{aligned} \tag{2.26}$$

and

$$\begin{aligned} \text{Error (two-step)} &= (\mathrm{d}V_1(j))_{(\cdot 2)} - (\mathrm{d}V_1(j))_{\mathrm{true}} \\ &= B_j(V_2^{\mathrm{I}})(\tfrac{1}{2}\,\mathrm{d}V_2) + B_j\left(V_2^{\mathrm{I}} + \tfrac{1}{2}\,\mathrm{d}V_2\right)\left(\tfrac{1}{2}\,\mathrm{d}V_2\right) \\ &\quad - \left[G_j(V_2^{\mathrm{I}} + \mathrm{d}V_2) - G_j(V_2^{\mathrm{I}})\right] \\ &= -\tfrac{1}{4}(\mathrm{d}V_2)' Q_j (\mathrm{d}V_2). \end{aligned} \tag{2.27}$$

[17] With Euler's method, we introduce small errors into the evaluation of the B matrix through errors in the evaluation of the endogenous variables. Nevertheless, this does not normally invalidate approximations such as (2.23). For a detailed mathematical discussion of Euler's method in the context of CGE modelling, see Dixon, Parmenter, Sutton and Vincent (1982, pp. 235–244).

Hence

$$\text{Error (two-step)} = \tfrac{1}{2}\,\text{Error (one-step)}.$$

More generally, we can show that if G_j is quadratic, then when we double the number of steps, we halve the error, i.e.

$$\text{Error } (2r\text{-step}) = \tfrac{1}{2}\,\text{Error } (r\text{-step}). \tag{2.28}$$

Findings such as these suggest that simple extrapolation procedures may produce accurate evaluations of $(\mathrm{d}V_1)_{\text{true}}$ based on just one- and two-step Johansen/Euler computations. For example, using (2.23) we find that

$$(\mathrm{d}V_1)_{\text{true}} \simeq 2(\mathrm{d}V_1)_{(\cdot 2)} - (\mathrm{d}V_1)_{(\cdot 1)}. \tag{2.29}$$

The use of the right-hand side of (2.29) to evaluate $\mathrm{d}V_1$ is an example of the application of Richardson's extrapolation.[18] Our experience has been that such extrapolations are highly effective in producing accurate simulation results in large models, using only a small number of Johansen/Euler steps.

What if the solution functions, G, are not well approximated by quadratic functions? Then we may require four or even eight-step Johansen/Euler computations. However, over a long period, working with many different models, we have found very few occasions in which it has been necessary to go beyond a two-step computation supplemented by an extrapolation.

Convenience: deriving the differential form. The third issue which we will consider in relation to the Johansen/Euler method is convenience. Is it difficult and time-consuming to do the total differentiation involved in taking a model from its initial form (2.12) into a differential form such as (2.13)?

In implementing the Johansen/Euler method, we have found it convenient to deal mainly with percentage changes in variables rather than changes.[19] That is, instead of solving the system (2.13) for $(\mathrm{d}V_1)$, we solve the system

$$F_1^*(V^{\mathrm{I}})v_1 + F_2^*(V^{\mathrm{I}})v_2 = 0, \tag{2.30}$$

[18] See Dalquist, Bjorck and Anderson (1974, pp. 269–273).

[19] For variables which pass through zero (e.g. the balance of trade), the percentage change form is not appropriate. For such variables, we continue, in the differential versions of our models, to use changes. Thus, in reality, the variables in systems such as (2.30) are usually a mixture of percentage changes and changes. There may even be a few levels variables. Nevertheless, we will, for ease of exposition, refer to all the variables in (2.30) as though they are percentage changes.

obtaining

$$v_1 = B_1^*(V^{\mathrm{I}})v_2 \tag{2.31}$$

where v_1 and v_2 are vectors of percentage changes in the variables in V_1 and V_2, and

$$F_1^*(V^{\mathrm{I}}) = F_1(V^{\mathrm{I}})\widehat{V}_1^{\mathrm{I}}, \tag{2.32}$$

$$F_2^*(V^{\mathrm{I}}) = F_2(V^{\mathrm{I}})\widehat{V}_2^{\mathrm{I}}, \tag{2.33}$$

and

$$B^*(V^{\mathrm{I}}) = -F_1^{*-1}(V^{\mathrm{I}})F_2^*(V^{\mathrm{I}}). \tag{2.34}$$

$\widehat{V}_1^{\mathrm{I}}$ and $\widehat{V}_2^{\mathrm{I}}$ are diagonal matrices formed from V_1^{I} and V_2^{I}.

As we will see later in this section and in Section 3, the components in the F_1^* and F_2^* matrices are often easy to interpret as cost and sales shares which can be evaluated as either column or row shares from input–output tables. A second advantage of the percentage-change version of the differential form is that the components of the solution matrix, B^*, are elasticities: the i, jth component of $B^*(V^{\mathrm{I}})$ is the elasticity of the ith endogenous variable with respect to the jth exogenous variable evaluated at the initial solution. Economists normally prefer to work with elasticities rather than with derivatives which depend on the units in which variables are measured.

Going from the levels representation, (2.12), of a model to a differential, percentage-change representation, (2.30), usually involves the application of only the three rules shown in Table 1.1. After some practice, application of these rules becomes

Table 1.1
Rules for deriving the percentage-change version of a model

	Representation in:		
	levels		percentage changes
multiplication rule	$X = YZ$	$\Rightarrow$	$x = y + z$
power rule	$X = Y^{\alpha}$	$\Rightarrow$	$x = \alpha y$
addition rules	$X = Y + Z$	$\Rightarrow$	$Xx = Yy + Zz$
			or $x = S_y y + S_z z$

X, Y and Z are levels of variables. x, y and z are percentage changes. α is a parameter and S_y and S_z are shares evaluated at the current solution. In the first step of a Johansen/Euler computation, the current solution is the initial solution. Hence $S_y = Y^{\mathrm{I}}/X^{\mathrm{I}}$ and $S_z = Z^{\mathrm{I}}/X^{\mathrm{I}}$. In subsequent steps, S_y and S_z are recomputed as X, Y and Z move away from their initial values.

quite straightforward and the resulting percentage-change representations are often more readily understood and interpreted than the corresponding levels representations.

In the case of demand and supply functions derived under the assumption that economic actors are optimizers, we can usually derive the percentage-change representation without bothering about the specification of the levels representation. For example, in most general equilibrium models, we assume that the demands for inputs $(X_{1j}, X_{2j}, \ldots, X_{nj})$ by producer j are chosen to minimize the costs of producing a given level of output. A popular specification for the production function is CES,[20] giving a cost minimizing problem of the form:

choose $X_{1j}, X_{2j}, \ldots, X_{nj}$

$$\text{to minimize } \sum_i P_i X_{ij} \tag{2.35}$$

$$\text{subject to } Z_j = A_j \left(\sum_i b_{ij} X_{ij}^{-\rho_j} \right)^{-1/\rho_j}, \tag{2.36}$$

where the P_i are input prices, Z_j is the level of output and A_j and b_{ij} are positive parameters with the bs summing to 1. ρ_j is parameter with value greater than -1 but not precisely zero.[21]

The Lagrangian conditions for a solution of problem (2.35)–(2.36) are

$$P_k = \Lambda_j A_j \left(\sum_i b_{ij} X_{ij}^{-\rho_j} \right)^{-(1+\rho_j)/\rho_j} b_{kj} X_{kj}^{-\rho_j - 1}, \quad k = 1, \ldots, n, \tag{2.37}$$

together with the production function constraint (2.36). From here, we can eliminate the Lagrangian multiplier, Λ_j, from the system (2.36), (2.37), eventually arriving at a representation of the input demand functions which could appear in a levels version of a CGE model:

$$X_{kj} = Z_j \frac{1}{A_j} \left[\sum_i b_{ij} \left(\frac{P_i b_{kj}}{P_k b_{ij}} \right)^{\rho_j/(1+\rho_j)} \right]^{1/\rho_j}, \quad k = 1, \ldots, n. \tag{2.38}$$

[20] The CES production function was introduced by Arrow, Chenery, Minhas and Solow (1961). For the derivation of percentage-change forms of demand and supply equations arising under different specifications of production, utility, transformation and unit cost functions (the duality approach), see Dixon, Parmenter, Powell and Wilcoxen (1992, pp. 124–148).

[21] As ρ_j approaches zero, (2.36) approaches the Cobb–Douglas form.

At this stage we can apply the three rules in Table 1.1 to (2.38) to obtain a percentage-change representation. Alternatively, we could move directly to a percentage-change representation by applying the rules in Table 1.1 to (2.37) and (2.36) giving

$$p_k = \lambda_j + (1+\rho_j)\left(\sum_i S_{ij}x_{ij}\right) - (1+\rho_j)x_{kj}, \quad k = 1,\ldots,n, \tag{2.39}$$

and

$$z_j = \sum_i S_{ij}x_{ij}, \tag{2.40}$$

where p_k, λ_j and x_{ij} are percentage changes in the variables denoted by the corresponding upper-case symbols, and

$$S_{tj} = \left(b_{tj}X_{tj}^{-\rho_j}\right)\Big/\left(\sum_i b_{ij}X_{ij}^{-\rho_j}\right), \quad \text{for} \quad t = 1,\ldots,n. \tag{2.41}$$

By multiplying (2.39) through by S_{kj} and summing over all k, we obtain

$$\lambda_j = \sum_t p_t S_{tj}. \tag{2.42}$$

Substituting (2.40) and (2.42) into (2.39) and rearranging gives a representation of the input demand functions which could appear in a percentage-change version of a CGE model:

$$x_{kj} = z_j - \sigma_j\left(p_k - \sum_t S_{tj}p_t\right), \quad t = 1,\ldots,n, \tag{2.43}$$

where σ_j is the elasticity of substitution between inputs and is given by $\sigma_j = 1/(1+\rho_j)$.

In interpreting (2.43), we start by noting that (2.41) and (2.37) imply that the Ss are cost shares, i.e.

$$S_{tj} = P_t X_{tj}\Big/\sum_k P_k X_{kj}, \quad t = 1,\ldots,n. \tag{2.44}$$

Now we can interpret (2.43) as follows. Reflecting the assumption of constant returns to scale underlying (2.36), (2.43) implies that in the absence of price changes producer j's demands for all inputs will change by the same percentage as its output. If the price of input k rises relative to a cost-share-weighted average of the movements in all input prices, then producer j will substitute away from input k, i.e. producer j's demand for k will rise less quickly than output. The strength of this price-substitution effect will depend on the value of the substitution parameter σ_j.

Not all equations from CGE models are more simply represented in percentage changes of variables than in levels. Some equations (e.g., those specifying total tax collections as the sum of collections of many different types of taxes) are straightforward summations in their levels representation. In their percentage-change representation, they involve some clumsy notation to represent various share coefficients (e.g., the share of total tax collections accounted for by the tax on the use of domestically produced good i by industry j). With recent versions of GEMPACK,[22] users can represent some of their equations in percentage-change form and some in levels form. The programs do the algebra to convert levels equations into differential forms before proceeding with the computations.

Inequalities and complementary slackness conditions. The final issue to be considered in relation to the Johansen/Euler method is the treatment of inequality and complementary slackness conditions. For example, what can we do if our model contains relationships such as

$$R \leqslant T, \tag{2.45}$$

$$I \geqslant 0, \tag{2.46}$$

and

$$I = 0 \quad \text{if} \quad R < T? \tag{2.47}$$

R, T and I are variables. They can be thought of as an industry's rate of return (R); its required or target rate of return (T); and its level of investment (I). Under (2.45), industries expand their capital stocks so that rates of return never exceed the target rates. Under (2.46), investment cannot be negative, and under (2.47), investment will be zero if the rate of return is below the target rate.

One approach to handling models containing relationships such as (2.45)–(2.47) is to solve a sequence of linear complementarity problems (LCPs). Assume that the original model can be written as:

$$f(x) \leqslant 0, \qquad x'f(x) = 0, \quad x \geqslant 0,$$

where x is the vector of endogenous variables. (Here we assume that the exogenous variables are part of the function f.) As described by Mathiesen (1985), we replace f by a linear approximation (e.g. a first-order Taylor approximation), thereby converting

[22] See Harrison, Pearson, Powell and Small (1993).

our model into an LCP. The solution to the LCP can be used in making a new linear approximation to f. After solving a sequence of LCPs, we can expect to arrive at an accurate solution to our original model.

In the Johansen/Euler framework, Horridge and Malakellis [see Malakellis (1992)] have used the following approach. First, they rewrite (2.45)–(2.47) as

$$R + S = T, \tag{2.48}$$

and

$$\min\{I, S\} = 0, \tag{2.49}$$

where S is a nonnegative slack variable. Then they include (2.49) in the gth step of an n-step Johansen/Euler computation as

$$D_{(g,n)}\big(I_{(g,n)} + (\mathrm{d}I)_{(g,n)}\big) + \big(1 - D_{(g,n)}\big)\big(S_{(g,n)} + (\mathrm{d}S)_{(g,n)}\big) = 0, \tag{2.50}$$

where $D_{(g,n)}$ is a coefficient defined by

$$D_{(g,n)} = 1 \quad \text{if} \quad I_{(g,n)} < S_{(g,n)}, \tag{2.51}$$

and

$$D_{(g,n)} = 0 \quad \text{if} \quad I_{(g,n)} \geqslant S_{(g,n)}. \tag{2.52}$$

To see how (2.50)–(2.52) works, assume that we are conducting an n-step computation of the effect of a 100 per cent reduction the tariff protecting an industry's domestic market from import competition. Assume that in the initial situation (i.e., before the tariff reduction), investment in the industry is positive, i.e.,

$$I_{(1,n)} > 0,$$

implying that

$$S_{(1,n)} = 0 \quad \text{and} \quad R_{(1,n)} = T_{(1,n)}.$$

With $I_{(1,n)} > S_{(1,n)}$, we have $D_{(1,n)} = 0$. Thus, in the first step of our computation, (the effect of reducing protection from its initial level to $(n-1)/n$ times that level), (2.50) reduces to

$$(\mathrm{d}S)_{(1,n)} = 0.$$

This means that R will continue to equal T, i.e.,

$$(\mathrm{d}R)_{(1,n)} = (\mathrm{d}T)_{(1,n)}.$$

With lowered protection, we would expect our model to imply that the industry's rate of return can be maintained at T only with a smaller capital stock and reduced investment, i.e., we expect

$$(\mathrm{d}I)_{(1,n)} < 0.$$

If $-(\mathrm{d}I)_{(1,n)} < I_{(1,n)}$ so that $I_{(2,n)} > 0$ and $D_{(2,n)} = 0$ $(0 = S_{(2,n)} < I_{(2,n)})$, then in the second step of our computation we would continue to fix R to T and we would continue to allow I to decline as we simulated the effect of a further reduction in protection. If I stays nonnegative over the n steps of our computation, i.e., if

$$I_{(g,n)} \geqslant 0 \quad \text{for} \quad g = 1, \ldots, n+1, \quad \text{where } I_{(n+1,n)} \text{ is the final value of } I,$$

then S will stay at zero, R will remain equal to T and our final result will be compatible with relations (2.45)–(2.47).

Now assume that in the $(g-1)$th step, $g - 1 < n$, investment becomes negative, i.e.,

$$I_{(g-1,n)} > 0 \quad \text{and} \quad S_{(g-1,n)} = 0$$

but

$$-(\mathrm{d}I)_{(g-1,n)} > I_{(g-1,n)},$$

so that

$$I_{(g,n)} < 0 \quad \text{and} \quad S_{(g,n)} = 0. \tag{2.53}$$

Under (2.53) we will have $D_{(g,n)} = 1$, reducing (2.50) to

$$(\mathrm{d}I)_{(g,n)} = -I_{(g,n)}. \tag{2.54}$$

Hence, in the gth step of the computation, investment will be nudged back to zero and S will be free to move (i.e., R will no longer be fixed to T). Because in the gth step we are both reducing protection and forcing investment to increase, we can expect $(\mathrm{d}R)_{(g,n)}$ to be negative. Assuming that T is fixed, this implies that

$$(\mathrm{d}S)_{(g,n)} > 0.$$

In the $(g+1)$th step of our computation, we will have

$$I_{(g+1,n)} = 0, \qquad S_{(g+1,n)} > 0 \quad \text{and} \quad D_{(g+1,n)} = 1.$$

Thus, investment will stay at zero while we can expect S to increase as we implement further reductions in protection. With the completion of our computations (i.e., with the simulation of a 100 per cent reduction in protection) we would expect to arrive at a solution, compatible with (2.45)–(2.47), in which $I_{(n+1,n)} = 0$ and $R_{(n+1,n)} < T$.

Horridge and Malakellis have found that their method allows the Johansen/Euler approach to be implemented quite easily via GEMPACK in models containing a small number of inequality constraints. Unfortunately they find that extrapolation procedures (e.g., Richardson's extrapolation) are no longer effective. In applying their method, we also need to exercise care to ensure that the differential form of the model stays well defined when, during the computations, variables stray into illegitimate regions (e.g., negative investment).

In our own applications of the Johansen/Euler approach we have usually avoided running up against nonnegativity conditions and other inequality constraints by using: utility functions implying large marginal utility for any commodity consumed at close to the zero level; production functions implying large marginal products for any input close to the zero level; and investment specifications implying reductions in required rates of return as investment levels approach zero. Nevertheless, Horridge and Malakellis have shown that model builders wishing to use the Johansen/Euler approach should not feel compelled to eschew theoretical specifications involving a few inequality constraints.

2.3. *Solving a multi-period model*

We consider four cases, each concerned with a model in which capital stocks available for use in year $t+1$ are determined by investment which takes place before year $t+1$ begins.

In Case 1 investment is exogenous. In Case 2, investment and capital accumulation in year $t+1$ depend on expected rates of return for year $t+2$, which we assume are determined by actual returns to and costs of capital in year $t+1$. In Cases 1 and 2, the models are recursive, i.e. they can be solved for year 1 and then for year 2 and so on.[23]

In Case 3 we assume that expected rates of return for year $t+2$ are the actual rates of return for year $t+2$. That is, we assume that expectations are rational or model

[23] Until recently, nearly all multi-period CGE models were recursive. Leading examples of recursive models are Hudson and Jorgenson's (1974) energy model for the US and the Norwegian model, MSG-4 documented in Longva, Lorentsen and Olsen (1985).

consistent. Under this assumption, the model is no longer recursive.[24] Relative to the recursive models in Cases 1 and 2, solution of our Case 3 model requires a more sophisticated computational approach. The approach we describe is a Johansen/Euler method for handling the computations for all of the years simultaneously.

In Case 4, the behavior of investors is explicitly optimizing. We continue to assume model consistent expectations. The solution method described for Case 3 is still applicable. However, we can also use various shooting methods.

Case 1. Exogenous investment, a recursive model. We start with a model of the form:

$$H\big(\widetilde{V}_1(t), \widetilde{V}_2(t), Q(t), \Pi(t), I(t), K(t-1)\big) = 0, \qquad t = 1, 2, \ldots, T, \tag{2.55}$$

and

$$K(t) = (I - D)K(t-1) + I(t), \quad t = 1, 2, \ldots, T, \tag{2.56}$$

where

- $Q(t)$ is a vector giving industry rentals or profits per unit of capital in year t ($Q_j(t)$ is the rental per unit of capital in industry j);
- $\Pi(t)$ is a vector giving the costs in year t of constructing units of capital for the different industries;
- $I(t)$ is a vector of investment levels in year t for the industries;
- $K(t-1)$ is a vector of industry capital stocks at the end of year $t-1$ and available for use during year t;
- D is a diagonal matrix of depreciation rates;

and

- $\widetilde{V}_1(t)$ and $\widetilde{V}_2(t)$ are other variables for year t. $\widetilde{V}_1(t)$ is the vector of endogenous variables such as domestic prices and outputs and $\widetilde{V}_2(t)$ is the vector of exogenous variables such as world commodity prices, taxes and technological coefficients. $\widetilde{V}_1(t)$ and $\widetilde{V}_2(t)$ could have been defined to include $K(t-1)$, $Q(t)$, $\Pi(t)$ and $I(t)$. However, these latter variables have important roles in our description of multi-period modelling and we prefer to represent them explicitly.

For any given value of t, say $t = \tau$, Eq. (2.55) specifies a typical one-period CGE model. It imposes conditions such as demands equal supplies, prices equal costs and

[24] An early example of a non-recursive CGE model is Dervis (1975). More recent examples include Ballard and Goulder (1985), Goulder and Summers (1989), Bovenberg and Goulder (1991), Jorgenson and Wilcoxen (1994) and Mercenier and Sampaio de Souza (1994).

demands and supplies are consistent with optimizing behaviour by various economic actors. $K(\tau - 1)$, capital availabilities in year τ, can be thought of as a vector of exogenous or pre-determined variables in the year-τ CGE model.

Equation (2.56) says that capital available for use in industry j in year $t+1$ [i.e., $K_j(t)$] equals capital available in year t depreciated at rate D_j [i.e., $(1-D_j)K_j(t-1)$] plus investment in year t [i.e., $I_j(t)$]. Figure 1.2 illustrates the timing of events.

	year t		year $t+1$		year $t+2$	
	$Q(t)$		$Q(t+1)$		$Q(t+2)$	
$K(t-1)$	$\Pi(t)$	$K(t)$	$\Pi(t+1)$	$K(t+1)$	$\Pi(t+2)$	$K(t+2)$
	$I(t)$		$I(t+1)$		$I(t+2)$	
	$\widetilde{V}(t)$		$\widetilde{V}(t+1)$		$\widetilde{V}(t+2)$	

Figure 1.2. Timing in the multi-period model.

We assume that the year-τ model contains no theory of investment, but that if $I(\tau)$ is set exogenously, then the year-τ model [together with predetermined values for $K(\tau-1)$ and exogenously given values for $\widetilde{V}_2(\tau)$] is sufficient to determine the other variables for year τ: $Q(\tau)$, $\Pi(\tau)$ and $\widetilde{V}_1(\tau)$. This means that if we know $K(0)$ and we have an exogenously specified timepath for investment $\{I(1), I(2), \ldots, I(T)\}$, then model (2.55)–(2.56) can be solved as a series of one-period CGE computations. First we use (2.56) to compute the time path for capital stocks $\{K(1), K(2), \ldots, K(T)\}$. Then given $\widetilde{V}_2(\tau)$, we can, in principle, compute $\widetilde{V}_1(\tau)$, $Q(\tau)$ and $\Pi(\tau)$ by solving the one-period CGE model specified by (2.55) with $t = \tau$.

To do these computations we can use the Johansen/Euler approach discussed in the previous subsection. Recognizing that (2.55) holds in each year, we see that growth rates from year t to year $t+1$ satisfy, to a first-order approximation, the system:

$$\begin{aligned}&H_1(t)\,\tilde{v}_1(t+1) + H_2(t)\,\tilde{v}_2(t+1) + H_q(t)\,q(t+1) + H_\pi(t)\,\pi(t+1)\\&\quad + H_i(t)\,i(t+1) + H_k(t)\,k(t) = 0, \quad t = 1, 2, \ldots, T-1. \qquad (2.57)\end{aligned}$$

Equation (2.57) is a percentage change version of (2.55) with the coefficients (H_u, $u = 1, 2, q, \pi, i$ and k) evaluated at the solution for year t, i.e., the H_us are evaluated at

$$V(t) = \big(\widetilde{V}_1(t), \widetilde{V}_2(t), Q(t), \Pi(t), I(t), K(t-1)\big). \qquad (2.58)$$

The variables denoted by lower-case symbols in (2.57) are percentage growth rates in the corresponding upper-case variables. For example, $q(t+1)$ is the vector of percentage growth rates between years t and $t+1$ in rentals, i.e.

$$q_j(t+1) = 100\big(Q_j(t+1) - Q_j(t)\big)/Q_j(t).$$

Consistent with our discussion of the Johansen/Euler method in Subsection 2.2, the lower-case symbols in (2.57) can also be interpreted as percentage deviations from an initial solution for the year-$(t+1)$ model. This initial solution is $V(t)$ given by (2.58).

Under our assumption that the year-$(t+1)$ model (together with $K(t)$, $I(t+1)$ and $\widetilde{V}_2(t+1)$) is sufficient to determine $Q(t+1)$, $\Pi(t+1)$ and $\widetilde{V}_1(t+1)$, we can rearrange (2.57) as

$$v_1(t+1) = B(t)\, v_2(t+1), \quad t = 1, \ldots, T-1, \tag{2.59}$$

where

$$v_1'(t+1) = \big[\tilde{v}_1'(t+1), q'(t+1), \pi'(t+1)\big], \quad t = 1, \ldots, T-1, \tag{2.60}$$

$$v_2'(t+1) = \big[\tilde{v}_2'(t+1), i'(t+1), k'(t)\big], \quad t = 1, \ldots, T-1, \tag{2.61}$$

and

$$\begin{aligned} B(t) = -[H_1(t), H_q(t), H_\pi(t)]^{-1}[H_2(t), H_i(t), H_k(t)], \\ t = 1, \ldots, T-1. \end{aligned} \tag{2.62}$$

With the time paths for investment in each industry given exogenously, we can easily compute $i(2), i(3), \ldots, i(T)$, and with $K(0)$ known we can use (2.56) in computing $k(1), k(2), \ldots, k(T)$. Finally, we assume that our input–output data and other data for the base-period give us a solution to (2.55) for $t = 1$, i.e., we assume that $V(1)$ is known.

We can now proceed recursively. Using $V(1)$ we can evaluate $B(1)$. Then from (2.59), we can compute $v_1(2)$.[25] Next we compute $V(2)$ by using formulae of the form

$$V^{(j)}(2) = V^{(j)}(1)\big(1 + v^{(j)}(2)/100\big), \tag{2.63}$$

where $V^{(j)}(t)$ is the value of the jth variable in year t. With $V(2)$ in place, we can evaluate $B(2)$ and compute $v_1(3)$, and so on.

[25] Alternatively, $v_1(2)$ could be evaluated by a multi-step Johansen/Euler computation.

Case 2. Endogenous investment but still recursive. Investment depends on rates of return. As the first step in moving towards a multi-period model with endogenous investment, we add to our previous model [(2.55)–(2.56)] the following definition of the rate of return in year $t+1$ on capital in industry j:

$$R_j(t+1) = \frac{Q_j(t+1)/(1+r) - \Pi_j(t) + \Pi_j(t+1)(1-D_j)/(1+r)}{\Pi_j(t)}, \tag{2.64}$$

for all j and for $t = 1, \ldots, T-1$,

where r is the rate of interest, which we will treat as a parameter. In this definition, we assume that an outlay of $\Pi_j(t)$ in year t buys a unit of capital ready for use in year $t+1$. This earns a rental in year $t+1$ of $Q_j(t+1)$. The unit of capital depreciates at the rate D_j and can be sold in year $t+1$ for $\Pi_j(t+1)(1-D_j)$. In other words, $R_j(t+1)$ is the present value in year t of investing a dollar in industry j.

Next, we add the equation

$$K_j(t)/K_j(t-1) = F_k(t)F_{kj}(t)\big(1 + R_j^e(t, t+1)\big)^{\alpha_j}, \tag{2.65}$$

for all j and for $t = 1, \ldots, T$.

That is, we assume that the rate of growth of capital through year t depends positively ($\alpha_j > 0$) on the rate of return expected in year t to apply in year $t+1$. The two F variables in (2.65) are shift terms which can be used in various ways. For example, we could set

$$F_{kj}(t) = \big(1 + \mathrm{lrrg}(j)/100\big)/\big(1 + \mathrm{NRR}(j)\big)^{\alpha_j}, \tag{2.66}$$

where $\mathrm{lrrg}(j)$ is the long-run trend rate of growth of capital in industry j and $\mathrm{NRR}(j)$ is j's normal rate of return. F_k could be used to simulate the effects of an overall (not industry-specific) change in the level of business confidence. If F_k is set at 1 and $R_j^e(t, t+1)$ equals $\mathrm{NRR}(j)$, then under (2.66) capital growth in industry j will be, in year t, at its long-run trend rate.

One theory of the expected rate of return is static expectations. We take this to mean that

$$R_j^e(t, t+1) = \frac{Q_j(t)}{\Pi_j(t)}\,\frac{(1+\mathrm{inf})}{1+r} - 1 + \frac{(1+\mathrm{inf})(1-D_j)}{1+r}, \tag{2.67}$$

for all j and for $t = 1, \ldots, T$,

where inf is the rate of inflation. In deriving (2.67), we wrote out the formula for $R_j(t+1)$ with rental and price variables for year $t+1$ replaced by their levels in t multiplied by $(1+\text{inf})$. That is, we assumed that expectations concerning year $t+1$ are formed in year t by inflating all nominal variables by the general rate of inflation. By assuming that $r = \text{inf}$, we can simplify (2.67), obtaining

$$R_j^e(t, t+1) = (Q_j(t)/\Pi_j(t)) - D_j, \tag{2.68}$$

for all j and for $t = 1, \dots, T$.

An advantage of expectations assumptions such as (2.67) or (2.68) is that they give us a model with endogenous investment while still allowing us to solve recursively, applying the Johansen/Euler technique. To demonstrate this, we add to the Johansen/Euler system (2.57) the following:

$$\begin{aligned} &k_j(t+1) - k_j(t) \\ &\quad = f_k(t+1) + f_{kj}(t+1) \\ &\qquad + \alpha_j\big(Q_j(t)/(Q_j(t) + (1-D_j)\Pi_j(t))\big)\big(q_j(t+1) - \pi_j(t+1)\big), \end{aligned} \tag{2.69}$$

for all j and for $t = 1, \dots, T-1$

and

$$K_j(t)k_j(t+1) = (1-D_j)K_j(t-1)k_j(t) + I_j(t)i_j(t+1), \tag{2.70}$$

for all j and for $t = 1, \dots, T-1$.

Equation (2.69) is a percentage-change version of (2.65) incorporating (2.68), and Eq. (2.70) is a percentage-change version of (2.56). In this expanded Johansen/Euler system [i.e., (2.57) and (2.69)–(2.70)], the variables are $\tilde{v}_1(t+1)$, $\tilde{v}_2(t+1)$, $q(t+1)$, $\pi(t+1)$ and $i(t+1)$ for $t = 1, \dots, T-1$; and $k(t+1)$ for $t = 0, \dots, T-1$. All of these are vectors of growth rates connecting years t and $t+1$.

For $t = 1$, the addition of (2.69)–(2.70) expands the original system, (2.57), by $2h$ equations where h is the number of industries. The expanded system for $t = 1$ also contains $2h+1$ new variables: $k_j(2)$, $f_{kj}(2)$ and $f_k(2)$. Assuming that the fs are set exogenously, the expanded system for $t = 1$ can now determine growth rates for h previously exogenous variables: $i_j(2)$ for $j = 1, \dots, h$. After solving the expanded system at $t = 1$, we can, as before, compute $V(2)$. Then we can set $t = 2$ and solve the expanded system [(2.57), (2.69)–(2.70)] for growth rates in the endogenous variables for year 3, and so on.[26]

[26] In making these computations, we need to be clear about initial solutions. In the computations reported for our illustrative model in Section 3, the initial solution for year $(t+1)$ is the set of values for the variables

In our illustrative model in Section 3, we adopt Eqs (2.69)–(2.70). However, rather than exogenizing $f_k(t+1)$ for all t, we either exogenize aggregate investment in each period or fix aggregate investment in relation to aggregate consumption. For period $t+1$, $f_k(t+1)$ is determined endogenously to ensure sufficient growth in capital stocks through year $t+1$ to absorb the given aggregate level of investment.

Case 3. A non-recursive multi-period model. An alternative to (2.68) is

$$R_j^e(t, t+1) = R_j(t+1), \quad \text{for all } j \text{ and for } t = 1, 2, \ldots, T.^{27} \tag{2.71}$$

This is the assumption of model consistent or rational expectations. With (2.71) replacing (2.68), we write (2.65) as

$$K_j(t)/K_j(t-1) = F_k(t)F_{kj}(t)(1 + R_j(t+1))^{\alpha_j}, \quad \text{for all } j \text{ and for } t = 1, \ldots, T, \tag{2.72}$$

and replace (2.69) by

$$\begin{aligned} k_j(t+1) - k_j(t) &= f_k(t+1) + f_{kj}(t+1) \\ &+ \alpha_j\big(R_j(t+1)/\big(1 + R_j(t+1)\big)\big)r_j(t+2), \\ &\text{for all } j \text{ and for } t = 1, \ldots, T-1, \end{aligned} \tag{2.73}$$

where $r_j(t+2)$ is the percentage change in industry j's rate of return between years $t+1$ and $t+2$.

Now with investment endogenous, we no longer have a recursive model. Before we can work out the growth rates connecting years 1 and 2, we need to know $r_j(3)$. But this depends on $q_j(3)$ and $\pi_j(3)$ [see (2.64)]. Values for these variables cannot be found until we work out the growth rates connecting years 2 and 3. To work out the growth rates for year 3, we need to know $r_j(4)$. But this depends on $q_j(4)$ and $\pi_j(4)$, and so on.

CGE computer packages such as GEMPACK (Pearson, 1988) can handle linear systems containing millions of equations and variables.[28] Using the Johansen/Euler

in year t. In particular, in our year-$(t+1)$ computation, the initial values for $K(t)$ and $K(t+1)$, i.e., the beginning and ending capital stocks in year $t+1$, are given by the begining and ending capital stocks in year t, i.e., $K(t)_{\text{initial}} = K(t-1)$ and $K(t+1)_{\text{initial}} = K(t) = (1-D)K(t-1) + I(t)$. With this initial solution for year $(t+1)$, our percentage-change answers from the Johansen/Euler computation retain their interpretation as both deviations from a base-case solution and as growth rates through time. For example, $k(t+1)$ is the percentage deviation in $K(t+1)$ from its initial value, $K(t)$. Thus, $k(t+1)$ is also the growth in capital through year $t+1$.

[27]This equation calls for a value of $R_j(T+1)$ which is beyond the range of (2.64). In applications of a model such as the one we are describing, we would expect to set the $R_j(T+1)$s exogenously to reflect the assumption that in the long-run, rates of return settle down to normal levels.

[28]GEMPACK uses sparse matrix methods. It also has convenient facilities for reducing the size of a linear system by using some of the equations to eliminate some of the variables. For a discussion of this in the context of a large-scale CGE model, see Dixon, Parmenter, Sutton and Vincent (1982, pp. 207–229).

method, we can solve very large, one-period CGE models. This suggests that we can overcome the non-recursivity problem in multi-period models by using a Johansen/Euler approach with all years treated simultaneously.[29]

To describe this approach, we start by representing a multi-period CGE model as

$$F\big(V(1), V(2), \ldots, V(T)\big) = 0, \tag{2.74}$$

where $V(t)$ is the vector of variables applying to year t.

All the equations of the model are included, i.e. (2.74) includes the equations linking contemporaneous variables (e.g. demands at time t equal supplies at time t) and the equations, such as (2.56), (2.64) and (2.72), linking variables from different times.

Providing that we have an initial solution

$$V^{\mathrm{I}} = \big(V^{\mathrm{I}}(1), V^{\mathrm{I}}(2), \ldots, V^{\mathrm{I}}(T)\big),$$

then we can use the rules from Table 1.1 to form a percentage change version of (2.74):

$$F^*(V^{\mathrm{I}})v = 0, \tag{2.75}$$

where F^* is the Jacobian matrix of F evaluated at V^{I} and multiplied by $\widehat{V}^{\mathrm{I}}$, and v is the vector of percentage deviations in variables from their initial values. (It is worth emphasizing that components of v are not growth rates through time.)

Once we have system (2.75), we can divide the variables into endogenous and exogenous sets. Then we follow the steps described in (2.30)–(2.31) to compute, for example, the effect on output and employment in the footwear industry in all years of an anticipated change in the tariff in year t.[30]

How do we find an initial solution, V^{I}? Unlike the situation in a one-period model, we cannot, for a multi-period model, simply read a solution from our input–output database.

In some models, it is easy to find a steady state or a balanced-growth path: i.e. we can find a solution of the form

$$V^{\mathrm{I}} = \big(V^{\mathrm{I}}(1), \hat{g}V^{\mathrm{I}}(1), \hat{g}^2V^{\mathrm{I}}(1), \ldots, \hat{g}^{T-1}V^{\mathrm{I}}(1)\big), \tag{2.76}$$

where $\hat{g}$ is a diagonal matrix (possibly the identity matrix) of growth factors.[31] However, not all models have a solution of the form (2.76). In any case, a solution of

[29]To our knowledge, this potential was first recognized by Bovenberg (1985) and Wilcoxen (1985 and 1987).

[30]In models with rational expectations [i.e. with expectation assumptions such as (2.71)], any change in tariffs in year t is anticipated. It affects behavior in years $t-1$, $t-2, \ldots$ as well as in years t, $t+1, \ldots$.

[31]For example, Bovenberg (1985) uses this method.

this form may be rather far from a realistic solution around which we would want to compute deviations.

An alternative approach to finding a V^{I} involves an initial recursive simulation, followed by a correction.[32] For example, consider the model specified by (2.55), (2.56), (2.64) and (2.72). Initially, we delete (2.72) from the system and solve the reduced system with $[I(1), I(2), \ldots, I(T)]$ set exogenously. This solution can be made recursively, as described in Case 1, using data for the initial year and a series of one-period Johansen/Euler computations.

After completing the initial recursive computation, we will have found a solution $(V_R^{\mathrm{I}}(1), \ldots, V_R^{\mathrm{I}}(T))$ to the reduced system. This includes values for $R_j(t+1)$, $t = 1, \ldots, T-1$, and $K_j(t)$, $t = 0, \ldots, T$. These (together with a suitable value for $R_j(T+1)$) can be substituted into (2.72) to obtain implied values for $F_{kj}(t)$, $t = 1, \ldots, T$. (We assume that $F_k(t)$ is set at one for all t.)

At this stage, we have a solution V^{II} (an initial, initial solution) to the full system (2.56), (2.57), (2.64) and (2.72). Using V^{II}, we can set up a percentage deviation version of the full system:

$$F^*\left(V^{\mathrm{II}}\right)v = 0. \tag{2.77}$$

The last step is to run a correction simulation using (2.77). In this simulation, we include percentage changes in investment $[i_j(t+1),\ t = 1, \ldots, T-1]$ among the endogenous variables while the $f_{kj}(t+1)$, $t = 1, \ldots, T-1$, are among the exogenous variables. The correction simulation consists of shocking the F_{kj}s from their values in V^{II} to realistic values, e.g. those specified in (2.66). After adjusting V^{II} for the effects of moving the F_{kj}s, we arrive at V^{I}. This is a solution to the full model containing economically sensible relationships between the paths of capital stocks and rates of return.

Case 4. A non-recursive multi-period model with optimizing investment behavior. In Section 1, we claimed that a strength of CGE modelling is its reliance on optimizing theories of behavior by different economic actors. Equation (2.72) rests uneasily with this claim. It was not derived from any explicit optimizing specification.

An optimizing specification which is often used in the derivation of investment equations for multi-period CGE models is as follows:[33] industry j chooses $I_j(t+1)$

[32]This method was suggested by Mark Horridge and is related to the homotopy concept [see Zangwill and Garcia (1981, Chapter 1)]. It has been developed and applied by his student Michael Malakellis (1992, 1994).

[33]The optimization problem is usually specified in continuous time with an infinite time horizon [see, for example, Bovenberg (1985), Bovenberg and Goulder (1991) and Dixon, Parmenter, Powell and Wilcoxen (1992, Chapter 5)]. Solution of the problem then involves the use of optimal control techniques and the specification of transversality conditions. By adopting a discrete-time optimization problem with a finite time horizon, we can use the Lagrangian method. Rather than imposing transversality conditions, we impose a value on terminal capital stocks.

and $K_j(t+1)$ for $t = 1, \ldots, T-1$ to maximize

$$\sum_{t=0}^{T-1} \left\{ \frac{Q_j(t+1)K_j(t)}{(1+r)^t} - \frac{\Pi_j(t+1)(I_j(t+1) + \theta I_j^2(t+1))}{(1+r)^t} \right\}$$
$$+ \left\{ \frac{Q_j(T+1)}{(1+r)^T} + (1-D_j)\Lambda_j(T+1) \right\} K_j(T) \tag{2.78}$$

subject to

$$K_j(t) = (1-D_j)K_j(t-1) + I_j(t), \tag{2.79}$$
$$\text{for all } j \text{ and for } t = 1, \ldots, T,$$

with $K_j(0)$ given.

The only new symbols in (2.78)–(2.79) are θ and $\Lambda_j(T+1)$. Both denote positive parameters. The remaining notation is the same as that used earlier in this subsection. The timing of events is also the same. That is, we assume that $K_j(t)$ can be used in production in year $t+1$ (Fig. 1.2).

Two features of the objective function (2.78) need explanation. The first is the term θI_j^2. This is often called a costs-of-adjustment term.[34] It makes rapid expansion of the capital stock very costly. With no explicit recognition of risk in (2.78)–(2.79), the inclusion of the costs-of-adjustment term plays a useful dampening role. Without it, behavioral specifications such as (2.78)–(2.79) are inclined to imply unrealistically large responses in investment to small changes in anticipated rentals and construction costs.

The second feature of (2.78) which may be puzzling is the final term. This gives units of capital a value at the end of the industry's planning period. If they were given no value, then (2.78)–(2.79) would probably imply unrealistically low investment levels for years close to T. We have chosen for notational reasons to represent this terminal value by

$$TV_j = \frac{Q_j(T+1)}{(1+r)^T} + (1-D_j)\Lambda_j(T+1). \tag{2.80}$$

As we will see, the use of this notation simplifies the presentation of the Lagrangian conditions for a solution of (2.78)–(2.79).

These Lagrangian conditions are

$$Q_j(t+1)/(1+r)^t - \Lambda_j(t) + (1-D_j)\Lambda_j(t+1) = 0, \tag{2.81}$$
$$\text{for all } j \text{ and for } t = 1, \ldots, T,$$

[34] The costs-of-adjustment term takes different forms in different models. The form used here is close to that in Dixon, Parmenter, Powell and Wilcoxen (1992, Chapter 5). Bovenberg and Goulder (1991) use $\theta I^2/K$.

$$-\Pi_j(t)(1+2\theta I_j(t))/(1+r)^{t-1}+\Lambda_j(t)=0, \quad \text{for all } j \text{ and for } t=1,\dots,T, \tag{2.82}$$

and

$$K_j(t)-(1-D_j)K_j(t-1)-I_j(t)=0, \quad \text{for all } j \text{ and for } t=1,\dots,T, \tag{2.83}$$

where the $\Lambda_j(t)$ for all j and for $t = 1,\dots,T$ are the Lagrangian multipliers[35] associated with the constraint (2.79). Notice in (2.81) that with our notational choice (2.80) we do not have to make a special case for $t = T$.

How can we deal with (2.81)–(2.83) in a multi-period CGE model? To answer this question, we consider the model formed by (2.81)–(2.83) together with (2.55).[36] As we have done earlier, we will assume that for any given value of t, say $t = \tau$, (2.55) can be solved for $\widetilde{V}_1(\tau)$, $Q(\tau)$, and $\Pi(\tau)$ in terms of $\widetilde{V}_2(\tau)$, $I(\tau)$ and $K(\tau-1)$. We also assume that we have a base-period solution for (2.55), i.e. we know $V(1) = (\widetilde{V}_1(1)$, $\widetilde{V}_2(1)$, $Q(1)$, $\Pi(1)$, $I(1)$, $K(0))$. With given values for $I(1)$ and $K(0)$, we will treat $K(1)$ as known.

Our aim is to solve the system (2.55), (2.81)–(2.83) with investment determined endogenously in a way which is consistent with the optimizing specification

[35] Remember that $\Lambda_j(T+1)$ is a parameter of problem (2.78)–(2.79), not a Lagrangian multiplier.

[36] This model is the same as that considered in Case 3 [i.e. (2.55), (2.56), (2.64) and (2.72)] except that (2.72) is replaced by

$$R_j(t+1)=2\theta\left(I_j(t)-\frac{1-D_j}{1+r}\,\frac{\Pi_j(t+1)}{\Pi_j(t)}I_j(t+1)\right), \quad \text{for all } j \text{ and for } t=1,\dots,T-1, \tag{2.84}$$

and

$$TV_j=\Pi_j(T)[1+2\theta I_j(T)]/(1+r)^{T-1}, \quad \text{for all } j. \tag{2.85}$$

(2.84) and (2.85) can be derived by using (2.82) to eliminate $\Lambda_j(t)$, $t = 1,\dots,T$, from (2.81) and then calling on the definitions of the rates of return in (2.64) and the terminal value of units of capital in (2.80). Formulation (2.84)–(2.85) not only helps to relate the present model to that studied in Case 3, but it helps us to understand the relationship between models with and without adjustment costs. Models without adjustment costs [e.g., Jorgenson and Wilcoxen (1992, 1994) and Malakellis (1994)] use arbitrage equations of the form

$$\frac{Q_j(t+1)}{1+r}-\Pi_j(t)+\Pi_j(t+1)\frac{1-D_j}{1+r}=0, \quad \text{for all } j \text{ and for } t=1,\dots,T-1. \tag{2.86}$$

(2.86) is what we obtain from (2.84) and (2.64) if we set $\theta = 0$. This implies that the models with adjustment costs can be regarded as generalizations of the models without adjustment costs.

(2.78)–(2.79). If we happen to know the values of $\Lambda_j(2)$ for all j, then we can do this recursively. For year 2, we form the system:

$$H\big(\widetilde{V}_1(2), \widetilde{V}_2(2), Q(2), \Pi(2), I(2), K(1)\big) = 0, \tag{2.87}$$

$$-\Pi_j(2)(1 + 2\theta I_j(2))/(1 + r) + \Lambda_j(2) = 0 \tag{2.88}$$

and

$$K_j(2) = (1 - D_j)K_j(1) + I_j(2), \quad \text{for all } j. \tag{2.89}$$

With the $\Lambda_j(2)$s known, (2.88) provides the extra equation to enable $I(2)$ to be determined endogenously in the system (2.87)–(2.88), while (2.89) allows us to calculate $K(2)$. Once we have found all the year-2 variables from (2.87)–(2.89), then we can move to year 3. For year 3 we have

$$H\big(\widetilde{V}_1(3), \widetilde{V}_2(3), Q(3), \Pi(3), I(3), K(2)\big) = 0, \tag{2.90}$$

$$-\Pi_j(3)(1 + 2\theta I_j(3))/(1 + r)^2 + \Lambda_j(3) = 0, \quad \text{for all } j, \tag{2.91}$$

$$Q_j(3)/(1 + r)^2 - \Lambda_j(2) + (1 - D_j)\Lambda_j(3) = 0, \quad \text{for all } j, \tag{2.92}$$

and

$$K_j(3) = (1 - D_j)K_j(2) + I_j(3), \quad \text{for all } j. \tag{2.93}$$

Equations (2.91) and (2.92) provide the extra conditions to enable $I(3)$ and $\Lambda(3)$ to be determined in the system (2.90)–(2.92), and $K(3)$ can be calculated from (2.93). Having found values for all the year 3 variables, we can move onto year 4, and so on. With a given base-period solution, $V(1)$, all these calculations could be carried out in a recursive, Johansen/Euler, year-to-year, computation of the type already described in Cases 1 and 2.

The main problem we face with a recursive approach is how to set $\Lambda_j(2)$ for all j. In any case, how do we know whether we have set the right values or not?

The question of whether the vector $\Lambda(2)$ was set correctly is answered when we do the calculations for year T. The year T calculation produces values for $\Lambda_j(T)$ for all j. We denote these by $\Lambda_j^{(g)}(T)$ where the superscript (g) refers to guess number g. That is $\Lambda^{(g)}(T)$ is the vector of values for $\Lambda(T)$ obtained in a recursive calculation based on the gth guess of the vector $\Lambda(2)$. If $\Lambda_j^{(g)}(T)$ equals the exogenously given value of TV_j for all j, i.e. if

$$\Lambda_j^{(g)}(T) = Q_j(T+1)/(1+r)^T + (1-D_j)\Lambda_j(T+1), \quad \text{for all } j, \tag{2.94}$$

then we conclude that our gth guess of $\Lambda(2)$ was correct and that we have now found a solution to the model (2.55), (2.81)–(2.83). If (2.94) is not satisfied, then we can revise our guess of $\Lambda(2)$, re-do the recursive calculations and hope that we manage to satisfy (2.94) while g is still quite small.

The approach we have just described to solving the model (2.55), (2.81)–(2.83) is a shooting algorithm. (We guess a value for $\Lambda(2)$ and shoot forward, trying to hit a terminal target value.) As explained in Dixon, Parmenter, Powell and Wilcoxen (1992, Chapter 5, pp. 333–340), simple shooting algorithms often work poorly in economic models. The difficulty is that small errors in the guess of $\Lambda(2)$ can result in very large differences between the left and right hand sides of (2.94). More success has been achieved with multiple shooting methods[37] (where each set of recursive calculations uses guesses of $\Lambda(t)$ for several values of t, not just $t = 2$) and with the Fair–Taylor method[38] (where each set of recursive calculations uses guesses of $\Lambda(t)$ for all values of t).

In our forecasting and policy work for businesses and government departments in Australia, we have not adopted the assumption of rational expectations. We solve a large (112 industry) recursive model incorporating externally supplied, realistic macro forecasts. Our approach, which is illustrated in Section 3 and discussed further in Section 4, is an application of Case 2. However, our colleague, Michael Malakellis (1994), has built a 13-sector, 30-period model for Australia along the lines of Case 4. Rather than adopting shooting methods, he has preferred to use in his computations the non-recursive, simultaneous, Johansen/Euler approach described in Case 3. Under this method, all the $\Lambda(t)$s, $Q(t)$s and other variables appear in the computations simultaneously, with $\Lambda(T+1)$ and $Q(T+1)$ treated as exogenous variables. His experience suggests that there would be no serious computational difficulties in applying this simultaneous method in the solution of very large multi-period models. The real issue now is the empirical relevance of the rational expectations assumption.

3. An illustrative CGE model

In this section we describe the theory and data of an illustrative CGE model. We show how CGE models can be used for comparative-static policy analysis and for forecasting. The illustrative model has just three sectors. Its equation system uses only

[37] See Lipton, Poterba, Sachs and Summers (1982) and Roberts and Shipman (1972).
[38] See Fair (1979) and Fair and Taylor (1983).

simple functional forms. With the same techniques as are employed for the illustrative model, models for real-world applications can be constructed which are much larger and which have equation systems based on more general functional forms.

In describing the illustrative model, we begin with the input–output database (Section 3.1). By examining this, we can set out the structure of the hypothetical economy to be modelled. Then we proceed to the model's equation system (Section 3.2). In Section 3.3 we describe the calibration of the equation system using the input–output accounts, some elasticities and other data. We show how this data set constitutes an initial solution to the model. Closure of the model is discussed briefly in Section 3.4. Our illustrative simulations are described in Section 3.5.

3.1. Input–output database

The basic structure of the model is revealed by Table 1.2, the model's input–output database. The columns identify the following purchasing agents:

(1) domestic producers in each of 3 industries;
(2) investors divided into 3 industries;
(3) a single representative household; and
(4) an aggregate foreign purchaser of exports.

The entries in the columns show the purchases made by these agents. Each of the 4 commodity types identified in the model can, in principle, be purchased locally or imported from overseas. In our data there are no imports of commodity 3 and no domestic supplies of commodity 4. The source-specific commodities are used by industries as inputs to current production and capital formation, consumed by households and exported. These commodity flows (in the first 8 rows of the table) are shown at basic values, i.e., at the prices received by the sellers not those paid by the purchasers. In the case of imports, basic values include import duties. Import duties are assumed to be levied at rates which vary by commodity but not by user. The revenue obtained is shown in the tariff vector labelled "(−) duty".

One domestically produced commodity (commodity 3) is used as a margins service[39] which is required to transfer commodities from their sources to their users. Commodity taxes are also payable on the purchases. The margins services and commodity taxes applying to the flows of domestic and imported commodities are shown in rows 9–24 of the table. By adding the margins and commodity taxes to the corresponding basic commodity flows, we can compute the purchasers' values of those flows.

As well as intermediate inputs, current production requires inputs of two categories of primary factors: labour and fixed capital.

[39]This could be thought of as trade and transport services.

Table 1.2
Input–output database for the illustrative model

		Inputs to current production in industries			Inputs to capital formation in industries			H'hold consn	Exports	(–)duty	Total sales
		1	2	3	1	2	3				
Domestic	1	10.00	20.00	10.00	0.25	0.13	0.62	4.00	35.00		80.00
commodities	2	15.00	15.00	10.00	3.80	1.90	9.30	55.00	10.00		120.00
	3	18.00	8.00	58.00	0.00	0.00	0.00	35.00	0.00		119.00
	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		0.00
Imports	1	0.00	5.00	10.00	0.51	0.25	1.24	8.00	0.00	−4.00	21.00
	2	0.00	6.00	0.00	1.52	0.76	3.72	18.00	0.00	−3.00	27.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4	5.00	10.00	4.00	0.76	0.38	1.86	8.00	0.00	−10.00	20.00
Margins on	1	1.00	4.00	3.00	0.00	0.00	0.00	2.00	6.00		16.00
domestic	2	3.00	5.00	2.00	1.27	0.63	3.10	21.00	4.00		40.00
commodities	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		0.00
	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		0.00
Margins on	1	0.00	1.00	3.00	0.25	0.13	0.62	1.00	0.00		6.00
imports	2	0.00	2.00	0.00	0.76	0.38	1.86	6.00	0.00		11.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		0.00
	4	1.00	2.00	1.00	0.51	0.25	1.24	2.00	0.00		8.00
Taxes on	1	1.00	4.00	1.50	0.00	0.00	0.00	0.00	10.00		16.50
domestic	2	3.00	3.57	3.00	0.72	0.36	1.77	18.08	−1.00		29.50
commodities	3	0.00	0.00	0.00	0.00	0.00	0.00	5.00	0.00		5.00
	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		0.00
Taxes on	1	0.00	1.00	1.50	0.00	0.00	0.00	0.00	0.00		2.50
imports	2	0.00	1.43	0.00	0.29	0.14	0.71	5.92	0.00		8.50
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		0.00
	4	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00		2.00
Labour		22.00	14.00	64.00							100.00
Capital		11.00	8.00	29.00							48.00
Total costs		90.00	110.00	200.00	10.63	5.32	26.05	191.00	64.00	−17.00	680.00
		MAKE MATRIX:									
Domestic	1	60.00	20.00	0.00							
commodities	2	30.00	90.00	0.00							
	3	0.00	0.00	200.00							
	4	0.00	0.00	0.00							
Total output		90.00	110.00	200.00							

In principle, each industry is capable of producing any of the 4 commodity types. The make matrix at the bottom of Table 1.2 shows the basic value of the output of each commodity by each industry. In our data, industries 1 and 2 both produce commodities 1 and 2. Industry 3 is a single-product industry and the sole producer of commodity 3. Commodity 4 is not produced domestically.

3.2. Equations

The model's theoretical structure describes the purchasing decisions of the industries, investors, households and foreigners; the production decisions of the industries; price formation; market clearing, capital accumulation; wage determination; and the definition of a number of macroeconomic variables. The equations of the model are set out in Table 1.3, both in the levels of the variables and in their percentage changes. The levels of the variables can be thought of as referring to year t, and the percentage changes to either comparative-static deviations or to growth rates connecting years t and $t + 1$. The percentage-change equations follow straightforwardly from the levels equations by application of the rules described in Table 1.1. The variables are defined in Table 1.4. The other notation appearing in the equations is defined in Table 1.5. The notational conventions should become apparent as we proceed through the equations.

Equations 3.1–3.6 describe the demands by users[40] for source-specific inputs and for composite inputs[41]. All users are assumed to be price takers. As can be seen from Table 1.2, producers use commodities and primary factors as inputs but other users (investors, households and purchasers of exports) use commodities only. Commodities can be sourced domestically or imported, although purchasers of exports use the domestic source only. Labour and capital are the sources of primary factors.

Industries are assumed to choose inputs to current production and capital formation to minimise the costs of these activities. Input–output separability is imposed on the current-production functions so that the composition of inputs is independent of the composition of output. The structure of Eqs 3.1, 3.2 and 3.5 reflects the nested structure of the input side of the production functions assumed for current production and capital creation. The top level of the nests, describing the technology for the use of composite inputs, are fixed-proportions (Leontief) functions leading to demand

[40] Users are denoted by the superscript (u) taking the values: $(1j)$, producers in industry j; $(2j)$, investors in industry j; (3) households; or (4) purchasers of exports.

[41] Inputs are denoted by the first subscript taking the values: $1, \ldots, g$, commodities; or $g + 1$, primary factors. The sources of the inputs are denoted by the second subscript taking the values: 1, domestic supplies in the case of commodities and labour in the case of primary factors; or 2, imports and capital. A "$\cdot$" in place of this second subscript indicates aggregation over sources, i.e., a composite commodity or primary factor.

Table 1.3
Equations of the illustrative model

Identifier	Equation	No.
	Substitution between domestic and imported products	
3.1(a)	$X_{(is)}^{(u)} = X_{(i\cdot)}^{(u)} \Psi_{(is)}^{(u)} \left(P_{(i1)}^{(u)}; P_{(i2)}^{(u)}\right)$	
3.1(b)	$x_{(is)}^{(u)} = x_{(i\cdot)}^{(u)} - \sigma_i^{(u)} \left[p_{(is)}^{(u)} - \sum_{t=1,2} \left\{V(i,t,(u))/V(i,\cdot,(u))\right\} p_{(it)}^{(u)}\right]$,	
	$i = 1,\ldots,g$; $s = 1$ and 2; and $(u) = (3)$ and (kj) for $k = 1$ and 2 and $j = 1,\ldots,h$	$4gh + 2g$
	Substitution between labour and capital	
3.2(a)	$X_{(g+1,s)}^{(1j)}/A_{(g+1,s)}^{(1j)} = X_{(g+1\cdot)}^{(1j)} \Psi_{(g+1,s)}^{(1j)} \left(P_{(g+1,1)}^{(1j)}/A_{(g+1,1)}^{(1j)}; P_{(g+1,2)}^{(1j)}/A_{(g+1,2)}^{(1j)}\right)$	
3.2(b)	$x_{(g+1,s)}^{(1j)} - a_{(g+1,s)}^{(1j)} = x_{(g+1\cdot)}^{(1j)} - \sigma_{g+1}^{(1j)} \Big(p_{(g+1,s)}^{(1j)} - a_{(g+1,s)}^{(1j)} - \sum_{t=1,2} \left\{V(g+1,t,(1j))/V(g+1,\cdot,(1j))\right\} \left(p_{(g+1,t)}^{(1j)} - a_{(g+1,t)}^{(1j)}\right)\Big)$,	
	$j = 1,\ldots,h;\ s = 1,2$	$2h$
	Household demands for composite commodities	
3.3(a)	$P_{(i\cdot)}^{(3)} X_{(i\cdot)}^{(3)} = \gamma_i P_{(i\cdot)}^{(3)} Q + \beta_i \Big(C - \sum_{j\in G} \gamma_j P_{(i\cdot)}^{(3)} Q\Big)$	
3.3(b)	$V(i,\cdot,(3))\left(p_{(i\cdot)}^{(3)} + x_{(i\cdot)}^{(3)}\right) = \gamma_i P_{(i\cdot)}^{(3)} Q \left(p_{(i\cdot)}^{(3)} + q\right) + \beta_i \Big(C_c - \sum_{j\in G} \gamma_j P_{(i\cdot)}^{(3)} Q \left(p_{(i\cdot)}^{(3)} + q\right)\Big), \quad i = 1,\ldots,g$	g
	Prices of composite commodities to households	
3.4(a)	$P_{(i\cdot)}^{(3)} X_{(i\cdot)}^{(3)} = \sum_{t=1,2} P_{(it)}^{(3)} X_{(it)}^{(3)}$	
3.4(b)	$p_{(i\cdot)}^{(3)} = \sum_{t=1,2} \left\{V(i,t,(3))/V(i,\cdot,(3))\right\} p_{(i\cdot)}^{(3)}, \quad i = 1,\ldots,g$	g
	Intermediate and investment demands for composites, commodities and primary factors	
3.5(a)	$X_{(i\cdot)}^{(u)} = Z^{(u)} A_i^{(u)}$ — $u = (kj)$ for $k = 1,2$ and $j = 1,\ldots,h$; If $(u) = (1j)$ then $i = 1,\ldots,g+1$	$2gh + h$
3.5(b)	$x_{(i\cdot)}^{(u)} = z^{(u)}$ — If $(u) = (2j)$ then $i = 1,\ldots,g$	

Table 1.3
(continued)

Identifier	Equation	No.
	Foreign demands (exports) for domestic goods	
3.6(a)	$P^{(4)}_{(i1)}E = F^{(4)}_{(i1)}\left(X^{(4)}_{(i1)}\right)^{-1/\eta_i}$	
3.6(b)	$p^{(4)}_{(i1)} + e = f^{(4)}_{(i1)} - (1/\eta_i)x^{(4)}_{(i1)}, \quad i = 1,\ldots,g$	g
	Margins demands for domestic goods	
3.7(a)	$X^{(is)(u)}_{(r1)} = X^{(u)}_{(is)}A^{(is)(u)}_{(r1)}, \quad r, i = 1,\ldots,g$ $(u) = (3), (4)$ and (kj) for $k = 1, 2$ and $j = 1,\ldots,h$	
3.7(b)	$x^{(is)(u)}_{(r1)} = x^{(u)}_{(is)}$ If $(u) = (4)$ then $s = 1$ If $(u) \neq (4)$ then $s = 1, 2$	$4g^2h + 3g^2$
	Composition of output by industries	
3.8(a)	$X^{(0j)}_{(i1)} = Z^{(1j)}\Psi^{(0j)}_{(i1)}\left(P^{(0)}_{(11)}, P^{(0)}_{(21)},\ldots,P^{(0)}_{(g1)}\right)$	
3.8(b)	$x^{(0j)}_{(i1)} = z^{(1j)} + \sigma^{(0j)}\left[p^{(0)}_{(i1)} - \sum_{t\in G}\{Y(t,j)/Y(\cdot,j)\}p^{(0)}_{(t1)}\right],$ $j = 1,\ldots,h, \; i = 1,\ldots,g$	gh
	Demand equals supply for domestic commodities	
3.9(a)	$\sum_{j\in H} X^{(0j)}_{(t1)} = \sum_{(u)\in U} X^{(u)}_{(t1)} + \sum_{i\in G}\sum_{s=1,2}\sum_{(u)\in U^*} X^{(is)(u)}_{(t1)} + \sum_{i\in G} X^{(i1)(4)}_{(t1)}$	
3.9(b)	$\sum_{j\in H} Y(t,j)x^{(0j)}_{(t1)} = \sum_{(u)\in U} B(t,1,(u))x^{(u)}_{(t1)}$ $+ \sum_{i\in G}\sum_{s=1,2}\sum_{(u)\in U^*} M(t,i,s,(u))x^{(is)(u)}_{(t1)}$ $+ \sum_{i\in G} M(t,i,1,(4))x^{(i1)(4)}_{(t1)}, \quad t = 1,\ldots,g$	g
	Industry revenue equals industry costs	
3.10(a)	$\sum_{t\in G} X^{(0j)}_{(t1)}P^{(0)}_{(t1)} = \sum_{t\in G^*}\sum_{s=1,2} X^{(1j)}_{(ts)}P^{(1j)}_{(ts)}$	
3.10(b)	$\sum_{t\in G} Y(t,j)p^{(0)}_{(t1)} = \sum_{t\in G^*}\sum_{s=1,2} V(t,s,(1j))p^{(1j)}_{(ts)}, \quad j = 1,\ldots,h$	h

Table 1.3
(continued)

Identifier	Equation	No.
	Basic price of imported commodities	
3.11(a)	$P^{(0)}_{(i2)} = \left(P^{(w)}_{(i2)}/E\right)T^{(0)}_{(i2)}$	
3.11(b)	$p^{(0)}_{(i2)} = p^{(w)}_{(i2)} - e + t^{(0)}_{(i2)}, \quad i = 1, \ldots, g$	g
	Purchasers' price related to basic prices and taxes	
3.12(a)	$P^{(u)}_{(is)} = P^{(0)}_{(is)}T(i,s,(u)) + \sum_{r\in G} P^{(0)}_{(r1)}A^{(is)(u)}_{(r1)}$	
3.12(b)	$V(i,s,(u))p^{(u)}_{(is)} = \left(B(i,s,(u)) + T(i,s,(u))\right)\left(p^{(0)}_{(is)} + t(i,s,(u))\right) + \sum_{r\in G} M(r,i,s,(u))p^{(0)}_{(r1)},$ $i = 1,\ldots,g$; $(u) = (3), (4)$ and (kj) for $k = 1,2$ and $j = 1,\ldots,h$ If $(u) = (4)$ then $s = 1$. If $(u) \neq (4)$ then $s = 1,2$	$3g + 4gh$
	Investment behaviour	
3.13(a)	$X^{(1j)}_{(g+1,2)}(1)/X^{(1j)}_{(g+1,2)} = F_k F^{(j)}_k \left[1 + \left(P^{(1j)}_{(g+1,2)}/P^{(1j)}_k\right) - \delta_j\right]^{\alpha_j}$	
3.13(b)	$x^{(1j)}_{(g+1,2)}(1) - x^{(1j)}_{(g+1,2)} = f_k + f^{(j)}_k + \alpha_j\left[P^{(1j)}_{(g+1,2)}/\left(P^{(1j)}_{(g+1,2)} + (1-\delta_j)P^{(1j)}_k\right)\right]\left(p^{(1j)}_{(g+1,2)} - p^{(1j)}_k\right),$ $j = 1,\ldots,h$	h
	Capital accumulation	
3.14(a)	$X^{(1j)}_{(g+1,2)}(1) = X^{(1j)}_{(g+1,2)}(1-\delta_j) + Z^{(2j)}$	
3.14(b)	$X^{(1j)}_{(g+1,2)}(1)x^{(1j)}_{(g+1,2)}(1) = X^{(1j)}_{(g+1,2)}(1-\delta_j)x^{(1j)}_{(g+1,2)} + Z^{(2j)}z^{(2j)},$ $j = 1,\ldots,h$	h
	Costs of constructing units of capital for industries	
3.15(a)	$P^{(1j)}_k Z^{(2j)} = \sum_{i\in G}\sum_{s=1,2} P^{(2j)}_{(is)}X^{(2j)}_{(is)}$	
3.15(b)	$V(\cdot,\cdot,(2j))p^{(1j)}_k = \sum_{i\in G}\sum_{s=1,2} V(i,s,(2j))p^{(2j)}_{(is)}, \quad j = 1,\ldots,h$	h

Table 1.3
(continued)

Identifier	Equation	No.
	Wage determination	
3.16(a)	$P^{(1j)}_{(g+1,1)} = (\mathrm{CPI})F^{(1j)}_{(g+1,1)}F_{(g+1,1)}$	
3.16(b)	$p^{(1j)}_{(g+1,1)} = \mathrm{cpi} + f^{(1j)}_{(g+1,1)} + f_{(g+1,1)},\quad j = 1,\ldots,h$	h
	Consumer price index	
3.17(a)	$\mathrm{CPI} = \prod_{i\in G}\prod_{s=1,2}\left(P^{(3)}_{(is)}\right)^{\overline{V}(i,s,3)/\overline{V}(\cdot,\cdot,(3))}$	
3.17(b)	$\mathrm{cpi} = \sum_{i\in G}\sum_{s=1,2}\left(\overline{V}(i,s,(3))/\overline{V}(\cdot,\cdot,(3))\right)p^{(3)}_{(is)}$	1
	Tax rates on sales to households	
3.18(a)	$T(i,s,(3)) = T_b(i,\cdot,(3))F_t(3)$	
3.18(b)	$t(i,s,(3)) = t_b(i,\cdot,(3)) + f_t(3),\quad i = 1,\ldots,g,\ s = 1,2$	$2g$
	Ratio of real investment to real consumption	
3.19(a)	$I_{\mathrm{R}}/C_{\mathrm{R}} = \mathrm{FIC}$	
3.19(b)	$i_{\mathrm{R}}/c_{\mathrm{R}} = \mathrm{fic}$	1
	Other equations defining	
• GDP, real GDP, price deflator for GDP		3
• Real consumption (the definition of nominal consumption is implied by (3.3) and the price deflator for consumption is defined in (3.17))		1
• Investment, real investment, price deflator for investment		3
• Absorption (i.e. gross national expenditure), real absorption, price deflator for absorption		3
• Supplies of domestic commodities and volumes of imports by commodity		$2g$
• Total employment, total usage of capital (rental-weighted sum of industry usage)		2
• Total values of imports (c.i.f) and exports (f.o.b.), and the balance of trade		3
• Indexes of import prices (c.i.f.) and export prices (f.o.b.), and the terms of trade		3
• Total tax collections, total collection of consumer taxes, total collection of tariff revenue		3
• Total tax collections in real terms (deflated by the price of absorption)		1
• Ratio of economy-wide average wage rate to average rental per unit of capital		1

Total number of equations is $4g^2h + 3g^2 + 11gh + 14g + 8h + 25$

Table 1.4
Variables in the illustrative model*

Variables	Index ranges	Description	No.
$x^{(u)}_{(is)}$ and $p^{(u)}_{(is)}$	$(u) = (3), (4)$ and (kj) for $k = 1, 2$ and $j = 1, \ldots, h$. If $(u) = (4)$ then $s = 1$; if $(u) \neq (4)$ then $s = 1, 2$. If $(u) = (1j)$ then $i = 1, \ldots, g+1$; if $(u) \neq (1j)$ then $i = 1, \ldots, g$.	Demand by user (u) for good or primary factor (is); and price paid by (u) for (is)	$6g + 8gh + 4h$
$x^{(u)}_{(i\cdot)}$	$u = (3)$ and (kj) for $k = 1, 2$ and $j = 1, \ldots, h$. If $(u) = (1j)$ then $i = 1, \ldots, g+1$; if $(u) \neq (1j)$ then $i = 1, \ldots, g$.	Demand for composite good or primary factor i by user (u)	$g + 2gh + h$
$a^{(1j)}_{(g+1,s)}$	$j = 1, \ldots, h$, $s = 1, 2$.	Primary-factor saving technological changes	$2h$
c		Total expenditure by households	1
q		Number of households	1
$p^{(3)}_{(i\cdot)}$	$i = 1, \ldots, g$.	Price to households of composite goods	g
$z^{(u)}$	$(u) = (kj)$ for $k = 1, 2$ and $j = 1, \ldots, h$.	Activity levels: current production $(k = 1)$ and investment $(k = 2)$ by industry	$2h$
$f^{(4)}_{(i1)}$	$i = 1, \ldots, g$.	Shift in foreign demand curves	g
e		Exchange rate (\$foreigh/\$domestic)	1
$x^{(is)(u)}_{(r1)}$	$r, i = 1, \ldots, g$. $(u) = (3), (4)$ and (kj) for $k = 1, 2$ and $j = 1, \ldots, h$. If $(u) = (4)$ then $s = 1$; if $(u) \neq (4)$ then $s = 1, 2$.	Demand for commodity $(r1)$ to be used as a margin to facilitate the flow of (is) to (u)	$4g^2h + 3g^2$
$x^{(oj)}_{(i1)}$	$i = 1, \ldots, g$; $j = 1, \ldots, h$.	Output of domestic good i by industry j	gh
$p^{(o)}_{(is)}$	$i = 1, \ldots, g$; $s = 1, 2$.	Basic price of good i from source s	$2g$
$p^{(w)}_{(i2)}$	$i = 1, \ldots, g$.	Foreigh-currency c.i.f. price of imported commodity i	g
$t^{(o)}_{(i2)}$	$i = 1, \ldots, g$.	Power of the tariff on imports of i	g

Table 1.4
(continued)

Variables	Index ranges	Description	No.
$t(i, s, (u))$	$i = 1, \ldots, g$. $(u) = (3)$, (4) and (kj) for $k = 1, 2$ and $j = 1, \ldots, h$. If $(u) = (4)$ then $s = 1$; if $(u) \neq (4)$ then $s = 1, 2$.	Power of the tax on sales of commodity (is) to user (u). The power of a tax is 1 plus the rate of the tax	$4gh + 3g$
$f_k^{(j)}$	$j = 1, \ldots, h$.	Industry-specific capital shift terms	h
f_k		Capital shift term	1
$x_{(g+1,2)}^{(1j)}(1)$	$j = 1, \ldots, h$.	Capital stock in industry j at the end of the year, i.e., capital stock available for use in the next year	h
$p_k^{(1j)}$		Cost of constructing a unit of capital for industry j	h
$f_{(g+1,1)}^{(1j)}$	$j = 1, \ldots, h$.	Industry-specific wage shift term	h
$f_{(g+1,1)}$		Wage shift term (often the real wage rate)	1
cpi		Consumer price index	1
$t_b(i, \cdot, (3))$	$i = 1, \ldots, g$.	Base value of power of consumer tax on good i, both domestic and imported	g
$f_t(3)$		Shift term allowing uniform percentage increase in powers of consumer taxes	1
i_{R}		Real aggregate investment	1
c_{R}		Real aggregate consumption	1
fic		Ratio of real investment to real consumption	1
Other	See list at the end of Table 1.3. Note that real investment and real consumption have already appeared earlier in this table.		$21 + 2g$
Total number of variables: $4g^2h + 3g^2 + 15gh + 19g + 13h + 31$			

* We list the variables only in their percentage-change form, i.e., in the lower-case notation in which they appear in the (b)-system of Table 1.3.

Table 1.5
Other notation used in the equations of the illustrative model

Symbol	Appears in equation:	Description
$\Psi^{(u)}_{(is)}$	3.1(a), 3.2(a), 3.8(a)	Functions determining composition of composite commodities and primary factors, and composition of industry outputs. They are the outcome of CES-cost-minimizing and CET-revenue-maximizing problems
$\sigma^{(u)}_i$	3.1(b), 3.2(b)	Parameter: elasticity of substitution for user (u) between alternative sources of commodity or factor i
$\sigma^{(0j)}$	3.8(b)	Parameter: elasticity of transformation in industry j between outputs of different commodities
$V(i, t, (u))$	3.1(b), 3.2(b), 3.10(b), 3.12(b), 3.15(b)	Input–output flow: purchasers' value of good or factor i from source t used by user (u)
$V(i, \cdot, (u))$	3.1(b), 3.2(b), 3.3(b), 3.4(b), 3.15(b)	Input–output flow: $V(i, s, (u))$ summed over s
$V(\cdot, \cdot, (u))$	3.15(b)	Input–output flow: $V(i, s, (u))$ summed over i and s
γ_i	3.3	Parameter: subsistence parameter in linear expenditure system
β_i	3.3	Parameter: marginal budget shares in linear expenditure system
$A^{(u)}_i$	3.5(a)	Parameter: demand for composite i per unit of activity by user (u)
η_i	3.6	Parameter: foreign elasticity of demand
$A^{(is)(u)}_{(r1)}$	3.7(a), 3.12(a)	Parameter: use of $(r1)$ as a margin per unit of flow of (is) to user (u)
$Y(t, j)$	3.8(b), 3.9(b)	Input–output flow: basic value of output of domestic good t by industry j
$Y(\cdot, j)$	3.8(b)	Input–output flow: sum of $Y(t, j)$ over t, i.e., basic value of output by industry j
$B(t, s, (u))$	3.9(b), 3.12(b)	Input–output flow: basic value of (ts) used by (u)
$M(t, i, s, (u))$	3.9(b), 3.12(b)	Input–output flow: basic value of domestic good t used as a margin to facilitate the flow of (is) to (u)
$T(i, s, (u))$	3.12(b)	Input–output flow: collection of taxes on the sale of (is) to (u)
δ_j	3.13, 3.14	Parameter: rate of depreciation of industry j's capital
α_j	3.13	Parameter: sensitivity of capital growth to rates of return
$\left.\begin{array}{l}\overline{V}(i, s, (3))\\ \overline{V}(\cdot, \cdot, (3))\end{array}\right]$	3.17	Parameters: initial values of $V(i, s, (3))$ and $V(\cdot, \cdot, (3))$

Table 1.5
(continued)

Symbol	Appears in equation:	Description
G	3.3, 3.8(b), 3.9, 3.12, 3.15, 3.17	Set: $\{1, 2, \ldots, g\}$, g is the number of composite goods
G^*	3.10	Set: $\{1, 2, \ldots, g+1\}$, $g+1$ is the number of composite goods and primary factors
H	3.9	Set: $\{1, \ldots, h\}$, h is the number of industries
U	3.9	Set: $\{(3), (4), (kj)$ for $k = 1, 2$ and $j = 1, \ldots, h\}$
U^*	3.9	Set: $\{(3), (kj)$ for $k = 1, 2$ and $j = 1, \ldots, h\}$

Eqs 3.5. The second level allows CES substitution between sources in the formation of composites, leading to the source-specific demand Eqs 3.1 and 3.2.[42]

The household is assumed to maximise a nested utility function subject to an aggregate-expenditure constraint. The top level of the utility function, describing preferences for composite commodities, is a Klein-Rubin (1948–1949)[43] function, leading to the linear-expenditure system, Eq. 3.3. The second level allows for CES substitution between sources of commodities as was the case for inputs to current production and capital formation. The consequent source-specific demand equations are included in 3.1.

Equation 3.6 specifies constant-elasticity foreign demand curves for exports.

Equation 3.7 specifies demands for margins services.[44] We assume that margins must be used in fixed proportions to the basic flows which they facilitate.

We assume that producers choose their output mixes, given their activity levels and output prices, to maximise revenue subject to CET transformation frontiers [Powell and Gruen (1968)]. This leads to the supply functions 3.8.[45]

Equation 3.9 imposes market clearing for domestic commodities. On the LHS it sums over producers of commodities and on the RHS over uses of commodities. Direct and margin uses are included.

[42]We include in the factor-demand Eqs 3.2 technology coefficients which we use to introduce labour-saving technical change in the simulations reported in Section 3.5(c). In models for real-world applications rather than illustration, we include a wider range of technology and taste coefficients.

[43]See also Geary (1950–1951) and Stone (1954).

[44]We denote the type and source of margin service in the double subscript, assuming by the value "1" for the second subscript that all margin services are domestically sourced. In our data (Table 1.2), only commodity 3 is used as a margin service. The basic flow which the margin service facilitates is specified by the triple superscript, the three components of which show, in turn, the basic flow's commodity, source and user.

[45]The "o" in the superscripts in these equations denotes output or, in the case of prices, basic values. The second component of the superscript in the symbols denoting outputs and activity levels specifies the producing industry.

Equations 3.10–3.12 constitute the model's pricing system. Equation 3.10 relates basic values to unit costs. In view of the constant returns to scale assumed in the model's production functions, output and input quantities can be eliminated from the percentage-change form leaving a relationship between percentage changes in the basic prices of outputs and percentage changes in the purchasers' prices of inputs. Equation 3.11 defines the basic prices of imports as their c.i.f., duty-paid prices. In 3.12 purchasers' prices are defined as the sums of basic values, margins costs and commodity taxes.

Equations 3.13 and 3.14 have been discussed already in Section 2.3 (see (2.65) and (2.68) which together correspond to 3.13(a); (2.69) which corresponds to 3.13(b); (2.56) which corresponds to 3.14(a); and (2.70) which corresponds to 3.14(b)). Notice that in 3.2(a) we used $X^{(1j)}_{(g+1,2)}$ to represent the flow of capital services to industry j. In 3.13(a) and 3.14(a), $X^{(1j)}_{(g+1,2)}$ is the capital stock in use in industry j. We assume that each unit of capital stock in existence at the beginning of a year is capable of providing one unit of capital services during the year and that the available capital services are always fully used. Hence, in our notation we need not distinguish between the capital stock and capital services.

Equation 3.15 specifies the unit costs of constructing capital. With constant returns to scale in the capital production functions, quantities can be eliminated from the percentage-change form, leaving percentage changes in the unit cost as functions of percentage changes in input prices only (3.15(b)).

Equation 3.16 allows for indexation of nominal wage rates to the CPI, defined by 3.17. Together, the shift variables (F) in 3.16 represent industry-specific real wage rates. In the percentage-change form, $f_{(g+1,1)}$ can be used to introduce shifts in the overall real wage rate and the $f^{(1i)}_{(g+1,1)}$ can accommodate changes in industrial wage relativities.

Equation 3.18 allows us flexibility in setting rates of commodity taxes on households. This is required in the revenue neutral tariff-reform simulations which we report in Section 3.5(b). Similar equations could be added to allow flexibility in the treatment of other tax rates if required.

Equation 3.19 is included to allow us to exogenize the ratio of real aggregate investment to real aggregate consumption, an option which we choose in the short-run comparative-static simulations reported in Sections 3.5(a) and (b).

As well as the equations which we have set out in detail in Table 3.2, the model includes definitions of the macroeconomic variables listed at the end of the table. The definitions of these variables are orthodox and straightforward. We omit the details for the sake of brevity.

3.3. *Coefficients, parameters, zero problems and initial solution*

To form a computable model, we must assign values to the parameters appearing in the system of equations that we choose to use. If we choose a differential system (the (b)-system in Table 1.3), then we must also give initial values to the coefficients. These are cost shares, sales shares or other functions of the model's variables. In each step of a Johansen/Euler computation, the coefficients are held constant but they are moved from step to step.

In this subsection we explain how we assigned values to the parameters and coefficients in the differential form of the illustrative model. Then we consider briefly the problem of setting parameters for the (a)-system. In our applications of the illustrative model, we use only the (b)-system – we adopt the Johansen/Euler approach. Nevertheless, it is still worthwhile to look at the (a)-system. This will allow us to illustrate a key point of Section 2: that for CGE models we almost always have a readily available initial solution or at least a readily available solution for an initial year.

Most of the information required to implement the (b)-system of our illustrative model is in the input–output data in Table 1.2. For example, a coefficient appearing in Eq. 3.1(b) is $V(1,1,(12))/V(1,\cdot,(12))$, i.e., the share of domestically produced good 1 in industry 2's expenditure on composite good 1 to be used as an input to current production. The initial value of this share (for the first step of a Johansen/Euler computation) can be calculated from Table 1.2 as

$$\left[\frac{V(1,1,(12))}{V(1,\cdot,(12))}\right]_{\text{initial}} = \frac{20+4+4}{(20+4+4)+(5+1+1)} = 0.8.$$

Similar coefficients, each consisting of the share of a sourced input in the user's expenditure on the relevant composite, appear in Eqs 3.2(b), 3.4(b) and 3.17(b).[46]

Equation 3.8(b) contains a different type of share coefficient: $Y(t,j)/Y(\cdot,j)$. This is the share of the *basic* value of industry j's output of good t in the basic value of industry j's total output. From Table 1.2, we see, for example, that the initial value of $Y(2,2)/Y(\cdot,2) = 90/110$.

Rather than being input–output shares, many of the coefficients in the (b)-system of Table 1.3 are input–output levels. These are purchasers' values of flows (the Vs in 3.3(b), 3.10(b), 3.12(b) and 3.15(b)); basic values of flows (the Bs and Ys in 3.9(b), 3.10(b) and 3.12(b)); tax collections (the Ts in 3.12(b)); and margins flows (the Ms in 3.9(b) and 3.12(b)). The initial values of all these can be read directly from Table 1.2 or calculated by a small number of additions.

Equation 3.3(b) contains the coefficient $P^{(3)}_{(i\cdot)}Q$. Setting the initial value of this coefficient requires a decision about units. The approach we use is to assume that quantity units for all composite commodities are chosen so that their initial purchasers' prices are unity

$$\text{i.e., } \left[P^{(u)}_{(i\cdot)}\right]_{\text{initial}} = 1, \quad \text{for all} \quad u \in U,\ i = 1,\dots,g \text{ and } s = 1,2. \qquad (3.20)^{47}$$

[46]In 3.17(b), the share coefficients are weights in the consumer price index. They are not moved in a multi-step Johansen/Euler computation.

We define population units so that the initial value of Q is also 1.

The final group of coefficients are those in the differential forms of the investment and capital accumulation Eqs 3.13(b) and 3.14(b). The data, beyond those in input–output tables, required for setting the initial values of such coefficients are depreciation rates and measures of rates of return. In our illustrative model, we assume that all depreciation rates (the parameters δ_j) are 10 per cent and that net rates of return (i.e., rental/capital-value ratios less depreciation rates) are 5 per cent. We assume that capital units are chosen so that $P_k^{(1j)}$ has an initial value of 1 for all j. Then for industry 1, we have

$$\left\{\left[P_{(g+1,2)}^{(11)} X_{(g+1,2)}^{(11)} / P_k^{(11)} X_{(g+1,2)}^{(11)}\right] - \delta_j\right\}_{\text{initial}} = 0.05.$$

From Table 1.2, we see that the initial gross earnings of capital in industry 1 are 11. Hence,

$$\left(11/X_{(g+1,2)}^{(11)}\right)_{\text{initial}} - 0.10 = 0.05$$

giving

$$\left(X_{(g+1,2)}^{(11)}\right)_{\text{initial}} = 73.33.$$

Similarly, we find that capital stocks available in the initial year for industries 2 and 3 are 53.33 and 193.33. We can now compute the initial values for the rental rates $(P_{(g+1,2)}^{(1j)})$. They are 11/73.33, 8/53.33 and 29/193.33, i.e., 0.15 for all three industries. From here, we find that the initial value of the coefficient in square brackets in 3.13(b) is 0.143 for all industries.

For Eq. 3.14(b), we have already found the initial values of the coefficients $X_{(g+1,2)}^{(1j)}$. With the initial values of $P_k^{(1j)}$ being 1, we can read the initial investment levels $(Z^{(2j)})$ from Table 1.2. They are 10.63, 5.32 and 26.05. Now we can calculate the initial quantity of next year's capital stock for industry 1 as

$$\left(X_{(g+1,2)}^{(11)}(1)\right)_{\text{initial}} = 73.33(1 - 0.10) + 10.63 = 76.63.$$

Similarly, we find that next year's capital stocks for industries 2 and 3 are, initially, 53.32 and 200.05.

[47] We usually assume that the quantity units of all sourced commodities are chosen so that their basic prices are initially 1. Can (3.20) then be satisfied without violating conditions such as 3.4(a)? For each user, u, composite i is a CES combination of $X_{(i1)}^{(u)}$ and $X_{(i2)}^{(u)}$, with each of the CES functions having its own "A" parameter [see (2.36)]. By suitable choices for the values of these $A_{(i\cdot)}^{(u)}$s we can ensure that (3.20) is fulfilled while still satisfying the condition: composite price times composite quantity equals the sum of expenditures on sourced commodities.

Most of the parameters in the differential representation of our model are either substitution or transformation elasticities [the σs in 3.1(b), 3.2(b) and 3.8(b)]. Ideally, these should be estimated econometrically. In practice, they are often assigned values based on literature search. For our illustrative model we chose values typical of those estimated for the ORANI model [Dixon, Parmenter, Sutton and Vincent (1982, Chapter 4)]. We set the Armington elasticities[48] (i.e., the elasticities describing domestic/import substitution, $\sigma_i^{(u)}$ for all (u) and for $i = 1, \ldots, g$) at 2; the labour/capital elasticities ($\sigma_{g+1}^{(1j)}$ for $j = 1, \ldots, h$) at 0.5; and the transformation elasticities ($\sigma^{(0j)}$, $j = 1, \ldots, h$) at 0.5.

In Eq. 3.6(b) we set the foreign demand elasticities (η_i) at 5 for commodity 1 (the main export commodity) and at 20 for all other commodities. Again these numbers are typical of those used in Australia's ORANI model. The ORANI foreign demand elasticities are consistent with Australia's export volumes having a minor influence on world prices of Australia's main exports (agricultural and mineral products), and having barely any influence on all other world prices.

In the household demand Eqs, 3.3(b), the parameters are marginal budget shares (β_i) and subsistence parameters (γ_i). In our illustrative model, the four βs were set at 0.0785, 0.5446, 0.3141 and 0.0628, and the four γs were set at 6.76, 66.85, 7.04 and 5.41. These parameter settings were chosen to give typical values for household expenditure elasticities and for the ratio of subsistence expenditure to total expenditure.[49] In conjunction with the data in Table 1.2, our chosen parameter values imply initial values for the four expenditure elasticities of 1, 0.84, 1.5 and 1, and for the subsistence ratio of 0.45.

The last set of parameters are the α_js in the investment Eqs, 3.13(b). These control the sensitivity of capital growth in each industry to variations in rates of return. Guided by our experience with ORANI, we assigned all of the α_js in the illustrative model the values of 2.0.

With initial values assigned to the coefficients and with the parameters set, we are almost ready to compute Johansen/Euler solutions for our illustrative model. One minor practical problem remains: zero input–output flows. These can cause difficulties in equations such as 3.12(b). Assume that $p_{(is)}^{(u)}$ is endogenous. Then if $V(i, s, (u))$ is zero, Johansen/Euler computations will fail because the value of $p_{(is)}^{(u)}$ will be indeterminate. One cure is to modify the input–output database by replacing zero flows with very small numbers. Another is to modify equations such as 3.12(b) to read

$$(V(i, s, (u)) + \text{TINY})p_{(is)}^{(u)} = \text{etc.}$$

where TINY is a parameter assigned a very small value.

[48]This type of elasticity is named in recognition of the contribution of Armington (1969, 1970).

[49]Lluch, Powell and Williams (1977) is a valuable source of estimates of expenditure elasticities and subsistence ratios for many countries.

To complete this subsection, we consider briefly how to assign values to the parameters of the (a)-system in Table 1.3.[50] We start with Eq. 3.1(a) for the cases where $i = 1$, $s = 1$ and 2 and $(u) = (12)$, that is, we look at the parameters of the demand functions for domestic and imported good 1 to be used as inputs to current production in industry 2.

From (2.38), we see that these demand functions have the form

$$X^{(12)}_{(1s)} = \left(X^{(12)}_{(1\cdot)}/A^{(12)}_{(1\cdot)}\right)\left[\sum_{t=1,2} b^{(12)}_{(1t)}\left(\frac{P^{(12)}_{(1t)}b^{(12)}_{(1s)}}{P^{(12)}_{(1s)}b^{(12)}_{(1t)}}\right)^{\rho^{(12)}_1/\left(1+\rho^{(12)}_1\right)}\right]^{1/\rho^{(12)}_1}, \quad (3.21)$$

$$s = 1, 2.$$

We have already set $\sigma^{(1j)}_1$ at 2, implying that $\rho^{(12)}_1$ is -0.5. As indicated in footnote 47, we assume that all basic prices are initially 1. Now from Table 1.2 we find that

$$\left(P^{(12)}_{(11)}\right)_{\text{initial}} = (20 + 4 + 4)/20 = 1.4$$

and

$$\left(P^{(12)}_{(12)}\right)_{\text{initial}} = (5 + 1 + 1)/5 = 1.4.$$

With our convention that the purchasers' prices of composites are initially 1, we have

$$\left(X^{(12)}_{(1\cdot)}\right)_{\text{initial}} = 28 + 7 = 35.$$

Remembering that $b^{(12)}_{(11)}$ and $b^{(12)}_{(12)}$ sum to 1, we can solve (3.21) to obtain the values of the three parameters, $b^{(12)}_{(11)}$, $b^{(12)}_{(12)}$ and $A^{(12)}_{(1\cdot)}$:

$$20 = \left(35/A^{(12)}_{(1\cdot)}\right)\left(\sum_{t=1,2} b^{(12)}_{(1t)}\left(\frac{1.4b^{(12)}_{(11)}}{1.4b^{(12)}_{(1t)}}\right)^{-1}\right)^{-2}$$

and

$$5 = \left(35/A^{(12)}_{(1\cdot)}\right)\left(\sum_{t=1,2} b^{(12)}_{(1t)}\left(\frac{1.4b^{(12)}_{(12)}}{1.4b^{(12)}_{(1t)}}\right)^{-1}\right)^{-2}$$

[50]This is what many CGE modellers refer to as calibration, see for example, Shoven and Whalley (1984).

giving

$$b^{(12)}_{(11)} = 0.666, \quad b^{(12)}_{(12)} = 0.333$$

and

$$A^{(12)}_{(1\cdot)} = 2.520.$$

By similar methods, we could assign values to all the other parameters in the (a)-system. However, we use only the (b)-system in our computations. Consequently, we will not go any further with the process of assigning parameter values in the (a)-system. Our main reason for considering the topic at all is its relationship to the idea, emphasized in Section 2, that the input–output database for most CGE models provides an initial solution.

Given the way we have set the parameters in (3.21), it is clear that the equation is satisfied by the initial values for $X^{(12)}_{(11)}$, $X^{(12)}_{(12)}$, $X^{(12)}_{(1\cdot)}$, $P^{(12)}_{(11)}$ and $P^{(12)}_{(12)}$ implied by our input–output data in combination with our conventions on quantity units and initial prices. Similarly, all the other equations in the (a)-system are satisfied by an initial solution generated in a straightforward way from the database. This is an implication of the way the parameter values and the initial values of some of the shift variables[51] are determined.

3.4. Closure of the illustrative model

From Tables 1.3 and 1.4 it can be seen that the illustrative model contains $(4gh + 5g + 5h + 6)$ more variables than equations.[52] Hence, to close the model this number of variables must be set exogenously. A strength of working with the linearized percentage-change version of the model and the GEMPACK software is that it is easy to run simulations under a variety of closures. In Section 3.5 we report four different simulations, each with a different closure. The closures are listed in Table 1.6. As we will demonstrate in explaining the simulations, by changing the closure we are able to use the model in many different modes.

[51] For example, from Table 1.2 we have $(X^{(4)}_{(11)})_{\text{initial}} = 35$ and $P^{(4)}_{(11)} = 51/35 = 1.457$. The parameter η_1 is set at 5. If we set the initial value of the exchange rate at 1, then we can satisfy 3.6(a) by setting the initial value of the shift variable $F^{(4)}_{(11)}$ at $(51/35)(35)^{0.2} = 2.97$.

[52] As shown by Table 1.2, in our data g (the number of commodities) is 4 and h (the number of industries) is 3. Hence, the excess of variables over equations in the implemented version of the model is 89.

Table 1.6
Numbers of exogenous variables in the simulations reported in Tables 1.7 to 1.9

	Variable	Standard short run (Table 1.7 cols 1, 2)	Short run for macro package (Table 1.7 col. 3)	Revenue-neutral short run (Table 1.8)	Forecasting (Table 1.9)
	Always exogenous				
q	number of households	1	1	1	$1^{\#}$
$a^{(1j)}_{(g+1,s)}$	tech. change	$2h$	$2h$	$2h$	$2h^{\#}$
$t^{(0)}_{(i2)}$	tariffs	g	g	g	$g^{\#}$
$t(i,s,(kj))$	sales taxes on current and capital inputs	$4gh$	$4gh$	$4gh$	$4gh$
$f^{(4)}_{(i1)}$	export demand shifts	g	g	g	$g^{\#}$
$p^{(w)}_{(i2)}$	import prices (cif, $foreign)	g	g	g	$g^{\#}$
$t_b(i,\cdot,(3))$	base powers of consumer tax	g	g	g	g
	Macro targets and instruments				
$f_{(g+1,1)}$	wage shift	$1^{\#}$	end	1	$1^{\#}$
c_{R}	real consumption	$1^{\#}$	end	1	$1^{\#}$
no symbol	total employment	end†	$1^{\#}$	end	end
no symbol	balance of trade as share of GDP	end	1	end	end
	Revenue constraint and wage setting				
no symbol	real tax collection (CPI deflated)	end	end	1	end
$f_t(3)$	consumer tax shifter	1	1	end	1
$p^{(1j)}_{(g+1,1)}$	nominal wage rates by industry	end	end	h	end
$f^{(1j)}_{(g+1,1)}$	industry specific wage shifters	h	h	end	h
	Short-run comparative statics vs forecasting				
$x^{(1j)}_{(g+1,1)}$	capital available for use in current production	h	h	h	$h^{\#}$
i_{R}	real investment	end	end	end	$1^{\#}$
fic	investment/consumption	1	1	1	end
$f_k^{(j)}$	industry-specific capital shift terms	h	h	h	h
f_k	capital shift term	end	end	end	end
e	exchange rate	1	1	1	end
cpi	consumer price index	end	end	end	$1^{\#}$

Table 1.6
(continued)

Variable		Standard short run (Table 1.7 cols 1, 2)	Short run for macro package (Table 1.7 col. 3)	Revenue-neutral short run (Table 1.8)	Forecasting (Table 1.9)
$x^{(4)}_{(i1)}$ or $t(i,1,(4))$	export volume or export tax	$g - b^*$ b	$g - b$ b	$g - b$ b	$g^{\#}$ end

Total number of exographic variables in all columns is $4gh + 5g + 5h + 6$

† The entry "end" in the table means endogenous.
# These variables are shocked in the relevant simulations.
* In the simulations reported in Tables 1.7 and 1.8, b is 1. Export volumes are exogenous for commodities 2, 3 and 4 and the export tax rate is exogenous only for commodity 1.

Table 1.7
Short-run comparative-static effects of policies for employment stimulation: 1-step Johansen/Euler solutions (percentage changes)

	Variable	Real wage cut	Demand expansion	Macro package
1.	Real wage rate	**−1.00**	**0.00**	−3.67
2.	Real aggregate absorption	**0.00**	**1.00**	3.09
3.	Aggregate employment	0.98	0.45	**5.00**
4.	Wage/rental ratio	−1.39	−0.88	−9.96
5.	Terms of trade	−0.34	0.22	−0.58
6.	GDP price index	−0.77	0.64	0.87
7.	Consumer price index	−0.68	0.58	−0.71
8.	Exports of commodity 1	2.14	−1.36	3.66
	Activity levels			
9.	Sector 1	1.56	−0.64	3.79
10.	Sector 2	0.19	0.61	2.59
11.	Sector 3	0.45	0.57	3.42
12.	100 (Balance of trade)/GDP	0.47	−0.56	**0.00**
13.	Import volume index	−0.31	1.12	2.34

Note: Numbers in bold type are exogenous

3.5. Simulations

This section contains some simulations results from the illustrative model, computed using GEMPACK [Codsi and Pearson (1988) and Pearson (1988)] applied to the b-system in Table 1.3. We present results of three types: comparative-static results

Table 1.8
Simulations of the effects of abolishing tariffs (percentage change)

	Variable	1-step	2-step	1, 2-step extrapolation	8, 16, 32-step extrapolation
1.	Tariff revenue	−94.92	−97.30	−99.69	−99.99
2.	Revenue from taxes on h'holds	59.01	60.79	62.57	62.88
3.	Import volume index	5.40	5.82	6.25	6.32
4.	Exports of commodity 1	12.09	12.54	13.00	13.02
5.	Terms of trade	−1.93	−1.94	−1.95	−1.95
6.	(Balance of trade)/GDP	0.01	0.01	0.00	0.00
	Activity levels				
7.	Sector 1	1.22	1.24	1.27	1.26
8.	Sector 2	0.58	0.62	0.65	0.65
9.	Sector 3	−0.27	−0.25	−0.24	−0.23

computed by the 1-step Johansen/Euler method (Table 1.7), comparative-static results computed by multi-step Johansen/Euler procedures (Table 1.8) and five-year forecasts comprising annual 1-step Johansen/Euler computations (Table 1.9). Closures for the simulations are listed in Table 1.6.

(a) Structural effects of macro employment strategies: 1-step Johansen/Euler simulations

As an example of comparative-static simulations, we have used the illustrative model to project the effects of alternative strategies for employment generation. We used ORANI-model results like these as inputs to the debate in Australia about macroeconomic policy, concentrating in particular on the structural effects of different strategies [Dixon, Powell and Parmenter (1979)]. As we illustrate here, our conclusions were that a combination of demand stimulation and wage moderation would increase employment without adverse consequences for the trade balance and without disruption to the structure of the economy. Corden and Dixon (1980) explore the prospects for implementing such a strategy by use of wage-tax bargains with the trade unions. Analysis such as this underpinned the macroeconomic policy adopted by the Australian government through the middle 1980s.

Selected results from our illustrative macro-strategy simulations are reported in Table 1.7. The first column shows the effects of a one-per-cent cut in the CPI-deflated wage rate with real domestic absorption held fixed. The second shows the effects of a one-per-cent increase in real domestic absorption (consumption plus investment) with the CPI-deflated real wage rate held fixed. In the third column are results from a simulation in which we computed percentage changes in the real wage rate and

Table 1.9
Five-year forecasts: Annual percentage growth rates

Variable	Year 1	Year 2	Year 3	Year 4	Year 5
(a) Exogenous scenario					
Vertical shifts in export demand schedules*					
commodity 1	1.00	10.00	11.00	2.00	2.00
commodity 2	4.00	4.00	4.00	4.00	4.00
World prices of imports					
commodity 1	4.00	4.00	4.00	4.00	4.00
commodity 2	4.00	4.00	4.00	4.00	4.00
commodity 4	4.00	4.00	4.00	4.00	4.00
Export volumes					
commodity 1	3.00	4.50	3.00	2.50	2.00
commodity 2	10.00	11.50	10.00	8.00	7.00
Real wage rate	0.80	0.80	1.50	1.50	1.00
Labour-saving tech. change					
sector 1	2.00	4.00	2.00	1.50	1.00
sector 2	2.00	4.00	2.00	1.50	1.00
sector 3	0.00	0.00	0.00	0.00	0.00
Aggregate real investment	2.00	7.20	6.80	0.00	−5.00
Aggregate real consumption	2.50	3.50	2.30	2.00	2.00
Powers of tariffs					
commodity 1	−1.00	−1.00	−1.00	0.00	0.00
commodity 2	−4.00	−4.00	−4.00	0.00	0.00
commodity 4	0.00	0.00	0.00	0.00	0.00
Consumer price index	2.90	4.10	3.90	3.00	3.00
Numbers of households	1.40	1.40	1.40	1.40	1.40
(b) Endogenous variables					
Terms of trade	−2.97	3.86	4.88	−2.04	−1.92
Wage/rental ratio	1.28	−2.51	1.73	4.73	4.75
Aggregate employment	2.15	3.58	2.31	1.31	0.87
Capital stock in use	3.13	2.96	3.50	3.94	3.41
Real GDP	2.77	4.24	3.08	2.35	1.73
Export volume index	4.42	6.03	4.54	3.71	3.17
Import volume index	2.97	5.44	4.43	1.47	0.01
Nominal devaluation	0.55	0.72	0.95	−0.50	0.07
GDP price index	2.02	5.13	5.30	2.37	2.42
Real devaluation	2.53	−0.41	−0.35	1.13	1.65

Table 1.9
(continued)

Variable	Year 1	Year 2	Year 3	Year 4	Year 5
Supplies of domestic goods					
commodity 1	3.12	4.35	3.05	2.72	2.27
commodity 2	2.40	3.95	2.98	2.70	2.00
commodity 3	2.90	4.38	3.23	2.39	1.76
Capital in use					
sector 1	4.50	3.69	3.32	3.57	3.38
sector 2	−0.02	1.09	1.98	3.16	3.01
sector 3	3.47	3.21	3.99	4.29	3.53
Activity levels					
sector 1	3.98	4.82	3.20	2.87	2.52
sector 2	1.62	3.52	2.85	2.58	1.76
sector 3	2.90	4.38	3.23	2.39	1.76
Employment					
sector 1	1.72	1.37	1.13	1.02	1.10
sector 2	0.57	0.93	1.39	0.73	0.01
sector 3	2.65	4.90	2.89	1.53	0.98
Capital growth through year					
sector 1	3.69	3.32	3.57	3.38	2.63
sector 2	1.09	1.98	3.16	3.01	1.81
sector 3	3.21	3.99	4.29	3.53	2.34
Investment					
sector 1	−1.28	0.90	5.25	2.06	−2.37
sector 2	11.20	9.13	12.02	1.99	−6.51
sector 3	1.46	9.27	6.21	−1.22	−5.63

* These are the percentage changes in the world prices of exports which would occur in the absence of changes in export volumes.

real domestic absorption which together give a five-per-cent increase in employment with no change in the ratio of the trade balance to the GDP. The results are reported as percentage changes in the variables. These are to be interpreted as percentage differences between the values which the variables would take in some target year if the shocks had been applied and the values which the variables would take in the same target year in the absence of the shocks.

Our experience suggests that with shocks of the sizes used for these simulations linearization errors arising in 1-step Johansen/Euler solutions are not serious. Hence, we relied on the 1-step method in compiling Table 1.7.

Short-run comparative-static closures. Columns 1 and 2 of Table 1.6 give the closures used for the simulations reported in Table 1.7. The first group of variables, which are exogenous in all of our simulations, begins with the number of households, the rates of factor-saving technical progress, and the rates of duty on imports and of commodity taxes on inputs to current production and capital formation. The next two variables allow us to impose on the single-country model shifts in export demand schedules and in the foreign-currency prices of imports. The last variable in the group is from the RHS of the Eq. 3.18 which determines our setting of the rates of commodity taxes on household consumption.

The treatment of the next block of variables (headed "Macro targets and instruments") distinguishes the simulations reported in the first two columns of Table 1.7 from that reported in its last column. In the first two columns, we shock "instruments" (the overall real wage rate, $f_{(g+1,1)}$, and real aggregate consumption, c_{R}) and report the effects on "targets" (aggregate employment and the trade-balance/GDP ratio). In the final column we assign values to the targets and report the changes in the instruments required to attain these targets.

No revenue constraint is imposed in the simulations reported in Table 1.7. Hence, real tax-revenue is endogenous and the consumption-tax shift variable $f_t(3)$ is exogenous. Nominal wage rates $(p_{(g+1,1)}^{(1j)})$ are endogenous in all the simulations in Table 1.7 but sectoral wage relativities are held constant. To implement this, the sector-specific wage-shift variables $(f_{(g+1,1)}^{(1j)})$ are exogenous.

In the first two columns of Table 1.6, the exogenous-endogenous assignments of the variables included in the final block implement some other features of our short-run, comparative-static environment. The availability of capital in each sector is assumed to be unaffected by the shocks in the simulations reported in Table 1.7 – an orthodox short-run assumption. Hence, capital stocks $(x_{(g+1,2)}^{(1j)})$ are exogenous.

Aggregate investment (i_{R}) is formally endogenous, the exogeneity of the variable "fic" ensuring that real investment moves at the same (exogenous) rate as real consumption. The allocation of investment between sectors, reflecting movements in relative rates of return, is determined by Eqs 3.13 and 3.14. The shift variable $f_k^{(j)}$ is exogenous but the variable f_k is endogenous, adjusting to ensure that sector-specific investment changes are consistent with the movement in aggregate investment.

The nominal exchange rate (e) is the numeraire in the comparative-static simulations with domestic prices, including the CPI, endogenous.

In the comparative-static simulations, we set the rate of export tax on commodity 1, the main exported commodity, exogenously and allow the model to determine movements in the volume of exports. The basic price of domestically produced commodity 1 is then linked tightly to the world export price [cf., Eq. 3.12]. Exports of the other three commodities are exogenous, with their export-tax rates endogenous, breaking the link between movements in their domestic and world prices.[53]

Results. The results in row 3 of the first two columns of Table 1.7 show that, in the standard short-run closure, employment is generated both by a cut in the CPI-deflated wage rate (with aggregate real absorption held constant) and by an increase in aggregate real absorption (with the CPI-deflated wage rate held constant). Both shocks reduce the real wage from the employers' point of view, that is, they both reduce the ratio of the wage to the average rental price of capital (row 4).

In the case of the increase in absorption, the main mechanism which triggers the reduction in the employers' real wage is an increase in the terms of trade (row 5) which raises the GDP price index (row 6) relative to the CPI (row 7). The increase in absorption improves the terms of trade because it crowds out exports of commodity 1 (row 8), driving up its world price. Note that under the wage-cut shock exports expand and the terms of trade deteriorate. This reduces the GDP price index relative to the CPI, moderating the fall in the producers' real wage.

The elasticity of aggregate employment to the wage/rental ratio is greater for the wage-cut shock than for the demand-expansion shock. This is explained by differences in the compositional effects of the shocks. Relative to the wage cut, the demand expansion stimulates sector 2 and inhibits sector 1 (the main producer of the exportable commodity 1). Sector 2 is less labour-intensive than sectors 1 and 3.[54]

As is implicit in our discussion of the results so far, the balance-of-trade effects which accompany employment generation differ sharply between the two shocks (row 12). Because it reduces domestic costs relative to world prices, the wage cut stimulates exports (row 8) and inhibits imports (row 13), causing an increase in the ratio of the balance of trade to the GDP. Demand expansion has the opposite effect. It raises domestic costs relative to world prices, crowds out exports and stimulates imports. Hence, it causes a deterioration of the trade-balance/GDP ratio.

A package of the two policies could avoid balance-of-trade movements and produce a more balanced expansion of the economy. The computation of such a package is reported in the third column of Table 1.7. We computed the package via a closure switch in which aggregate employment and the trade-balance/GDP ratio replace the CPI-deflated wage rate and real aggregate consumption as exogenous variables. We then use the model to compute the changes in the wage rate and in consumption which are required to produce a 5 per cent increase in employment with no change in the trade-balance/GDP ratio. As can be seen from column 3 of Table 1.7, a wage cut of

[53]This expedient reflects our inadequate understanding of what determines export volumes for the economy's minor exports and a recognition that domestic prices of these products tend to move quite independently of world prices. Some care is needed to ensure that collection of the endogenous export taxes does not distort unduly a model's public-finance results.

[54]For a more extensive discussion of the employment effects of demand expansion in ORANI-style models, see Malakellis and Peter (1991).

3.67 per cent and an absorption increase of 3.09 per cent is the required combination.[55] This produces an expansion of the economy in which all three sectors participate quite evenly.

With results like these [reported in Dixon, Powell and Parmenter (1979)], we were able to counter the argument that in Australia's recessed economy of the late 1970s assistance measures targeted at particular industries were required. Our argument was that macroeconomic policy, suitably designed, would stimulate aggregate employment without leaving structural problems.

(b) Effects of tariff changes: Multi-step simulations

Another topic which we have analyzed via comparative-static simulations with ORANI is the short-run effects of reductions in tariffs on imports. The significance of this issue in the history of CGE modelling and as a policy issue in Australia is discussed in Section 4.

A closure suitable for short-run tariff simulations is given in the third column of Table 1.6. It is identical to the closure used for the simulations underlying columns 1 and 2 of Table 1.7 (see the first column of Table 1.6) except for the treatment of the variables listed in the section of Table 1.6 headed "Revenue constraint and wage setting". In computing the effects of tariff changes we impose a real tax-revenue constraint. By adjusting the consumption-tax rate, the model ensures that the tariff changes are revenue-neutral. Hence, the real tax-revenue variable is switched with the consumption-tax shifter ($f_t(3)$) as exogenous. We also choose to hold constant nominal, rather than real, wage rates. Hence, $p_{(g+1,1)}^{(1j)}$ is exogenous, with Eq. 3.16 playing no role other than to determine the value of $f_{(g+1,1)}^{(1j)}$.

Results of simulations of the abolition of tariffs under this closure are in Table 1.8. Given our database (Table 1.2), abolishing the tariffs requires percentage reductions in the powers of the tariff rates on the model's four commodities of, in turn, 16.00, 10.00, 0, 33.33. These shocks are quite large, raising doubts about the accuracy of 1-step Johansen/Euler solutions. Using Table 1.8 we can consider whether such doubts are justified. Column 1 contains 1-step results, column 2 contains results computed via a 2-step procedure with no extrapolation, column 3 contains results extrapolated by the Richardson method from the 1-step and 2-step solutions, and column 4 contains results extrapolated from 8-step, 16-step and 32-step solutions. We regard the results in column 4 as not significantly different from the full non-linear solution. This is confirmed by row 1 of the table which shows the percentage change in aggregate tariff revenue asymptoting to -100 as we increase the accuracy of the solution. With a negative relationship between the c.i.f. value of imports (the revenue base) and the tariff rates, it is clear that the 1-step solution will understate the revenue effects of

[55]Column 3 is just a weighted sum of columns 1 and 2 with the weights being 3.67 and 3.09 respectively.

reducing tariff rates. It also understates the revenue effects of the offsetting increase in the rate of taxes on consumption (row 2 of the table).

For all variables, the results in column 3 are close to those in column 4. This confirms our experience that, in most cases, use of the Johansen/Euler procedure with extrapolation allows very accurate solutions to be obtained with only modest computing effort. To produce column 3 required just two solutions to the model (a one-step solution and a two-step solution). The accuracy of even the 1-step solution in column 1 is such that the policy conclusions which could be drawn from it are not substantially different from those which could be drawn from the full non-linear solution.

The simulations suggest that abolishing tariffs would stimulate imports (row 3) but also exports (row 4). The increase in exports, given our assumptions about the elasticities of the world demand schedules, would erode the terms of trade. The sectoral effects (rows 7–9) show sector 1, the main exporter, as expanding strongly. Sector 2 is held back by the large share of the main import-competing commodity in its sales structure but enjoys a significant cost reduction because of the relatively large share of imports in its input structure. Sector 3 contracts as consumers substitute away from the non-traded commodity 3, the price of which has risen relative to the prices of commodities 1 and 2.

(c) Five-year forecasts

Table 1.9 contains hypothetical five-year forecasts made with our illustrative model. In forecasting mode, the model is recursive, using the investment specification described for Case 2 in Section 2.3. The variables in the percentage change equations are to be interpreted as year-on-year percentage growth rates. The forecasts in Table 1.9 are designed to show how we are using the MONASH model[56] to make forecasts for the Australian economy. They comprise five annual simulations each of which was made with a 1-step Johansen/Euler computation.[57] After each annual simulation we use the results to update the data and use the updated data as the basis for the next annual simulation.

In forecasting with MONASH we drive the very detailed CGE model with exogenously specified scenarios for most macroeconomic variables and for some structural variables. The macro scenarios are taken from a business forecasting group and the structural forecasts from expert bodies like the Australian Bureau of Agricultural and Resource Economics (ABARE). The role of macroeconomics in forecasting with CGE models is discussed further in Section 4.

[56] MONASH is a multi-period forecasting version of ORANI, see Adams, Dixon, McDonald, Meagher and Parmenter (1994).

[57] If we are concerned about linearization errors in the annual simulations, multi-step procedures could be substituted for the 1-step computations.

Closure for recursive forecasts. The closure for the forecasting simulations is given in the final column of Table 1.6. It differs from the standard short-run closure (column 1 of the table) only in the treatment of variables in the last of the four blocks.

In each of the annual forecasts, the $x^{(1j)}_{(g+1,2)}$, i.e., the capital stocks used in the forecast year, are exogenous, as they are in short-run comparative statics. Unlike in comparative statics, for the forecasts we shock the $x^{(1j)}_{(g+1,2)}$. Except for the first year, the shocks are results for $x^{(1j)}_{(g+1,2)}(1)$ obtained from the forecasts for the previous year. For the first year, the shocks are calculated from the data.

Aggregate investment (as well as aggregate consumption) is exogenous in the forecasts. This requires fic to be endogenous, breaking the link in Eq. 3.19 between the growth rates of investment and consumption. The allocation of investment between sectors is determined in the forecasts by Eqs 3.13 and 3.14, as in our comparative statics. Equation 3.13 determines through-the-year growth in the capital stock in each sector according to the rate-of-return movement in the sector. The accumulation relationship 3.14 then determines the sectoral investment growth required to support the capital growth. The role of the endogenous variable f_k is to ensure that the sectoral capital growth rates are consistent with the exogenous growth rate of aggregate investment.

The numeraire in the forecasts is the rate of inflation of domestic consumer prices rather than the nominal exchange rate. In the forecasts the export volumes of all commodities are exogenous.

Forecast scenario for exogenous variables. The scenario which we have adopted for the illustrative forecasts is listed in part (a) of Table 1.9. It is similar to the scenarios which we have used for recent forecasts with MONASH [Adams, Dixon, McDonald, Meagher and Parmenter (1994), Syntec (1993c)], showing the economy emerging from a trough in the business cycle.

The illustrative scenario begins with projections of shifts in the foreign demand schedules for the two exported commodities, i.e., projections of the changes in the world prices which would occur if export volumes remain unchanged. The scenario also includes projections of the growth rates of the world prices of the three imported commodities and of export volumes. For our forecasts with MONASH, we get information like this from ABARE. The projections for exports of commodity 1 exhibit a cyclical pattern typical of the commodity markets which account for the bulk of Australia's exports. For commodity 2, the projections show high but gradually falling export growth rates. This is typical of the situation facing Australia's non-traditional exports (manufacturing and services). In recent years these have exhibited very rapid growth from a low base. Exports are expected to continue to be strong but as the base grows it is unlikely that the very high growth rates will be sustained. Note in Section (b) of the table that the world-price projections imply a cyclical movement in the terms of trade.

Projections for the real wage rate are for continuation of wage moderation. Wage moderation was the cornerstone of the Australian government's macroeconomic policy in the 1980s. With the high levels of unemployment produced by the recession of the early 1990s, wage moderation is likely to continue. The projections incorporate mild pro-cyclical movements in the rate of growth of the real wage rate.

Technical change is a particularly difficult component of the scenario required for the forecasting simulations. For the MONASH forecasts, we set technical-change scenarios in the light of past patterns of technical change. At the level of disaggregation at which MONASH works, generating evidence about these past patterns requires a major research effort [cf. Dixon and McDonald (1993a)]. The hypothetical scenario in Table 1.9 is limited to projections of labour-saving technical change.[58] We have built into it two stylised facts: pro-cyclical movements in labour productivity; and slow measured technical progress in services (sector 3) relative to other sectors.

The scenario includes projections for the domestic demand aggregates. These follow the cyclical pattern dictated by the export projections and their implications for the terms of trade. Typical of historical experience, investment is projected to be significantly more volatile than consumption.

Our hypothetical scenario includes a program of tariff reductions which is assumed to be complete by the end of the third year.

The final two variables in the exogenous scenario are the CPI and the demographic variable, the number of households. The first of these is the numeraire. Our projections for import prices reveals our implicit assumption about the world rate of inflation. Note that we are assuming that on average the domestic rate of inflation will be a little lower than the world rate. This has been a feature of Australia's recent economic history. The only role played in the forecasts by the demographic variable is in the determination of the commodity composition of aggregate consumption (see Eq. 3.3).

Results for endogenous variables. Part (b) of Table 1.9 reports forecasts for selected endogenous variables. Space precludes a very extensive discussion of these results. Our strategy for demonstrating the type of insights which this forecasting technique allows is first to explain the results for year 1 in some detail. The type of explanation which we give for the year-1 column should readily be applicable to the other columns. We then point to some features of the across-years pattern of the results which illustrate the implications of the underlying dynamics of the model.

In looking at the results for year 1, we start by noting that the terms-of-trade forecast is a direct implication of the exogenous scenario. The modest upward shift of the foreign demand schedule of the major export commodity together with the forecast expansion of the export volume imply relatively slow growth in its world price. Hence, the terms of trade fall. The terms-of-trade decline implies that the GDP deflator must

[58]This is not to suggest that other aspects of technical change are unimportant. Dixon and McDonald (1993a) find, for example, that input biases in the patterns of intermediate-input-saving technical change were very important in explaining the growth of imports in the Australian economy in the late 1980s.

rise less rapidly than the CPI. Because we have assumed that the nominal wage rate rises faster than the CPI, the wage/rental ratio rises, reducing the labour/capital ratio. For year 1, growth in the capital stock is predetermined by our data. The rates of growth of capital, together with the forecast rates of growth of employment and the of rates of labour-saving technical change included in the exogenous scenario, imply real GDP growth of a little less than 3 per cent.

As with the terms of trade, the rate of growth of the export volume index is implied directly by our exogenous scenario. Since the rates of growth in the exogenous scenario for domestic investment and consumption are less than the forecast rate of growth of GDP, imports are forecast to grow less rapidly than exports, i.e., an improvement in the real trade balance is implied. With no adjustment in the real exchange rate, imports would grow more rapidly than the forecast rate, due mainly to the tariff reductions which are included in the exogenous scenario. Hence, the model forecasts a real depreciation. With the domestic rate of inflation assumed to be less than the foreign rate only a small nominal depreciation is required.

Forecasts of structural variables are reported at the bottom of Table 1.9. We begin with growth rates for outputs of domestically produced commodities. Output growth for commodity 1 is forecast to be strong relative to GDP growth for two reasons: because of the rate of export growth included in the exogenous scenario and because the real devaluation is (notwithstanding the tariff cut) strong enough to allow domestic production to increase its share vis-à-vis imports in the domestic market. Commodity 2 faces a larger tariff cut and loses market share to imports. Exports of commodity 2 grow very rapidly but are only a small share of its total sales and make a only a small contribution to its aggregate output growth. As might be expected, the growth rate forecast for the output of the non-traded commodity 3 is close to the growth rate forecast for real GDP.

Growth of capital used in year 1 depends entirely on initial conditions in our data. As implied in footnote 26, growth rates in capital used in year 1 are given by through-the-year growth rates in the data (year 0). The main feature of the forecasts for growth of capital in use in year 1 is that the capital stock in sector 1 is growing much more rapidly than that of sector 2. Recall that both these sectors produce commodities 1 and 2. The faster rate of capital growth in sector 1 allows it to gain market share against sector 2 in the production of both these commodities. This is apparent in forecasts of growth in sectoral activity levels. Activity in sector 3, a single-product sector which is the sole producer of commodity 3, must expand at the same rate as the output of commodity 3.

The employment forecasts are implied directly by the forecasts for growth in capital usage, growth in activity, and technical change. With rapid capital growth and labour-saving technical change, employment growth in sector 1 is quite modest despite its strong activity growth. In sector 2 also, labour-saving technical change is forecast to keep employment growth slower than activity growth, despite the absence of growth in the capital stock. According to our exogenous scenario, sector 3 enjoys no technical improvement. Hence, even with quite strong capital growth, employment growth at a rate not much less than activity growth is required.

Our sectoral results for through-the-year capital growth in year 1 are quite similar to our results for growth in capital usage in year 1. Both in the through-the-year and usage results, sectors 1 and 3 have much higher growth rates than sector 2. The similarity in the through-the-year and usage growth rates reflects sluggishness in the adjustment from year to year in through-the-year growth rates. (Recall that the growth rates in capital usage in year 1 are the through-the-year growth rates in year 0.)

This sluggish adjustment is implemented via our treatment of the shift variables in Eq. 3.13. According to Eq. 3.13(a), growth in sector j's capital stock depends on its net rate of return and on the product of the shift variables F_k and $F_k^{(j)}$. As mentioned in Section 3.3, the net rate of return on capital in the data for year 0 is 5 per cent for all sectors. With capital use for sectors 1 and 3 growing strongly in year 1, we can see that a property of our base solution must be that $F_k(0)F_k^{(2)}(0)$ has a low value compared with $F_k(0)F_k^{(j)}(0)$ for $j = 1$ and 3. In our simulation for year 1, the percentage movement in $F_kF_k^{(j)}$ is determined by the percentage movement in F_k and is, therefore, the same for all j.[59] Hence, in year 1, $F_kF_k^{(2)}$ remains low relative to $F_kF_k^{(j)}$, $j = 1$ and 3. This explains the relatively low growth through year 1 in sector 2's capital stock. Scarcity of capital raises the rate of return in sector 2 so that in later years capital growth in sector 2 is not far below that in sectors 1 and 3.

Despite having the lowest capital growth through year 1, sector 2 has the highest growth in investment. Growth in a sector's investment reflects the *change* in the sector's rate of growth in capital. Between years 0 and 1, sector 2's capital growth increases (from -0.02 to 1.09 per cent) whereas capital growth in both the other sectors declines. As explained already, capital growth in sector 2 increases relative to that of the other sectors because the rate of return in sector 2 increases relative to that of the other sectors.

We now move to the second part of our explanation strategy. That is, by looking across some rows in Table 1.9 we describe some aspects of the dynamic operation of our forecasting model.

Starting with the GDP results we see strong growth in years 2 and 3. This reflects improving terms of trade, higher rates of growth of export volumes, and rapid growth in investment. In years 4 and 5, all these weaken as determinants of growth.

Employment follows this cycle closely, despite the pro-cyclical movement in labour-saving technical change which we assume. The effects of the increase in the rate of technical change in year 2 can be seen, nevertheless, in the employment forecasts and productivity changes for sectors 1 and 2. Contributing to the strength of employment growth in year 2 is the sluggish adjustment of the capital stock. Investment is assumed to boom in years 2 and 3 but its growth in year 1 is quite low. With growth of capital

[59] In Table 1.6, $F_k^{(j)}$ is exogenous and unshocked.

used in year 2 depending on investment in year 1, capital growth in year 2 is unable to keep pace with output growth. This results in a fall in the wage/rental ratio and very strong employment growth, mainly in sector 3, which experiences no labour-saving technical change. On the other hand, in years 3–5, following the investment boom, capital growth exceeds output growth, increasing the wage/rental ratio and slowing the rate of growth of employment.

Finally, we note that a number of rows in Table 1.9 show the effects of our between-year updates of the model's data. The export volume index is one example. In years 1 and 3 the shocks to export volumes in the exogenous scenario are identical but the export volume index grows more rapidly in the second of these two years. The reason is that in year 3 the weight in the index of commodity 2 (the most rapidly growing export) is greater than it was in year 1.

4. Concluding remarks: Success, partial success and potential of CGE modelling

This section contains three propositions:

(1) CGE models have provided useful insights on the likely effects of disturbances in one part of the economy on activity in other parts;

(2) CGE-based analyses of the welfare effects of proposed policy changes have been only partially successful; and

(3) CGE models are yet to fulfill their potential for providing guidance to people concerned with investment and other business decisions.

4.1. Success: Quantifying linkages between different parts of the economy

Before CGE models there were input–output models. These emphasize input–output linkages between industries. They imply that stimulation of the motor vehicle industry, for example, perhaps from the imposition of a tariff, stimulates the sheet metal industry. In turn, this stimulates the steel industry and so on.

Input–output computations imply that stimulation of any one industry stimulates all industries with widespread employment gains. Not surprisingly, input–output models have been and remain a popular tool of lobbyists seeking government favours for their industries.

CGE models go beyond input–output models by linking industries via economy-wide constraints. These include: constraints on the size of government budget deficits; constraints on deficits in the balance of trade; constraints on the availability of labour, capital and land; and constraints arising from environmental considerations such as air and water quality. With these constraints in place, the economy-wide implications of stimulation of one industry can be negative and a favourable outcome for some industries can be at the expense of others.

For many years, CGE models have provided results of this type for the effects of changes in protection.[60] As in Table 1.8, these show protection as favouring import-competing industries while harming export-oriented industries. For example, since the mid-seventies, ORANI simulations for Australia have generated results with the following flavour.[61]

- An increase in protection for textiles, clothing footwear and motor vehicles saves jobs in these industries.
- However, it increases the prices of their products, thereby increasing the CPI.
- With wage rates being linked to the CPI, there is an increase in nominal wage rates.[62]
- This causes cost increases throughout the economy with a profit squeeze and job losses in those industries which are poorly placed to increase their selling prices.
- Industries in this category are those relying largely on exports for their sales. Selling prices for these industries are determined on world markets, independently of their costs.
- Thus in ORANI, with the protected sector and the exporting sector linked through the labour market, the initial stimulation of TCF and motor vehicles arising from an increase in protection is translated into a contraction for agriculture, mining and other export-oriented activities.
- With the real wage rate fixed economy-wide, ORANI implies that increases in protection have little effect on aggregate employment. The number of jobs gained in protected industries is approximately balanced by the number of jobs lost in export-oriented activities.
- Changes in protection change the regional allocation of activity in Australia with Victoria gaining from increases in protection and Queensland and Western Australia losing. Similarly, changes in protection change the occupational composition of employment.

Apart from protection, there are many other issues for which adequate analysis requires recognition of linkages arising from economy-wide constraints. Some of these were indicated in the opening paragraph of Section 1. Here we give one more example, again drawing on an ORANI application [see Adams and Parmenter (1993 and 1995)].

The question to be answered was: what would be the implications for the states of a general stimulation of international tourism in Australia? Part of the ORANI-generated

[60]See, for example Srinivasan and Whalley (1986) and Whalley (1985).

[61]The main reference on ORANI is Dixon, Parmenter, Sutton and Vincent (1982). Chapter 7 of that book is a detailed report of an ORANI tariff simulation. For an overview of the role of ORANI in the Australian economic debate, see Powell and Snape (1993).

[62]In most ORANI simulations, it has been assumed that real (CPI deflated) pre-tax wage rates are fixed and that increases in tariff rates are accompanied by cuts in income taxes, not by CPI-reducing cuts in other indirect taxes.

answer was that Queensland (Australia's sunshine state and the main destination of many foreign tourists) would be a small loser. The explanation relies on linkages between different parts of the economy provided by constraints on the trade accounts.

Certainly international tourists spend money in Queensland, although not as much as we thought prior to becoming familiar with the relevant statistics. Although many tourists travel in Queensland, the bulk of their money is spent in New South Wales, especially on airline tickets for flights in and out of Sydney. Thus, the Queensland economy experiences moderate (not huge) gains from the expenditures of international tourists.

The downside for Queensland comes from the trade accounts. With stimulation of international tourism, there is, according to ORANI, a strengthening of the exchange rate. This impacts adversely on export-oriented activities including mining and agriculture.[63] With these activities representing a comparatively large share of its gross state product, Queensland is left as a net loser from general tourism stimulation.

It is in the tracing out of linkages arising from economy-wide constraints that CGE modelling has had its greatest successes. With the advent of CGE modelling, the input–output approach, with its exclusive reliance on linkages arising from flows of intermediate inputs, is no longer credible.

4.2. Partial success: Analysis of welfare effects

Much of CGE modelling has been concerned with the welfare implications of proposed policy changes, for example changes in protection, changes in taxes and changes in environmental regulations. Usually these welfare implications have been measured by calculating the variation in consumer income which would produce the same variation in consumer utility as that generated in the CGE simulation of the policy change under consideration.

Many interesting welfare results have been obtained, especially in the analysis of tax changes. For example, using a 19-sector, 12-consumer, multi-period, CGE model, Ballard, Shoven and Whalley (1985) calculated the marginal excess burden[64] (MEB) of US taxes on labour, capital, consumption, income and output. Under a variety of assumptions concerning the elasticity of saving with respect to the real after-tax rate of return, and the elasticity of labour supply with respect to the real after-tax wage rate, they found MEBs in the range 0.18 to 0.56. This has an important implication for the assessment of publicly funded projects. Because of the necessity of generating finance through increases in decision-distorting taxes, only those publicly funded projects

[63]This is an example of Dutch disease: a booming export sector (tourism) impacts adversely on other trading sectors, see, for example, Corden (1984).

[64]The MEB of a tax is x if an increase in the rate of the tax sufficient to increase government revenue by \$1 leaves households with the same level of welfare as the imposition of a lump-sum (non-distortionary) tax of \$$(1 + x)$.

which provide benefits valued by consumers well in excess of their costs should be undertaken.

Nevertheless, we rate CGE work in the area of welfare analysis as only partially successful. Typically, in the calculations supporting this work, it has been assumed that the proposed policy change does not affect: the levels of involuntary unemployment of labour and capital; the form of competition between firms; and rate of technological progress. It is assumed that welfare changes arise only from reallocations of consumer budgets between different goods (including possibly leisure and savings) and reallocations of scarce factors of production between different industries. Such CGE calculations of welfare changes often produce small and unconvincing numbers. We consider two examples: the costs of protection and the costs of reducing CO2 emissions.

(a) The costs of protection

Even for countries with high and non-uniform tariffs, typical CGE calculations show gains from moving to free trade of less than 1 per cent of GDP. This result could have been anticipated from pre-CGE work on the costs of protection. For example, in a theoretical article containing illustrative arithmetic, Johnson (1960) demonstrated that under competitive assumptions with normal settings for demand and supply elasticities, the costs of protection are likely to be a very small share of GDP. Dixon (1978b, p. 63) concluded that if the principal aim is to measure the costs of protection, "it would be pointless to apply a model which failed to recognize intra-industry specialization and economies of scale. Such a model is virtually certain to generate a paltry estimate for the costs of protection, whatever the true situation might be".

In Australia, where there have been considerable reductions in protection over the last 10 years, costs-of-protection (welfare) numbers derived from CGE models have been ignored. In implementing anti-protection measures, policy makers have referred to mechanisms not usually included in CGE calculations. Among these omitted mechanisms are the effects of increased competition from imports on the structure of industries and on the behaviour of both management and unions. With lower protection, policy makers have argued

(a) that there are likely to be reductions in numbers of firms and product lines allowing lower costs through economies of scale, and
(b) that management is likely to work more effectively and that unions are less likely to take actions imposing cost increases on firms.

Recognizing that they are missing the main motivations for reductions in protection, CGE modellers have sometimes enhanced their welfare calculations by assuming that tariff cuts are accompanied by exogenously given improvements in productivity. This can produce welfare numbers more in keeping with the views of anti-protectionists. However, such CGE calculations merely illustrate the implications of anti-protection arguments. They neither explain these arguments nor provide evidence in their support.

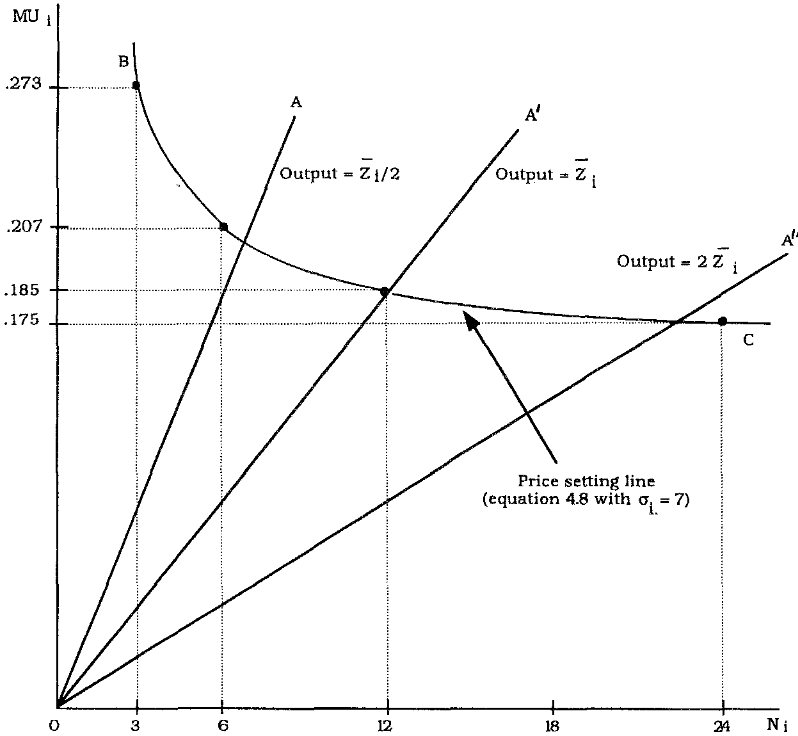

Figure 1.3. Zero pure profits and price-setting with imperfect competition and economies of scale.

The most celebrated CGE work incorporating some of the features required for a satisfactory analysis of the costs of protection is by Harris and Cox (1983) for Canada. They allowed for economies of scale, intra-industry specialization and non-competitive market structures. Their lead has been followed by Horridge (1987), Norman (1990), Mercenier (1994a) and others.

An illustration of the theoretical approach adopted in these studies is given in Fig. 1.3. The horizontal axis shows the number of firms, N_i, in industry i and the vertical axis shows the markup up in industry i over variable costs:

$$MU_i = P_i/V_i - 1 \tag{4.1}$$

where P_i is the price of good i and V_i is variable cost per unit of output.

We assume that firms in industry i incur an annual fixed cost, F_i, and that their variable cost, V_i, per unit of output is independent of their output level. This implies that each firm experiences increasing returns to scale. We also assume that the number of firms in the industry adjusts to ensure zero pure profits, i.e.,

$$P_i Z_i = V_i Z_i + F_i N_i, \tag{4.2}$$

where Z_i is industry output. By rearranging (4.2) we obtain

$$MU_i = (F_i/V_i)(N_i/Z_i). \tag{4.3}$$

Equation (4.3) is represented in Fig. 1.3 for different levels of industry output by lines OA, OA$'$ and OA$''$. In drawing these lines, we assumed that F_i/V_i is constant with respect to variations in N_i and Z_i.

The line BC in Fig. 1.3 is a price-setting line for a typical firm in industry i. It is drawn on the assumption that as the number of firms increases, the industry becomes more competitive, i.e., as N_i rises, each firm perceives a higher (larger negative) elasticity of demand for its product. With profit maximizing behaviour, the Lerner (1934) condition will apply. This can be written as[65]

$$MU_{iq} = -1/(1 + \varepsilon_{iq}) \tag{4.4}$$

where MU_{iq} is the markup by firm q in industry i, and ε_{iq} is q's perception of the elasticity of demand for its product.

Assuming all firms are alike, so that $MU_{iq} = MU_i$ and $\varepsilon_{iq} = \varepsilon_i$ for all q, we can rewrite (4.4) as

$$MU_i = -1/(1 + \varepsilon_i(N_i)). \tag{4.5}$$

With the perceived elasticity, ε_i, becoming a larger negative number as N_i increases, we see that MU_i is negatively related to N_i. This is reflected in the negative slope of BC.

For CGE modelling, we need a numerically specified relationship between ε_i and N_i. A possible starting point [Horridge (1987)] for obtaining such a relationship is to assume that demanders of product i have preferences for different varieties given by a CES function. This implies that the demand function for the product of the qth firm in industry i has the form:

$$x_{(iq)} = a_i - \sigma_i\left(p_{(iq)} - \sum_r S_{(ir)} p_{(ir)}\right) \tag{4.6}$$

[65]Profit maximization requires that marginal revenue equals marginal cost, i.e.,

$$\partial(P_{iq}Z_{iq})/\partial Z_{iq} \equiv P_{iq} + (\partial P_{iq}/\partial Z_{iq})Z_{iq} = V_{iq}.$$

By dividing through by P_{iq}, we arrive at (4.4). We assume that $\varepsilon_{iq} < -1$ so that $MU_{iq} > 0$.

where

$x_{(iq)}$ is the percentage change in the demand for product i of variety q (the variety produced by the qth firm);

a_i is the percentage change in the activity variable (e.g. household disposable income) relevant in the determination of overall demand for product i;

$p_{(iq)}$ is the percentage change in the price of product (iq);

$S_{(ir)}$ is the share of the total sales of product i accounted for by variety r;

and

σ_i is the elasticity of substitution by users of i between different varieties.

Now we assume that all firms in industry i are the same size ($S_{(ir)} = 1/N_i$ for all r) and that they are Bertrand rivals, i.e., they behave as if they expect changes in their prices to generate no price response from their competitors. Then, each firm's perceived elasticity of demand is

$$\varepsilon_i = -\sigma_i(1 - 1/N_i), \tag{4.7}$$

giving a numerically implementable equation for the BC line in Fig. 1.3 of the form[66]

$$MU_i = -1/\big(1 - \sigma_i(1 - 1/N_i)\big). \tag{4.8}$$

Another approach to specifying the price-setting line is to adopt a price-leadership model such as that of Eastman and Stykolt (1967). They assume that

$$P_{iq} = \alpha_i P_i^m(1 + T_i), \tag{4.9}$$

where P_i^m is the c.i.f. import price of commodity i, T_i is the tariff rate, and α_i is a parameter. Under (4.9), the price-setting line is horizontal. Irrespective of their number, all firms in industry i are led by the landed-duty-paid price of competitive imports. The markup for each firm is

$$MU_i = \alpha_i P_i^m(1 + T_i)/V_i - 1. \tag{4.10}$$

Harris and Cox (1983) and Horridge (1987) experimented with price-setting assumptions combining both the Lerner/Bertrand and the Eastman/Stykolt specifications. For example, Horridge assumed that the markup of firms in import competing industry i is

$$MU_i = W_1\big[-1/(1 - \sigma_i(1 - 1/N_i))\big] + W_2\big[\alpha_i P_i^m(1 + T_i)/V_i - 1\big], \tag{4.11}$$

where W_1 and W_2 are nonnegative weights summing to one.

[66]We assume that $\sigma_i > N_i/(N_i - 1)$, ensuring that $MU_i > 0$.

In an application of his model to the analysis of the effects of reductions in protection in Australia, Horridge found that when W_1 is close to 1 (the Lerner/Bertrand pricing specification) the results are similar to those obtained under competitive assumptions with constant returns to scale. In particular, calculated costs of protection are small. He also found that when W_2 is close to 1 (the Eastman/Stykolt specification), the calculated costs of protection are much larger than those obtained under competitive, CRS assumptions. Both these results can be understood by reference to our diagram.

For import-competing industries, Horridge assumed, on average, that N_i is about 12 and σ_i is about 7, giving a typical value for MU_i in (4.8) of 0.185. As shown in the diagram, variations in N_i from 6 to 24 cause relatively little variation (from 0.207 to 0.175) in MU_i. Because his price-setting lines were quite flat, Horridge found in the Lerner/Bertrand case that changes in protection could cause large changes in industry outputs without causing much change in markups. Consequently, he found that percentage movements in N_i were approximately equal to percentage movements in Z_i. (Notice in Fig. 1.3 that if output in industry i doubles, switching us from ray OA$'$ to OA$''$, then N_i approximately doubles. Similarly, if Z_i halves, then N_i approximately halves.) With N_i approximately proportional to Z_i, total fixed costs in industry i are also approximately proportional to Z_i. Thus, despite allowing for imperfect competition and for economies of scale at the firm level, the Lerner/Bertrand version of Horridge's model behaves in a similar way to a competitive model with CRS specified at the industry level.

With the Eastman/Stykolt specification ($W_2 = 1$), a cut in protection causes a downward shift in the price-setting line. Because the Eastman/Stykolt price-setting line is completely flat, a downward shift causes a reduction in MU_i, irrespective of what happens to Z_i. With a reduction in MU_i, there is an increase in output per firm and a reduction in fixed costs incurred in the industry per unit of output. Relative to the Lerner/Bertrand case, Horridge found that this saving of fixed costs per unit of output generates a considerably increased figure for the welfare gain from eliminating protection.

On the basis of the work by Harris and Cox, Horridge and others, we can conclude that the costs of protection depend critically on production technologies and on how firms in protected industries compete with each other. However, we already knew this from theoretical literature such as Corden (1974). While CGE modellers have made considerable progress in dealing with imperfect competition and economies of scale, they have, as yet, failed to incorporate sufficient empirical detail to allow a useful narrowing of the range of possible estimates for the costs of protection.

(b) The costs of reducing CO2 emissions

Our second example of the inadequacies of CGE-based welfare analysis concerns the costs to Australia of reducing CO2 emissions by 20 per cent by 2005, i.e., the costs of meeting the Toronto target. Using a version of ORANI, the Industry Commission (1991) concluded that the main action required in Australia to meet the Toronto target is the substitution in electricity generation of low CO2 fuels, such as oil and gas, for high CO2 fuels, especially brown coal. They found that this would involve an annual welfare cost of about 1.5 per cent of GDP.

As with most CGE welfare calculations, the Industry Commission calculations were comparative static. They compared two pictures of the Australian economy in 2005: one in which Australian electricity generation continued to rely mainly on coal with CO2 emissions being of no concern, and the other in which a major fuel switch had taken place to facilitate a sharp reduction in CO2 emissions. As the Industry Commission recognized, adjustment costs over the period between now and 2005 were omitted from their calculations. For example, no account was taken of the extra investment needed over this period to replace brown-coal-fired generation plants in the La Trobe valley (a brown-coal producing region with enormous investments in generation capacity).

The Industry Commission's work on CO2 emissions indicates that for convincing welfare analysis, we need to add dynamics and adjustments costs to the list of necessary features of the model. Unfortunately, the dynamics required are complicated. Because they do not deal adequately with scrappage, simple dynamic models, assuming perfectly malleable capital stocks, are inadequate. Dynamic analyses of the costs of meeting CO2 emission targets which have adopted the malleability assumption (brown-coal-fired generation capacity can be converted effortlessly into oil/gas capacity) include Jorgenson and Wilcoxen (1992 and 1994). These analyses, as with those based on comparative statics, may seriously underestimate the costs of adjusting to meet environmental objectives.

McKibbin and Wilcoxen (1993a and 1993b) analyze the effects of greenhouse-gas reductions in a dynamic model with adjustment costs of the type discussed in Subsection 2.3 (Case 4). While their work is an advance, it suffers from the following limitations: the form of the adjustment-cost specification (they use $\theta I^2/K$) has no clear theoretical or empirical justification; the critical parameters, the θs, are merely assigned, not estimated; and their model is highly aggregated (for example electricity is a single industry) meaning that the costs of moving resources within large sectors of the economy are ignored.

Perhaps the most promising approach to creating models capable of generating satisfactory estimates of the costs of environmental policies is that of Manne (1991). He is attempting to absorb a detailed energy model such as MARKAL (Fishbone et al. 1981) into a CGE framework. Under our definition (Section 1.1), MARKAL is not a CGE model: it treats prices exogenously and includes insufficient specification of the behavior of economic actors outside the energy sector. MARKAL's strength is that it can include specifications of dozens of energy-producing technologies (such as brown-coal-fired-electricity generation in the south-eastern area) based on engineering data. Associated with each technology is a non-malleable capacity constraint and a specification of the costs of creating additional capacity. Progress in taking MARKAL-like structures into a CGE framework has also been made by Adams et al. (1991) and Jones et al. (1991).

4.3. Potential: Disaggregated forecasting

Most CGE modelling has been concerned with the effects of proposed policy changes or the effects of exogenous events, e.g., the discovery of mineral deposits. However, there is strong demand for forecasts. Disaggregated forecasts are required to help policy makers, investors, trade unions and households to form realistic expectations concerning: real wage growth; the costs of capital relative to labour; the industrial composition of economic activity; employment growth in different occupations and industries; and growth rates in different regions.

CGE models have not yet proved themselves to be valuable forecasting tools. While their tight theoretical structure is an attractive feature, it is far from sufficient. In our efforts to transform ORANI into a forecasting tool we have identified the following areas as requiring major effort.

(a) Achieving good macro forecasts

The first attempt to use ORANI in forecasting mode was Dixon (1986). Forecasts were produced for the period 1986 to 1990. The main feature of these forecasts at the macro level was a sharp reduction in Australia in the costs of capital. This was supposed to follow from two sources: a reduction in real interest rates world-wide in response to a contraction in the US budgetary deficit; and the formation of market expectations that the Australian exchange rate would be strong through the forecast period.

The assumed reduction in the costs of capital produced in our forecasts an investment boom, rapid real wage growth and average annual GDP growth of over 5 per cent. At the industry level, we projected good prospects for investment-related industries such as construction.

In later papers, e.g., Dixon and Parmenter (1987), we argued that foreign financiers would insist that Australia stabilize its foreign debt as a share of GDP by the end of 1990. Through ORANI, we found that this implied a sharp real devaluation of the exchange rate with high real interest rates and costs of capital. This led to forecasts of only modest real GDP growth, poor prospects for real wage growth and poor prospects for investment and investment-related industries.

None of our early ORANI forecasts have been close to reality. It is now clear that we did not know enough about how to forecast the macro economy. Because our macro forecasts were inaccurate, our industry forecasts were unrealistic.

There are two approaches to macro forecasting in a CGE framework. One is to rely on the CGE-generated macro implications of assumptions concerning the future paths of variables such as aggregate employment, required rates of return on capital, technical change and changes in the terms of trade. This was the approach we used in our early forecasting exercises with ORANI. In the second approach, we rely on the CGE model only for structural forecasts, e.g. forecasts of the industrial composition of GDP and the occupational and regional composition of employment. Under this approach, we force the CGE model to produce results compatible with exogenously supplied macro forecasts. These can be derived from a conventional macro model emphasizing business cycle phenomena. Compatibility between the CGE model and the macro forecasts is achieved by endogenizing in the CGE model such variables as: an overall measure of technical change (allowing compatibility between exogenously specified levels for GDP and for aggregate inputs of capital and labour); an overall measure of import/domestic preferences (allowing compatibility between exogenously specified levels of aggregate imports and of the real exchange rate); the average propensity to consume (allowing compatibility between exogenous specified levels for consumption, GDP and tax rates); and the overall required rates of return on capital (allowing compatibility between exogenously specified levels for aggregate investment and for overall real unit labour costs).

Eventually, it may be possible to generate realistic macro forecasts in a CGE model without help from specialist macro forecasters. However, at this stage it seems sensible to exploit the advantages of division of labour. For example, in forecasting for Australia, it is necessary to pay close attention to overseas economies. This is because Australia's business cycle is closely connected to that of the US and other major countries. The explanation is that growth in the world economy is the main determinant of movements in Australia's terms of trade. These movements exert a strong influence on GDP, the exchange rate and other macro variables in the Australian economy. By building a CGE model capable of using exogenously given macro forecasts, we have been able to draw on the expertise of macro modellers and business forecasters specializing in the study of overseas economies and their macro economic influence on Australia. This leaves us free to specialize in CGE modelling of industries, regions and occupations.[67]

(b) Creating and maintaining up-to-date input–output data

In most countries, input–output tables are published by the official statistical bureau with a long lag. For example, until February 1994 the latest input–output tables published by the Australian Bureau of Statistics (ABS) were for 1986–1987.

Out-of-date input–output data do not usually pose major difficulties for comparative static applications of CGE models. For example, Dixon, Parmenter and Rimmer (1986) found that the simulated effects of a given tariff cut varied little as they changed the

[67] In generating CGE forecasts for industries, regions and occupations, we are currently using inputs from Murphy's (1988 and 1991) macroeconometric model and from the business forecasting group, Syntec (1993a and b). See Adams et al. (1994) and Syntec (1993c).

input–output database underlying their CGE model from 1969 to 1975. Nevertheless, timeliness of input–output data is vital for forecasting. This is especially true for forecasting the prospects of investment-supplying industries. Working from an out-of-date database, a CGE model may be able to produce satisfactory forecasts of growth in the housing stock reflecting demographic and income projections. However, for forecasting the prospects of residential construction, cement, bricks, glass and other industries closely associated with home building, we need current data on the level of activity in these industries. If the construction activities are currently subdued, then the achievement of a given growth path for the housing stock may imply that they have strong growth prospects. Alternatively, if their current level of activity is high, then the same path for the housing stock may imply a construction slowdown.

As explained in Dixon and McDonald (1993a), we have devoted considerable resources to updating input–output tables published by the ABS. Out initial motivation was to provide an up-to-date starting point for our CGE forecasts. A subsidiary benefit has been a detailed quantification for the second half of the 1980s of technological change and of changes in consumer tastes. This has helped us to develop forecasts of these variables for the 1990s. In addition, the update project has given us a framework for analysing structural changes in the Australian economy [see Dixon and McDonald (1993b)].

(c) Disaggregating and understanding what the statistics for the industries represent

Most published CGE models have less than 30 industries. For many purposes this provides inadequate disaggregation. For example, in Subsection 4.2(b) we argued that convincing analysis of the costs of limiting CO2 emissions requires greater disaggregation of the energy sector than is normal in CGE models. In forecasting, even with a 100-industry model it is difficult to meet the requirements of clients, both in the public and private sectors, seeking guidance in the allocation of funds between alternative investment possibilities.

The development of a relatively disaggregated CGE forecasting model is a major task. We are finding that it is necessary to think carefully about what the statistics for each industry really represent. It is not enough to follow the usual practice in CGE modelling of adopting the same specification (e.g. Leontief, CES, nested-CES, etc.) to describe each industry's technology, with only the parameter values differing between industries. Similarly, a uniform specification of how imported products complete with domestic products (e.g. the Armington specification) is adequate. We give two examples.

- *Communication.* In the input–output tables published by the Australian Bureau of Statistics this industry has considerable imports and exports. Does this mean that output and employment in the industry are highly sensitive to exchange rate movements and to costs in Australia relative to costs overseas? This is the conclusion that follows in ORANI under standard specifications of the behaviour

of trade flows. On getting to known about the nature of trade flows in communication, we find that this is not a sensible conclusion. Communication imports are mainly payments by Australia's Telecom to overseas telephone companies for facilitation of the transmission of calls from Australia. There is also a rental component for Australian use of foreign-owned communication satellites. Communication exports are mainly payments to Australia's Telecom for facilitating the transmission of calls coming from overseas. Given the nature of these trade flows, we expect future movements in exports to be approximately in line with those in imports (calls go to and fro). After modifying our specification of the industry to recognize the links between its imports and exports, we no longer find that its output and employment are highly sensitive to its international competitiveness.
- *Aircraft.* Does an upsurge of imports of aircraft harm employment and output in the Australian aircraft industry? This is the result that ORANI gives under standard specifications. However, on looking into what the industry does we find that its product is complementary with imports rather than competitive. The local industry specializes in aircraft repairs and the manufacture of parts. On changing the standard specification to reflect this, we find that the local industry is likely to prosper during a period of strong growth in the volume of imported aircraft.

The availability of programs such as GAMS and GEMPACK mean that computational difficulties are not currently a binding constraint in CGE modelling on either disaggregation or on the use of industry-specific specifications. What is now required for the creation of practical, decision-oriented CGE models is a willingness by model builders to increase the amount of information incorporated in their models. To do this, they will need to work closely with their national statistical agencies. They will also need to work in teams rather than as individuals. Research teams will be necessary to handle the work loads involved in implementing highly disaggregated CGE models containing thoughtful theoretical specifications for each industry.

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