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Social Accounting Matrices

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I Introduction

The theory of international trade is a general equilibrium affair: offer curves, Edgeworth–Bowley box diagrams, and Travis boxes are frequently used tools. For a long time, however, applied trade policy analysis was restricted to partial equilibrium techniques.[1] This was primarily due to constraints on data and computation for general equilibrium calibration and simulation. Nonlinear simulation software is now readily available, and general equilibrium techniques are often applied to trade policy problems. Surveys of these applications can be found in Shoven and Whalley (1984) and de Melo (1988).

Computable general equilibrium (CGE) models are calibrated to what are known as benchmark equilibrium datasets. The calibration process computes intercept and share parameters for the model's mathematical functions, given assumed or estimated values of behavioral elasticities, to reproduce the observed data as an equilibrium solution of the model. While much attention is devoted to the assumed or estimated behavioral elasticities, the calibrated intercept and share parameters are at least as important. Regardless of the quality of elasticity estimates, comparative static results have little empirical significance if they are not evaluated with respect to observed economic conditions.

A consistent and convenient means of compiling a benchmark equilibrium dataset is the social accounting matrix (SAM). This data framework has been extensively applied to developing countries under promotion by the World Bank and has been more recently applied to developed countries.[2] A SAM for 1984 underlies the CGE model of the United States

1 A modern presentation of these techniques is presented in Chapter 5 by Francois and Hall.

2 Applications to developing countries are reviewed in Pyatt and Round (1985). Hanson and Robinson (1991) and Reinert and Roland-Holst (1992) apply the SAM framework to the United States. Reinert, Roland-Holst, and Shiells (1993) apply the SAM framework to North America.

developed by de Melo and Tarr (1992). The CGE model in use at the U.S. International Trade Commission is currently based on a SAM updated biannually (Reinert and Roland-Holst, 1991). The analysis of the North American Free Trade Area conducted by Roland-Holst, Reinert, and Shiells (1994) is based on a three-country SAM for the region. The initial GATT assessment of the Uruguay Round was based on a SAM for nine regions of the world economy (Francois et al., 1994). These nonexhaustive examples indicate that SAMs are rapidly becoming the standard data construct for CGE models of trade policy.

This chapter will provide the reader with an introduction to SAMs, beginning in Section II with simple macroeconomic SAMs and moving on to SAMs with sectoral detail. Section III shows how SAMs are constructed for trade policy modeling. Section IV discusses the relationship between SAMs and CGE trade policy models. Finally, concluding comments are presented in Section V.

II What Is a SAM?

Economic accounting is based on a fundamental principle of economics: For every income or receipt there is a corresponding expenditure or outlay.[3] This principle underlies the double-entry accounting procedures that make up the macroeconomic accounts of any country. A SAM is a form of *single*-entry accounting. SAMs also embody the fundamental principle, but they record transactions between accounts in a square tableau or matrix format.[4] The transactors or accounts constitute the dimension of the square matrix. By convention, incomes or receipts are shown in the rows of the SAM while expenditures or outlays are shown in the columns. The utility of SAMs is that they can provide a *comprehensive* and *consistent* record of the interrelationships of an economy at the level of individual production sectors, factors, and general public and foreign institutions. They can be used to disaggregate the macroeconomic accounts, and they can reconcile these with the economy's input–output accounts.

Traditionally, the database for models with sectoral detail was the input–output accounting tableau, which captures linkages through flows of intermediate inputs. Although it provides sectoral disaggregation, an input–output model does not include enough institutional detail to provide a

3 Pyatt (1988), p. 329. The title of this section is taken from King (1985). The interested reader is referred to this source for further introductory material.

4 United Nations Statistical Office (1968) shows that national accounts can be presented in four ways: standard double-entry accounts, balance statements, matrices, and equations.

framework for considering the full impact of policy on the economy. The input–output accounts can be extended to capture income and expenditure flows between other institutions, such as households, government, and the rest of the world, in a SAM. Indeed, the development of SAMs was motivated in part by the desire for a unified framework that reconciled the input–output accounts with macroeconomic accounts. The SAM thus provides detail and an economywide policy perspective in a fully consistent accounting framework.

Algebraically, a SAM may be represented as a square matrix:

$$\mathbf{T} = \{t_{ij}\} \tag{4.1}$$

where t_{ij} is the value of the transaction with income accruing to account i from expenditure by account j.

Nominal flows cross the SAM from columns to rows. For transactions involving goods and services, there are corresponding real flows crossing the SAM from rows to columns. For financial transactions, there are corresponding flows of assets from rows to columns. For pure transfers, there are only the nominal flows from column accounts to row accounts.

The fundamental law of economics ensures that the corresponding row and column totals of a SAM, the income and expenditure for each account, must be equal. That is:

$$\sum_j t_{kj} = \sum_i t_{ik} \qquad \text{for all } k \tag{4.2}$$

As a consequence of this, SAMs satisfy a variant of Walras's Law. If all accounts but one balance, then the last account must also balance.[5] This property hints at the relationship between SAMs and neoclassical general equilibrium models.

Let us first consider a closed economy where economic activity is divided into three main types: production, consumption, and accumulation.[6] The representative accounts for this economy are presented in Table 4.1. Production receives its revenue from selling consumption goods in transaction t_{12} and investment goods in transaction t_{13}. The revenue from these sales passes to the consumption account as income paid to the factors of production in transaction t_{21}. The consumption-account income is spent in two ways. Part of it goes to purchase consumption goods in transaction t_{12}, and part is saved in transaction t_{32}. Savings is channeled to investment goods demand

5 Robinson (1989), p. 903.
6 The accumulation account is also known as the capital account. It can be thought of as a loanable funds market. This simple economy is addressed in Stone (1981, Chapter 1).

Table 4.1. *A closed-economy SAM*

	Expenditures			
Receipts	1	2	3	Totals
1. Production	-	C	I	Demand
2. Consumption	Y	-	-	Income
3. Accumulation	-	S	-	Savings
Totals	Supply	Expenditure	Investment	

Variables:

t_{12} = C = consumption

t_{13} = I = investment

t_{21} = Y = income

t_{32} = S = savings

Accounting Identities:

1. Y = C + I (GNP)
2. C + S = Y (Domestic Income)
3. I = S (Saving-Investment)

in transaction t_{13}, closing the macroeconomic system of income-expenditure flows.

The accounts of Table 4.1 reflect a functional classification. We next introduce an institutional classification. First, production and consumption accounts are redefined as the institutions "suppliers" and "households," respectively. Second, the government sector is included as an institution. Third, the economy is opened to the rest of the world. The resulting new accounts are set forth in Table 4.2.[7] Suppliers receive revenue by selling final consumption goods to households (transaction t_{12}) and government (transaction t_{13}), investment goods to the capital account (transaction t_{14}), and export goods to the rest of the world (transaction t_{15}). Revenue from production is

7 This five-account economy is addressed in Robinson and Roland-Holst (1988).

Table 4.2. *An open-economy SAM with a government sector*

Receipts	Expenditures 1	2	3	4	5	Totals
1. Suppliers	-	C	G	I	E	Demand
2. Households	Y	-	-	-	-	Income
3. Government	-	T	-	-	-	Receipts
4. Capital Accnt.	-	S_h	S_g	-	S_f	Savings
5. Rest of World	M	-	-	-	-	Imports
Total	Supply	Expend-diture	Expend-diture	Invest-ment	Foreign Exchange	

Additional Variables:

$t_{42} = S_h$ = private savings $t_{32} = T$ = tax payments

$t_{43} = S_g$ = government savings $t_{15} = E$ = exports

$t_{45} = S_f$ = foreign savings $t_{51} = M$ = imports

$t_{13} = G$ = government spending

Accounting Identities:

1. $Y + M = C + G + I + E$ (GNP)
2. $C + T + S_h = Y$ (Income)
3. $G + S_g = T$ (Government Budget)
4. $I = S_h + S_g + S_f$ (Saving-Investment)
5. $E + S_f = M$ (Trade Balance)

spent on value added (transaction t_{21}) and imports from the rest of the world (transaction t_{51}). Household outlays take the form of consumption expenditures (transaction t_{12}), tax payments (transaction t_{32}), and private domestic savings (transaction t_{42}). Government outlays take the form of consumption goods (transaction t_{13}) and government savings (transaction t_{43}). Inflows from the rest of the world take the form of export demand (transaction t_{15}) and foreign savings (transaction t_{45}). Foreign savings is the negative of the trade balance.

Actual SAMs typically include more detail than Table 4.2 in part because of a more detailed specification of the production side of the economy. The "suppliers" account of Table 4.2 is usually replaced with four accounts: activities, commodities, factors, and enterprises. The activities accounts buy intermediate inputs and hire factor services to produce commodities, generating value added in the process.[8] The goods sold by activities should be valued at producer prices in the SAM. The commodities accounts combine domestic supply with imports.[9] Commodities should be valued at purchaser prices in the SAM. Factors are a set of accounts for the expenditures and receipts of the factors of production: labor, land, and capital. Enterprises collect gross profits and government transfers and distribute them to other accounts.[10]

The new set of accounts is presented in Table 4.3. We describe the accounts from the receipts side and leave the reader to look at them from the expenditure side. Activities' receipts consist of payments at producer prices for the sales of goods to the commodity accounts (transaction t_{12}).[11] In an actual SAM, there would be many commodity and activity accounts (i.e., cell t_{12} would be a matrix). In the terminology of the input–output accounts, t_{12} is the "make table."[12] Commodities' receipts fall under five accounts. The first of these, transaction t_{21}, is from activities where commodities receive payments in purchaser prices for the sales of intermediate goods.[13] Input–output accounting refers to t_{21} as the "use table."[14] Transactions t_{25}, t_{26}, t_{27}, and t_{28} are

8 The United Nations' System of National Accounts (SNA) defines activity accounts as follows: "Production accounts of industries, producers of government services, producers of private non-profit services to households, and the domestic service of households, in respect of their gross output of goods and services and their intermediate consumption, primary inputs and indirect taxes *less* subsidies" (United Nations Statistical Office, 1968, p. 230).

9 The SNA defines commodity accounts as follows: "Accounts relating to the supply of commodities from domestic production and imports and their disposition to intermediate and final uses" (United Nations Statistical Office, 1968, p. 231). Hanson and Robinson (1988) describe the commodity account as "a giant department store" which "buys goods from domestic producers and foreigners (imports) down the column and sells them to demanders (including exports) along the row" (p. 218).

10 Hanson and Robinson (1991) describe the difference between activities and enterprises as follows: "(A)ctivities are aggregations of establishments within a sector. They purchase inputs on factor and product markets and sell output on product markets. They are different from enterprises which collect gross capital income and distribute it to other institutions. The distinction provides a framework for capturing an establishment-firm dichotomy, which exists in both data and theory" (p. 228).

11 "Producers' values are equal to the accumulation of factor costs, including the factor costs of the distribution and transport services embodied in inputs, and all indirect taxes" (United Nations Statistical Office, 1968, p. 54).

12 "The make table shows the value of each commodity produced by each industry. . . . The value of the primary product is shown in the diagonal cell. . . . The secondary products of the industry (products primary to other industries) are shown in the other cells along the row" (United States Department of Commerce, 1984, pp. 49–50). In terms of the vocabulary of this chapter, we can replace the word "industry" in this statement with "activity.'

13 "Purchasers' values are equal to producers' values plus the trade and transport margins appropriate to the purchaser in question" (United Nations Statistical Office, 1968, p. 54).

14 "The use table shows the value of each commodity used by each industry" (United States Department of Commerce, 1984, p. 48).

Table 4.3. *A more detailed SAM*

	Expenditures								
Receipts	1 Activities	2 Commodities	3 Factors	4 Enterprises	5 Households	6 Government	7 Capital Acct.	8 Rest of World	9 Total
1. Activities		gross outputs (make table)							total sales
2. Commodities	intermediate demand (use table)				household consumption	government consumption	investment	exports	aggregate demand
3. Factors	value added (net of taxes on activities)							factor service exports	factor income
4. Enterprises			gross profits			transfers			enterprise income
5. Households			wages	distributed profits		transfers		foreign remittances	household income
6. Government	indirect taxes	tariffs	factor taxes	enterprise taxes	direct taxes				government income
7. Capital acct.				retained earnings	household savings	government savings		capital transfers from abroad[a]	total savings
8. Rest of World		imports	factor service imports		transfers abroad	transfers abroad	capital transfers abroad		foreign exchange payments
9. Total	total costs	aggregate supply	factor expenditure	enterprise expenditure	household expenditure	government expenditure	total investment	foreign exchange receipts	

[a]Includes increase in reserves.

commodity receipts from sales (again at purchaser prices) of consumption goods to households and the government, of investment goods to the capital account, and of exports to the rest of the world. Factor receipts (transaction t_{31}) record the value-added payments from the activities accounts and factor-service exports (transaction t_{38}) from the rest of the world.[15]

Now consider the receipts of institutions. Enterprises receive payments from two sources. The first is gross profits from the factors account, transaction t_{43}; the second is transfers from the government account (t_{46}). Households receive payments from four sources; the first is wages from the factors account (t_{53}). The second and third are from other institutional accounts: distributed profits from enterprises (t_{54}) and transfers from the government (t_{56}). The fourth source is foreign remittances (t_{58}). The government receives payments from the first five accounts: indirect taxes from activities (t_{61}), tariffs from commodities (t_{62}), factor taxes (t_{63}), enterprise taxes (t_{64}), and direct taxes from households (t_{65}).

The capital account receives payments in the form of domestic and foreign savings. Transaction t_{74} comprises the retained earnings of enterprises, while t_{75} and t_{76} represent the savings of households and the government, respectively. Capital transfers from abroad, including any increase in reserves, are received from the rest of the world in transaction t_{78}.

Lastly, there are the receipts of the rest of the world. The first of these is import payments from the domestic commodity account, transaction t_{82}. The second is factor-service imports (t_{83}).[16] Finally, the rest-of-the-world account receives three types of transfers: transfers abroad from persons (t_{86}) and government (t_{85}) and capital transfers abroad (t_{87}).

To illustrate an actual macroeconomic SAM, we next consider a SAM of the United States based on the 1989 National Income and Product Accounts (NIPA). This macroeconomic SAM has twelve accounting categories. Accounts 1 and 2 are the activity and commodity accounts, respectively. There are two factor accounts: labor (account 3) and property (account 4). Gross national product or value added is allocated between accounts 2 and 3 in accordance with the conventions adopted by the Department of Commerce in their input–output accounts (U.S. Department of Commerce, 1984). That is, charges against GNP are broken up into three types: (1) compensation of employees, which is received by labor; (2) profit-type income, net interest, and capital consumption allowances, which are received by property;[17] and (3) indirect business taxes, which are received by government.

15 Factor-service exports consist of a flow of profits into the country in question from its foreign investments.

16 Factor-service imports consist of a flow of profits from the country in question to foreign investors.

17 Profit-type income consists of proprietors' income, rental income of persons, corporate profits, and business transfer payments, less (subsidies less current surplus of government enterprises).

Account 5 is the enterprise account. Accounts 6 and 7 are the household and government accounts, respectively. Account 8 is the capital account, which closes the system of income-expenditure flows. Account 9 is the rest-of-the-world account (ROW), which records international transactions. Account 10 collects tariffs and distributes them to the government. Accounts 11 and 12 are the errors account and the total account, respectively.

To construct the macroeconomic SAM requires a mapping between the NIPA account items and the twelve SAM accounts. The mapping used is detailed in Reinert and Roland-Holst (1992), and its implementation for the year 1989 is presented in Table 4.4.[18] The mapping is designed so that factor-service imports (transaction t_{94}) and factor-service exports (transaction t_{49}) are broken out of net output. We assume that all factor-service payments are for capital. In contrast to typical practice, we define gross domestic product (GDP) to be net of imports valued at market prices rather than border prices. Therefore, the \$5,145,736 transaction t_{12} represents the typically defined GDP *less* customs duties. We do this because it is government, not activities, that engages in tariff collection.

Property income is passed on to enterprises (t_{54}) and to the rest of the world in the form of factor-service imports (t_{94}), whereas some of labor income is passed to the government in the form of social insurance contributions (t_{73}). Enterprise income is distributed among households (t_{65}), government (t_{75}), and the capital account (t_{85}). Household income is distributed between commodities (t_{26}), enterprises (t_{56}), government (t_{76}), the capital account (t_{86}), and the rest of the world (personal transfer payments in t_{96}). Government receipts are spent on commodities (t_{27}) and transfers to enterprises (t_{57}) to households (t_{67}), and to the rest of the world (interest and transfer payments to foreigners in t_{97}). Capital account expenditures are divided between commodities (t_{28}) and the government deficit. This last item, \$87,832 million (\$87.8 billion) in transaction t_{78}, represents a net deficit for federal, state, and local governments combined. The rest of the world makes payments to the commodities account for exports of goods and nonfactor services (t_{29}), to the property account for factor-service exports (t_{49}), and to the capital account in the form of net foreign investment (t_{89}). The tariff account makes payments to the government ($t_{7,10}$).

SAMs which are actually used to calibrate CGE trade policy models have a sectoral structure instead of just a single sector as in macroeconomic SAMs such as the one presented in Table 4.4. To give the reader a sense of what a multi-sector SAM looks like, Table 4.5 presents a 1989 SAM for the United

18 The mapping is an adaptation of that presented in Hanson and Robinson (1991).

Table 4.4. *A macro SAM for the United States, 1989 (millions of dollars)*

	Expenditures											
Receipts	1 Activ.	2 Commod.	3 Labor	4 Prop.	5 Enter.	6 Hsehld.	7 Govt.	8 Capital	9 ROW	10 Tariff	11 Error	12 Total
1. Activities	0	5,145,736	0	0	0	0	0	0	0	0	0	5,145,736
2. Commodities	0	0	0	0	0	3,450,085	1,025,579	771,232	490,991	0	0	5,737,887
3. Labor	3,079,017	0	0	0	0	0	0	0	0	0	0	3,079,017
4. Property	1,687,273	0	0	0	0	0	0	0	135,235	0	0	1,822,508
5. Enterprises	0	0	0	1,724,858	0	102,175	93,057	0	0	0	0	1,920,090
6. Households	0	0	2,602,254	0	1,177,548	0	604,472	0	0	0	0	4,384,274
7. Government	396,494	0	476,763	0	135,092	658,754	0	87,832	0	17,481	0	1,772,416
8. Capital acct.	0	0	0	0	607,450	171,834	0	0	96,828	0	-17,048	859,064
9. Rest of World	0	574,670	0	97,650	0	1,426	49,308	0	0	0	0	723,054
10. Tariffs	0	17,481	0	0	0	0	0	0	0	0	0	17,481
11. Errors and Omissions	-17,048	0	0	0	0	0	0	0	0	0	0	-17,048
12. Total	5,145,736	5,737,887	3,079,017	1,822,508	1,920,090	4,384,274	1,772,416	859,064	723,054	17,481	-17,048	

Table 4.5. *A 1989 SAM with sectoral detail for the United States (millions of dollars)*

	1 agforfsh	2 mining	3 construct	4 ndurmfg	5 durmfg	6 trcomut	7 trade	8 fininsre	9 services
1 agforfsh	38,126	8	1,601	99,351	3,915	54	2,068	5,758	7,323
2 mining	91	12,581	2,415	94,532	9,239	30,424	1	32	38
3 construct	1,838	6,256	817	12,092	14,977	31,392	11,435	48,436	36,287
4 ndurmfg	28,226	1,790	31,129	380,908	94,362	31,703	26,253	12,765	171,496
5 durmfg	3,368	4,519	164,147	53,692	554,616	16,832	8,019	3,197	68,872
6 trcomut	3,965	2,094	16,987	74,988	67,178	89,804	55,069	23,890	100,551
7 trade	9,122	1,980	114,570	77,008	96,320	20,121	26,271	14,077	78,059
8 fininsre	7,301	4,639	6,425	19,202	25,768	14,058	54,642	186,314	92,169
9 services	2,326	3,000	66,338	83,685	88,934	34,296	165,043	90,875	287,060
10 labor	35,840	18,361	206,728	227,430	448,031	219,985	395,520	217,032	1,310,090
11 property	65,927	46,332	33,356	152,053	73,094	218,134	155,628	567,000	375,748
12 enterprise	0	0	0	0	0	0	0	0	0
13 household	0	0	0	0	0	0	0	0	0
14 government	8,472	10,109	7,624	28,363	19,242	37,254	135,160	117,844	32,427
15 capaccount	0	0	0	0	0	0	0	0	0
16 row	11,115	41,593	0	123,195	304,973	51,908	0	4,202	37,683
17 rowtaxes	169	212	0	8,860	8,240	0	0	0	0
18 error	-406	-276	-912	-1,502	-1,990	-1,751	-2,528	-3,322	-4,360

	10 labor	11 property	12 enterprise	13 household	14 govt.	15 capaccnt.	16 row	17 rowtaxes	18 error
1 agforfsh	0	0	0	24,532	9,261	-4,660	28,143	0	0
2 mining	0	0	0	307	827	-4,495	7,208	0	0
3 construct	0	0	0	0	124,985	362,709	0	0	0
4 ndurmfg	0	0	0	496,358	53,700	8,858	96,307	0	0
5 durmfg	0	0	0	228,777	139,146	323,428	238,286	0	0
6 trcomut	0	0	0	249,367	44,338	10,919	55,067	0	0
7 trade	0	0	0	518,776	15,675	60,601	0	0	0
8 fininsre	0	0	0	834,059	17,544	13,804	12,173	0	0
9 services	0	0	0	1,097,910	620,103	68	53,807	0	0
10 labor	0	0	0	0	0	0	0	0	0
11 property	0	0	0	0	0	0	135,235	0	0
12 enterprise	0	1,724,858	0	102,175	93,057	0	0	0	0
13 household	2,602,254	0	1,177,548	0	604,472	0	0	0	0
14 government	476,763	0	135,092	658,754	0	87,832	0	17,481	0
15 capaccount	0	0	607,450	171,834	0	0	96,828	0	-17,048
16 row	0	97,650	0	1,426	49,308	0	0	0	0
17 rowtaxes	0	0	0	0	0	0	0	0	0
18 error	0	0	0	0	0	0	0	0	0

note: Key to sectors: agforfsh–agriculture, forestry, and fishing; mining–mining and mineral resources; construct–construction; ndurmfg–nondurable manufacturing; durmfg–durable manufacturing; trcomut–transportation, communication, and utilities; trade–wholesale and retail trade; fininsre–finance, insurance, and real estate; services–personal, business, and public services; labor–labor value added; property–property value added; enterprise–enterprise; household–household final demand; government–government final demand; capaccount–savings and investment; row–rest of the world; rowtaxes–rest of the world taxes (tariffs).

States in which the economy has been divided into nine broad sectors. The reader will find that the transaction values in rows and columns 3–11 of Table 4.4 are repeated in rows and columns 10–18 of Table 4.5. The activity and commodity accounts of Table 4.4 (rows and columns 1 and 2) have been replaced by a set of nine commodity accounts.[19] Let us look at columns 1–9. Rows 1–9 of these columns give the interindustry transactions matrix. Rows 10 and 11 give the value added payments. Row 14 records the indirect business tax payments of each sector. Row 16 records imports, and Row 17 records tariff collections. Finally, Row 18 is a sectoral distribution of the macroeconomic accounting error.

Next, let us look at rows 1–9 of columns 13–16 in Table 4.5. In column 13, these rows give sectoral household expenditures. In columns 14 and 15, these give government and investment (including inventories) demand, respectively. Finally column 16 gives sectoral export demands.

In most countries, the United States included, the data which go into a SAM such as that presented in Table 4.5 come from a number of sources: macroeconomic accounts, input–output accounts, trade data, etc. The process of combining these data, which may not all be from the same year, into a consistent and balanced SAM is neither fully a science nor merely an art. The next section gives the reader some idea of how this process works.

III SAM Construction

Building a SAM of any size can be a tedious exercise. However, while there are few rewards in SAM construction *per se*, we have found the process well worth the effort in its contribution to trade policy analysis. This section will outline some general procedures we have used in SAM construction with the hope of minimizing difficulties for others embarking on similar efforts.[20]

Construction of a SAM begins with the transformation of the country's macroeconomic accounts into a macroeconomic tableau such as that presented in Table 4.4. This macroeconomic SAM provides control totals for each submatrix of the detailed SAM, as well as scalar interinstitutional transfers. The most recent year for which the macroeconomic data are available sets a limit on the choice of a base year. For a multi-country SAM, macroeconomic SAMs must be constructed for each of the member countries. Next, the individual macroeconomic SAMs must be joined into a single

19 Activity accounts are removed by using a procedure described in Section III.

20 The interested reader also may want to consult Keuning and de Ruijter (1988). Reinert and Roland-Holst (1992) describe the construction of a U.S. social accounting matrix for trade policy analysis in some detail, and Reinert, Roland-Holst, and Shiells (1993) describe the construction of a multi-country social accounting matrix of North America.

macroeconomic SAM. This involves converting the macroeconomic SAMs into a single currency using exchange rates, including trade flows among the member countries, and subtracting these from the respective rest-of-the-world accounts. Finally, factor-service flows and capital flows among the member countries might be added, with appropriate subtractions made from the respective rest-of-the-world acounts.

To construct the detailed accounts, the practitioner will need input–output tables for the country or countries in question and a host of other data sources to be mentioned later. It is typically the case that recent data are at a higher level of sectoral aggregation than less recent data. For example, detailed input–output accounts might be available five years prior to the chosen base period. To deal with this problem, we generally use the most recent data first for broad sectoral aggregates and then estimate more detailed sectoral transactions using shares from the less recent data. Often this procedure will evolve into a cascade of data steps progressing from high levels of aggregation and recent data to progressive disaggregation with less recent data, the latter being used in share form. While tedious, this approach attempts to make the SAM as timely as possible given the available data.

Construction of sectoral accounts can begin with activity output data, perhaps disaggregated to the required degree by the cascade procedure just described. To obtain commodity output data, the make matrix from the input–output accounts can be row normalized and premultiplied by a row vector of the activity outputs. The make matrix can then be updated to the base year by using a matrix balancing method, such as the RAS procedure, with activity and commodity output vectors as control totals.[21]

For the value-added submatrix, sectoral control totals are needed and may have to be estimated by a cascade procedure. The sectoral totals can be allocated among labor income, property income, and indirect business taxes based on shares from the input–output data. Finally, the value-added submatrix is balanced to the macro-SAM control totals by using the RAS procedure.

To estimate the import and export submatrices, control totals for both are taken from the macroeconomic SAM. Data on imports and exports for the base (or nearest) year should be available from the country's customs agency.[22] The resulting totals of imports and exports are unlikely to agree

21 Matrix balancing methods, including the RAS procedure, are discussed in the Appendix.
22 Customs agencies generally only collect data on merchandise trade. For an attempt to incorporate service trade data into a SAM of the United States, see the Appendix of Dighe, Francois, and Reinert (1994).

with the control totals taken from the macroeconomic SAM. Such discrepancies can be allocated among the sectors in proportion to their shares in the resulting totals.

Final demand submatrices can contain four types of vectors: household demand vectors, government demand vectors, investment demand vectors, and export demand vectors. In a trade model, it will be necessary to have separate export demand.[23] The simplest way to deal with domestic final demand is to aggregate the household, government, and investment final demands into a single vector.[24] Alternatively, one can maintain separate household, government, and investment accounts.[25] The former approach avoids a number of macroeconomic closure issues discussed in Section IV, but the latter is more complete and consistent with national income accounting conventions. For the purposes of this section, we assume the various final demand vectors are maintained.[26]

A dated final demand submatrix will be available from the input–output accounts. The export vector will be updated as described. The household, government, and investment vectors can be simply updated, using control totals from the macroeconomic SAM and shares from the dated vectors. Alternatively, more recent data on sectoral final demand by institution can be used if they are available. One wrinkle arises from those commodities without intermediate deliveries. In these cases, total final demand must be set equal to estimated commodity supply. In practice, the construction of the final demand submatrix can be difficult and may require ingenuity and selective matrix balancing.

The last step is to update the use matrix to the base year. For the row control vector, the estimated vector of commodity output, plus the import vectors, and less the final demand vectors will be used. For the column control vector, the activity output vector, less the value-added and error vectors, will be used. With these control vectors, the use matrix can be updated by using the RAS procedure.

23 Multi-country SAMs have more than one export demand vector, whereas a single-country SAM would have only one.
24 This is the approach taken by de Melo and Tarr (1992).
25 This was the approach taken by Reinert and Roland-Holst (1991).
26 One can go further in detailing final demand vectors. The disaggregation of the household final demand vector into income levels and the mapping of labor and property income into these household types would provide the opportunity to explore the relationship between trade policies and the household distribution of income. This disaggregation of the household sector is at the heart of the original purpose of the SAM framework (see Stone, 1985) but has not been applied very often in trade policy modeling. For exceptions, see Chapter 10 of this volume by Khan. See also Hanson and Reinert (1994).

Most trade policy applications of SAMs require that the make and use matrices be consolidated into a transactions matrix. There are two possible approaches: to eliminate the commodity accounts or to eliminate the activity accounts. The following example consolidates via elimination of activity accounts. We then explain why the resulting SAM may be more suitable for trade policy analysis than one obtained by elimination of commodity accounts.

Consider a simple case with two activity accounts ($A1, A2$), two commodity accounts ($C1, C2$), a demand account (D), and a rest-of-the-world account (R). We partition the SAM with the accounts to be retained ($C1, C2, D, R$) in the upper-left-hand corner:

	$C1$	$C2$	D	R	$A1$	$A2$
$C1$:	0	0	F_1	E_1	U_{11}	U_{12}
$C2$:	0	0	F_2	E_2	U_{21}	U_{22}
D:	0	0	0	0	V_1	V_2
R:	I_1	I_2	0	0	0	0
$A1$:	M_{11}	M_{12}	0	0	0	0
$A2$:	M_{21}	M_{22}	0	0	0	0

The use matrix has coefficients U_{ij}, and the make matrix has coefficients M_{ji}. Final demands are F_i, and the value added values are V_j. Imports and exports are I_i and E_i, respectively.

As Pyatt (1985) demonstrates, the consolidated accounts may be expressed in matrix form:[27]

$$T = \begin{bmatrix} 0 & 0 & F_1 & E_1 \\ 0 & 0 & F_2 & E_2 \\ 0 & 0 & 0 & 0 \\ I_1 & I_2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \underline{U}_{11} & \underline{U}_{12} \\ \underline{U}_{21} & \underline{U}_{22} \\ \underline{V}_1 & \underline{V}_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 \end{bmatrix} \qquad (4.3)$$

where the underbars denote column-sum normalized coefficients.

As can be seen from this expression, the trade accounts are preserved in their original form by commodity. This is *not* the case for the elimination of the commodity accounts under which the imports (including tariffs) and exports are apportioned. Therefore, elimination of activity accounts facilitates interpretation for trade policy analysis.

27 It can be shown that the use of the Pyatt apportionment method to eliminate either the commodity or the activity accounts implies the assumption of activity technology. On the distinction between commodity and activity technologies, see United Nations Statistical Office (1968, p. 39).

IV SAMs and the General Equilibrium Analysis of Trade Policy

By themselves, SAMs are insufficient to allow the analyst properly to assess the impacts of changes in trade policies. This limitation has been clearly expressed by Thorbecke (1985):

The SAM is clearly an essential tool in diagnosing the initial situation and in organizing data in a systematic way with respect to accounts and the classification and interrelationship of variables appearing in these accounts. At the same time, by itself, the SAM is nothing more than a snapshot in time, yielding base-year information in a consistent way among a whole set of variables. If the SAM is to be used for policy rather than purely diagnostic purposes, it has to be coupled with a conceptual framework that contains the behavioral and technical relationships among variables within and among sets of accounts or modules. In other words, the SAM as a data framework is a large-scale identity which, to come alive, should be linked to a model of the causal relationships among variables. (p. 207)

An exhaustive description of the transition from SAM to CGE model is beyond the scope of this chapter. Instead, we will touch on a few issues which the practitioner may face in moving from the one to the other. These issues are flexible aggregation, calibration, and closure.

IV.1 Flexible Aggregation

By its nature, trade policy often is directed at detailed sectors. Import quotas on cheese, export taxes on coffee, and high tariffs on machine tools are the focal points of commercial policies. It is often important, then, to have very detailed accounts to assess these policies. Unfortunately, implementing a CGE model at such a level of disaggregation would be difficult numerically and would generate vast amounts of information extraneous to the issue at hand. These considerations have led us to advocate a "flexible aggregation" approach to using SAMs for trade policy analysis. The base SAM is estimated for as many sectors as possible to address a broad spectrum of detailed industry issues. For a particular application, however, those detailed sectors which do not bear on the current problem are aggregated into fewer broad sectors (e.g., one-digit SITC).

The role of the SAM and flexible aggregation in supporting CGE modeling of trade policy is represented in Figure 4.1. The fully disaggregated base SAM is denoted as *SAM* I in this figure, and the initial exogenous behavioral parameter estimates are denoted *EPE* I. The aggregation procedure takes the information in this disaggregated data base and creates a

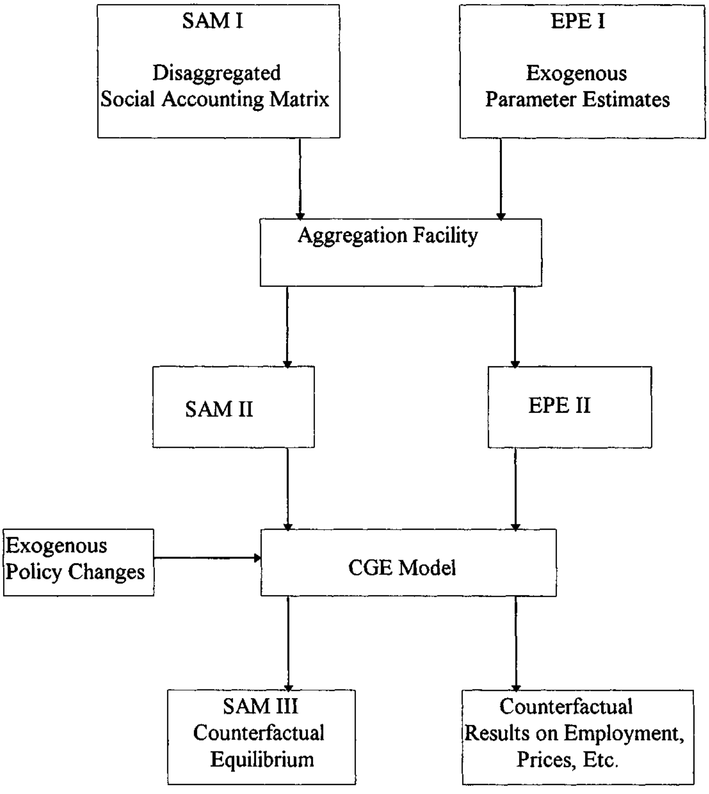

Figure 4.1. *The flexible aggregation procedure*

second SAM and corresponding parameter set at the level of aggregation specified for a given analysis. The resulting aggregates are labeled *SAM* II and *EPE* II in the figure. *SAM* II composes the benchmark equilibrium data set to calibrate the CGE model. The analyst introduces an exogenous, counterfactual policy change, such as a tariff cut, and the behavioral model simulates the response of the economy to such a policy change. This results in a counterfactual equilibrium which can be expressed as a new SAM, denoted *SAM* III. In this way, *the modeling exercise begins and ends with a SAM*. At the third stage, the model also produces a large volume of subsidiary counterfactual results on changes in employment, trade, production, etc.

Consider an example. Flynn and Reinert (1993) undertook a study of the U.S. dairy quotas. Their CGE model was structured around five detailed dairy sectors and nine aggregate sectors for a total of fourteen model sectors. The detailed dairy sectors were dairy farm products, butter, cheese, con-

Table 4.6. *A 1989 SAM for the analysis of the U.S. dairy quotas (millions of dollars)*

	1 dairyfarms	2 butter	3 cheese	4 condevap	5 fldmilk	6 agforfsh	7 mining	8 construct	9 ndurmfg	10 durmfg	11 trcomut
1 dairyfarms	0	669	3,769	2,372	10,194	400	0	0	1,155	7	0
2 butter	0	29	19	10	80	0	0	0	498	0	8
3 cheese	0	15	2,411	54	85	0	0	0	78	3	1
4 condevap	0	38	60	321	349	0	0	0	1,266	0	0
5 fldmilk	0	669	479	428	1,249	0	0	0	467	1	6
6 agforfsh	3,992	0	21	6	1,005	33,685	8	1,602	80,167	3,914	54
7 mining	0	0	2	0	0	90	12,585	2,420	94,613	9,262	30,416
8 construct	103	8	277	39	179	1,700	6,144	803	11,377	14,741	30,812
9 ndurmfg	4,129	134	998	503	2,603	24,021	1,787	31,122	367,569	94,391	31,610
10 durmfg	117	29	124	248	273	3,223	4,485	163,170	52,640	551,637	16,695
11 trcomut	389	40	263	142	444	3,564	2,089	16,982	73,996	67,197	89,580
12 trade	405	187	795	286	761	8,675	1,972	114,328	74,734	96,172	20,034
13 fininsre	362	16	64	63	267	6,916	4,628	6,421	18,759	25,768	14,019
14 services	71	41	270	100	673	2,246	2,992	66,288	82,447	88,917	34,194
15 labor	1,790	363	1,572	534	2,466	34,020	18,361	207,063	222,621	449,018	219,857
16 property	6,779	172	1,201	802	4,248	59,105	46,340	33,415	145,738	73,268	218,046
17 enterprise	0	0	0	0	0	0	0	0	0	0	0
18 household	0	0	0	0	0	0	0	0	0	0	0
19 government	531	20	60	19	189	7899	10,068	7,605	27,975	19,207	37084
20 capaccount	0	0	0	0	0	0	0	0	0	0	0
21 row	0	2	353	22	13	11,088	41,524	0	122,670	305,141	51,792
22 rowtaxes	0	0	34	0	0	168	211	0	8,818	8,247	0

	12 trade	13 fininsre	14 services	15 labor	16 property	17 enterprise	18 household	19 government	20 capaccount	21 row	22 rowtaxes
1 dairyfarms	0	0	0	0	0	0	103	0	0	0	0
2 butter	1	0	830	0	0	0	561	348	0	50	0
3 cheese	3	0	1,255	0	0	0	8,506	330	0	31	0
4 condevap	1	0	463	0	0	0	2,592	436	0	424	0
5 fldmilk	2	0	1,827	0	0	0	18,150	1,768	0	37	0
6 agforfsh	2,065	5,749	7,316	0	0	0	24,491	9,298	-4,804	28,231	0
7 mining	0	31	37	0	0	0	307	831	-4,637	7,236	0
8 construct	11,220	47,520	35,618	0	0	0	0	123,296	367,381	0	0
9 ndurmfg	26,170	12,727	166,712	0	0	0	467,106	50,957	9,118	95,931	0
10 durmfg	7,951	3,169	68,315	0	0	0	227,727	138,713	331,045	237,334	0
11 trcomut	54,911	23,819	100,305	0	0	0	249,636	44,451	11,239	55,159	0
12 trade	26,148	14,010	77,726	0	0	0	518,389	15,686	62,268	0	0
13 fininsre	54,470	185,717	91,918	0	0	0	834,724	17,584	14,205	12,190	0
14 services	164,492	90,567	286,225	0	0	0	1,098,576	621,397	70	53,871	0
15 labor	395,141	216,811	1,309,394	0	0	0	0	0	0	0	0
16 property	155,506	566,524	375,614	0	0	0	0	0	0	135,744	0
17 enterprise	0	0	0	0	1,724,729	0	102,170	93,190	0	0	0
18 household	0	0	0	2,602,196	0	1,177,398	0	604,679	0	0	0
19 government	134,493	117,257	32,280	476,820	0	135,094	658,100	0	90,225	0	17,481
20 capaccount	0	0	0	0	0	607,597	171,702	0	0	96,811	0
21 row	0	4,191	37,600	0	97,778	0	1,427	49,447	0	0	0
22 rowtaxes	0	0	0	0	0	0	0	0	0	0	0

densed and evaporated milk, and fluid milk. The nine aggregate sectors were broad groups utilized in the U.S. National Income and Product Accounts. The resulting SAM is presented in Table 4.6. Unlike the SAM in Table 4.5, the SAM in Table 4.6 has no error account. Since modeling an error account makes little economic sense, this account was removed and the matrix rebalanced. Therefore, the inter-institutional transactions in Table 4.6 will differ slightly from those in Table 4.5.

Row 1 of columns 2–5 in Table 4.6 illustrates the relevance of SAMs to trade policy analysis. The dairy farm sector is an important upstream supplier to the butter, cheese, condensed and evaporated milk, and cream sectors, all of which are protected by quotas. In fact, these quotas are in place to protect labor and property incomes in the dairy farm sector via these input–output relationships. The flexible aggregation approach allows the practitioner to focus on such detailed linkages without confusing the model with an unwieldy number of sectors.

IV.2 Calibration

As we stated in the introduction, the term "calibration" refers to the process of calculating intercept and share parameters of a CGE model's mathematical functions (given exogenously specified behavioral elasticities) so that the model will replicate the base year SAM as an equilibrium solution. Nearly every transaction in the SAM is used to calibrate a model function, calculate a policy parameter, or define a model constraint. Consider Table 4.6 as an example. The upper-left-hand submatrix formed by accounts 1–14 contains 196 transactions which will be involved in calibrating (typically fixed) input–output coefficients. Rows 15 and 16 of columns 1–4 contain value-added transactions which will be used to calibrate value-added functions such as Cobb–Douglas or constant elasticity of substitution (CES). Row 19 of columns 1–14 gives the indirect business tax collections which will be used to calculate *ad valorem* indirect business tax rates. Row 21 of columns 1–14 gives imports. These will be used in the calibration of (typically CES) import aggregation functions.[28] The tariff collections given in row 22 of columns 1–14 will be used to calibrate *ad valorem* tariff equivalents.

Final demand vectors are given in rows 1–14 of columns 18–21. The transactions in column 18 will be used in calibrating a household demand

28 See de Melo and Robinson (1989) and Chapter 6 of this volume by Devarajan, Go, Lewis, Robinson, and Sinko.

function such as a Cobb–Douglas, CES, or Linear Expenditure System (LES). The transactions in column 21 give export demands and will be used to calibrate export aggregation functions such as constant elasticity of transformation (CET).[29] Columns 19 and 20 give government and investment demands. The way these transactions are dealt with involves questions of macro closure discussed later.

The remaining submatrix defined by accounts 15–22 contains interinstitutional transactions. Foreign remittances, which transactions $t_{16,21}$ less transactions $t_{21,16}$ and comprise $t_{21,18}$, enter into the balance of payments constraint. Transaction $t_{20,21}$ is foreign savings, which also is an element of the balance of trade constraint as well as the savings–investment balance. Transaction $t_{21,19}$ is government lending and enters both the balance of payments and government budget constraints. The government deficit is represented in transactions $t_{19,20}$ and enters the government budget constraint and the savings–investment balance. Transactions $t_{17,19}$ and $t_{18,19}$ are government transfers, which enter the government budget constraint and the income equations. Private savings is given by the sum of transactions $t_{20,17}$ and $t_{20,18}$. This will be used to calculate a savings rate. Factor tax collections given in transactions $t_{19,15}$ and $t_{19,17}$ are used to calculate factor tax rates. The income tax rate will be calculated by transaction $t_{19,18}$.

The remaining interinstitutional transactions ($t_{17,16}$, $t_{17,18}$, $t_{18,15}$, $t_{18,17}$, and $t_{19,22}$) will not be directly used. The first four of these will be implicit in the model structure and the last (total tariff revenue) will be calculated as a sum of sectoral tariff collections.

IV. 3 Closure

The term "closure" refers to prescribing which variables are endogenous and exogenous in a general equilibrium system. For example, in a system of linear equations based on a SAM, closure is simply the choice of which accounts are to be the exogenous accounts and which are to be endogenous accounts.[30] A number of different closure issues must be addressed in building a CGE model, and we discuss only a few of them here, concentrating on those most closely related to the SAM.[31] Before proceeding, it will be helpful to say a word about the specification of a numéraire. A convenient numéraire is a weighted average index of the model's prices or a GDP deflator. This variable is then fixed exogenously, and the exchange rate of

29 Again, see de Melo and Robinson (1989) and Chapter 6 of this volume.
30 See Robinson (1989).
31 Other closure issues are discussed in Chapter 7 of this volume by Blonigen, Flynn, and Reinert.

the model behaves as a real exchange rate.[32] In what follows, we assume that such a specification has been implemented.

In the preceding calibration section, we mentioned that a foreign savings variable is given an initial value from the SAM. In one type of foreign sector closure, this variable is made exogenous and the real exchange rate is endogenous. Changes in the real exchange rate ensure that the base period current account position is maintained. Alternatively, this foreign savings variable can be made endogenous and the real exchange rate would then be held fixed exogenously. In this latter case, however, welfare calculations are difficult to interpret, since foreign exchange (a basic resource) in the hands of the economy in question will change. We also mentioned a government deficit variable. Typically, one fixes government demand in each sector in real terms, and the government deficit is thereby endogenous. This is a neoclassical approach. Alternatively, one could take a more structuralist view and fix this or other magnitudes in nominal terms.[33]

Perhaps the most important closure issue from a macroeconomic viewpoint is how the savings–investment balance is achieved. Private savings, foreign savings, and government savings all enter into this balance. A straightforward closure which facilitates welfare analysis based on the household sector account is one in which government and investment demands are fixed in each sector in real terms.[34] The numéraire price index is fixed. Foreign savings is fixed in terms of the foreign currency, and the household savings rate is fixed. The government transfer variable is specified as endogenous and maintains the savings–investment balance. This approach has been used by Devarajan and Rodrik (1991) and Flynn and Reinert (1993), among others. A savings-driven closure described by Robinson (1989, 1991) involves investment determined by savings. In this case, real investment demands are endogenous, and welfare changes cannot be properly measured on the basis of the household sector alone. This specification brings intertemporal considerations into any welfare analysis. Finally, alternative "structuralist" closures are available and are discussed in some detail by Taylor (1990).

32 We use the term "exchange rate" to refer to a conversion factor that translates world prices of imported or exported goods into domestic prices. See de Melo and Robinson (1989) and de Melo and Tarr (1992).

33 As Taylor (1990) states, "A . . . distinctive feature of structuralist models is that they are *not* set up in 'real' terms (i.e. with only relative prices). Rather, they explicitly include prices and income flows in nominal or money terms" (p. 4). For neoclassical models, homogeneity of degree zero can be checked by exogenously increasing the numéraire variable by a small amount. All prices and variables expressed in terms of prices (nominal magnitudes) should increase by the same proportionate amount. Real quantities should not change at all. See Condon, Dahl, and Devarajan (1986).

34 In a static trade-policy model, investment demand merely maintains accounting balance since it does not augment the capital stock.

A number of these closure issues can be avoided by aggregating the household, government, and investment accounts into a single final demand account (e.g., de Melo and Tarr, 1992). Turning to the accounting identities presented at the bottom of Table 4.2, the government budget and savings–investment balances are no longer operative. However, there will be an endogenous intra-institutional transfer from the final demand account to itself.

V Concluding Comments

The workhorse of the trade theorist is the 2×2×2 Heckscher–Ohlin–Samuelson model. Unfortunately for the trade policy analyst, this simple model offers little guidance to the effects of real-world policy changes. This has been stated quite clearly by two notable trade theorists, Dixit and Norman (1980):

> We should stress . . . that one should avoid drawing general conclusions from the two-good results. The moral . . . is that one cannot say much about the general equilibrium effects of changes in parameters without knowing the exact values of the parameters and the exact characteristics of demand and supply functions. If the theory is to be applied, therefore, it should be done by putting numerical values into the general formulae; not by applying qualitative results from the two-good case directly. (pp. 127–128)

If trade policy analysts are to provide accurate assessments of trade policy changes, they must "put numerical values in the general formulae" as carefully as they can. In our view, the SAM framework helps the analyst to do just that. It represents a comprehensive and consistent framework for developing databases for rigorous economic methods like applied general equilibrium analysis. Finally, it helps in the reconciliation of the numerous data sources to complete the detailed picture of economywide activity.

Appendix 4.1

SAM Balancing

One of the objectives of the Cambridge Growth Project was to estimate a detailed SAM of the United Kingdom for the year 1960. A transactions matrix was only available for 1954, so Stone (1962) suggested a procedure to update the matrix to 1960. This "RAS" method takes its name from the notation used in Stone's original equations. The RAS method estimated a transactions matrix for the year 1960 by starting with the 1954 transactions matrix, expressing it in 1960 prices, and adjusting rows and columns iteratively so that they add up to the 1960 totals.[35]

35 See Bacharach (1970, Chapter 3).

Let $\boldsymbol{R}_0$ be a known, initial matrix of transactions and let $\boldsymbol{R}$ be the unobservable transaction matrix for the year we desire to estimate. Let $\boldsymbol{p}$ be a vector whose elements are the ratios of desired period prices to initial period prices. Let $\langle z \rangle$ denote the diagonal matrix having vector z on its main diagonal. The $\boldsymbol{R}$ matrix in desired period prices then takes the form[36]

$$\boldsymbol{R}_0^* = \langle \boldsymbol{p} \rangle \boldsymbol{R}_0 \langle \boldsymbol{p} \rangle^{-1} \tag{A4.1}$$

The next step is to calculate a column vector of intermediate outputs for the desired year as the difference between gross outputs and final demands. Stone and Brown (1965) denote this vector $\boldsymbol{u}$. The row vector $\boldsymbol{v}$ of intermediate inputs for the desired year is the difference between gross outputs and value added.

The following constraints must be satisfied:

$$\boldsymbol{Ri} = \boldsymbol{u} \tag{A4.2}$$

$$\boldsymbol{i}'\boldsymbol{R} = \boldsymbol{v} \tag{A4.3}$$

where $\boldsymbol{i}$ is the conformal unit column vector. Equation A4.2 states that the rows of the new transaction matrix must sum to the observed row totals. Equation A4.3 states that the columns must sum to the observed column totals.

The problem is then to adjust $\boldsymbol{R}_0^*$ to obtain an estimate of $\boldsymbol{R}$. The RAS algorithm proceeds as follows:[37]

Step 0 (Initialization): Set $k = 0$ and $\boldsymbol{R}^k = \boldsymbol{R}_0^*$.

Step 1 (Row Scaling):

Define $\rho^k = \langle \boldsymbol{u} \rangle (\boldsymbol{R}^k \boldsymbol{i})^{-1}$

and update $\boldsymbol{R}^k$ as $\acute{\boldsymbol{R}} \leftarrow \langle \rho^k \rangle \boldsymbol{R}^k$

Step 2 (Column Scaling):

Define $\sigma^k = (\boldsymbol{i}'\acute{\boldsymbol{R}})^{-1} \langle \boldsymbol{v} \rangle$

and define R^{k+1} by $R^{k+1} = \acute{\boldsymbol{R}} \langle \sigma^k \rangle$

Step 3: Replace $k \leftarrow k+1$ and return to Step 1.

The algebraic RAS has a number of limitations. First, it cannot handle negative matrix elements. While this is not a problem for balancing the transactions matrix, it could be a problem for balancing other components of a SAM. Second, it is necessary to rescale the problem if any negative row or column totals appear. This rarely arises in practical work, however. Finally, the method assumes that the elements of the matrix are identically uniformly distributed random variables. This

36 The reader might multiply out a 2×2 example to elucidate this adjustment.
37 See Stone and Brown (1965) and Schneider and Zenios (1990).

may not always be the case if one is less certain about some elements of the matrix than others. It is this consideration that has led to research in new matrix balancing techniques.

Byron (1978) proposed the estimation of $\boldsymbol{R}$ by the minimizing of a constrained quadratic loss function. Let $\boldsymbol{r}_0^*$ denote the column vector created from the row vectorization of the nonzero elements of $\boldsymbol{R}_0^*$. Similarly, the column vector created from the row vectorization of estimates of the nonzero elements of $\boldsymbol{R}$ is denoted by $\boldsymbol{r}$. Now re-express (A4.2) and (A4.3) as

$$\boldsymbol{Gr}-\boldsymbol{h}=\boldsymbol{0} \tag{A4.4}$$

The objective will be for the estimates $\boldsymbol{r}$ to be as close to $\boldsymbol{r}_0^*$ as possible in a quadratic loss sense subject to the constraints in (A4.4). This can be accomplished by using the following constrained quadratic loss function:

$$Z=\frac{1}{2}\left(\boldsymbol{r}-\boldsymbol{r}_0^*\right)'\boldsymbol{V}^{-1}\left(\boldsymbol{r}-\boldsymbol{r}_0^*\right)+\lambda'\left(\boldsymbol{Gr}-\boldsymbol{h}\right) \tag{A4.5}$$

The term $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers. The diagonal matrix $\boldsymbol{V}$ consists of weights that indicate the degree of certainty (variance) in the original $\boldsymbol{r}_0^*$. The less the certainty, the less important are the differences between the estimated element and the original element. Byron proposes a conjugate gradient algorithm for minimizing (A4.5).

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