LEON WALRAS AND HIS ECONOMIC SYSTEM

A Review Article

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"Thus the system of the economic universe reveals itself, at last, in all its grandeur and complexity: a system at once vast and simple, which, for sheer beauty, resembles the astronomic universe." Leon Walras.[1]

The appearance of William Jaffé's loving translation of Leon Walras' *Elements of Pure Economics* offers an excuse for re-examining that great work some eighty odd years after its original publication. Though in so far as this is a review, it is a review of Walras and not of Jaffé, I cannot refrain from prefacing it with a word of thanks to Jaffé for his translation, which is a model of its kind: careful, accurate, and marked throughout by an unobtrusive attention to detail. His notes on the collation of editions are an important aid to research; his translator's notes illuminate many points of the text as well as directing the reader's attention to much recent writing that is relevant to its interpretation.

Though I regard as somewhat extravagant Schumpeter's judgment that, "so far as pure theory is concerned, Walras is . . . the greatest of all economists,"[2] there can be no doubt that the *Elements* is a great work which marked an important step forward in the development of economics as a science, and which still plays an important role in economic thinking. It is well worth having a translation even at this late date in order to make it more readily accessible both to the profession at large and particularly to students learning to become economists: it belongs on their "five foot shelf." The comments that follow deal with the book in this context, as a piece of living literature, rather than with its role in the history of economic thought.

On the broadest level of generality, there are two main themes in the *Elements:* the analysis of *rareté*, or marginal utility; and the theory of general equilibrium. Walras regarded the two as fitting together in one harmonious whole, which is certainly tenable; he also viewed the marginal utility analysis as indispensable for the study of general equilibrium, which seems much more dubious. The marginal utility analysis impresses the modern reader as "dated," as important primarily in understanding the development of economic ideas rather than in directly extending his horizons as a scientist. For this reason I

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shall discuss the marginal utility analysis first, in order to clear the ground for the theory of general equilibrium.

I. Rareté

Walras essentially completes his analysis of the "Theory of Exchange of Two Commodities for Each Other," the title of Part 2 of the *Elements,* before he introduces utility analysis at all. Prior to that point, he has derived demand curves and offer curves, discussed their typical shapes, and considered the meaning of their points of intersection, distinguishing stable from unstable equilibria. These topics are described as revealing the "nature of exchange"; and utility curves are then introduced in order to examine "the cause" of exchange. Similarly, at each successive stage in the analysis—the extension of the theory of exchange of two commodities to several commodities, and the expansion of the system to include successively production, capital formation and credit,[3] and circulation and money—utility considerations strike the reader as something introduced rather artificially, as being on a different level from the rest of the analysis and capable of being extracted from it bodily without in any way altering its essence—a step that Cassel took in his reformulation of the Walrasian system.

Yet this is clearly not the way it seemed to Walras or to his contemporaries, Jevons and Menger. Today, Walras' primary contribution would surely be regarded as general equilibrium theory, of which at best only pale reflections can be found in Jevons or Menger; yet the three linked themselves together and were linked together by others as the pioneers of "marginal utility."

[3] As Jaffé remarks, the analysis of utility considerations in connection with the theory of capital formation and credit is in parts "obscure to the point of almost complete incomprehensibility." Jaffé gives an extensive reconstruction in an attempt to render the argument intelligible in his translator's note [2] to Lesson 27 (pp. 536-39). I find it difficult to accept this reconstruction in one important respect, namely, Jaffé's interpretation of the argument as applying to a stationary state and his resulting assignment of an essential role to expenditures on the replacement of capital goods. I am inclined to go to the opposite extreme. It seems to me that Walras here, as elsewhere, thought initially in terms of capital goods that were permanent, required no maintenance or replacement, and gave rise to a permanent flow of services. The question he seems to me to be asking in the section at issue is: given a certain amount of productive power to be used in producing an additional set of permanent capital goods of this kind, what bundle of capital goods will produce the additional stream of consumer goods having the greatest utility. His proof is correct, provided one consistently treats the capital goods as permanent and interprets his differential coefficients or ratios between them as rates of substitution—which seems to me also required in Lesson 26 and to explain what puzzles Jaffé in his translator's note [1] to that lesson (p. 533). The equations labeled [ε] at the bottom of p. 297 are not simultaneously valid; they are alternatives, showing that if (A) is substituted for (T), and all other quantities are unchanged, the quantity of (A) acquired must equal in value the quantity of (T) given up; and so on for every possible pair. That this is intended seems to me even clearer from the wording of earlier editions.

The tendency for Walras to work his argument out initially in terms of permanent capital goods requiring no replacement seems to me to explain also how the difficulty arose which Jaffé deals with in his translator's note [3] to Lesson 27 (pp. 539-41). Having arrived at a result for this case, Walras generalized it without full proof to the case of nonpermanent capital goods, in the process making what Jaffé terms—correctly, I believe—a "slip."

Walras writes as an italicized theorem, "The exchange of two commodities for each other in a perfectly competitive market is an operation by which all holders of either one, or of both, of the two commodities can obtain the greatest possible satisfaction of their wants consistent with the condition that the two commodities are bought and sold at one and the same rate of exchange throughout the market," and goes on to say, "The main object of the theory of social wealth is to generalize this proposition. . . . We may say . . . that this proposition embraces the whole of pure and applied economics" (p. 143).

It is hard now for us to understand why this marginal utility analysis should have been regarded as so vital and revolutionary. We can repeat the formulae of the histories of economic thought that it gave a meaningful solution to the diamond-water paradox and so permitted demand to be assigned its proper role and the shackles of the cost of production or, even worse, labor theory of value to be overthrown. But I do not believe that such formulae carry real conviction or understanding. Partly, this is for the usual reason that an error, once pointed out, seems obvious to those who never held it, though it may have taken a real stroke of genius to discover the error and though simply pointing it out did not make it obvious to those who had the error imbedded in the fabric of their thought. But I suspect the main reason is quite different, namely, the change in our general philosophical and methodological outlook that has been wrought, though by no means directly, by the developments in physical science, in particular, by the replacement of the physics of Newton by the physics of Einstein. Surely this is why a chapter title like that of Lesson 10, "*Rareté*, the Cause of Value in Exchange," strikes us as an anachronism.

The almost purely metaphysical role of *rareté* in Walras is brought out very well by his discussion of measurement:

> The above analysis is incomplete; and it seems impossible, at first glance, to pursue it further, because intensive utility, considered absolutely, is so elusive, since it has no direct or measurable relation to space or time, as do extensive utility [the quantity that will be taken at a price zero] and the quantity of a commodity possessed. Still, this difficulty is not insurmountable. We need only assume that such a direct and measurable relationship does exist, and we shall find ourselves in a position to give an exact, mathematical account of the respective influences on prices of extensive utility, intensive utility and the initial stock possessed. I shall, therefore, assume the existence of a standard measure of intensity of wants or intensive utility . . . (p. 117).

In a modern writer, one would expect this to be followed by a statement that such an assumption, combined presumably with others, has observable implications of a kind that will enable utility, though "it has no direct or measurable relation to space or time," to be assigned numerical values that are inferred from what are regarded as its manifestations. Walras, of course, does not take this line. He says nothing more on the subject and simply proceeds to take for granted that there is something called *rareté* which has numerical values that can be plotted, averaged, and so on, and can be identified with "satisfaction" in a sense that is relevant for welfare purposes.

In a way, Walras' ready acceptance of the nonmeasurability by physical operations of his *rareté* is somewhat ironical. For, like the other pioneers of marginal utility, he made a subsidiary assumption about utility functions that, if accepted, gives a relatively straightforward method of assigning numbers to utility. Walras throughout assumes that the total utility of a collection of commodities can be written as the sum of functions, each containing as a variable the quantity of only one commodity. Indeed, one gets the impression that it may well have been this feature of his utility function that was to him the main justification for regarding *rareté* as *the* cause of value in exchange; for *rareté* was "absolute," depending only on the quantity of the one commodity itself, whereas value in exchange was "relative," the ratio of two such absolutes; and along the same lines, the utility curve for a particular commodity was more fundamental, because a function of only one variable, than the demand function which had to be regarded as depending on several. However, if a consumer's preferences can be validly represented by a sum of one-variable functions, a convenient measuring rod for utility is at hand; one need only take the utility added by some specified unit of one commodity, say the utility added by the tenth slice of bread, as the basic unit, and the utility of all other commodities can be expressed in terms of it—essentially the procedure that both Fisher and Frisch experimented with at a later date.

The reason why this method of measuring utility has not been adopted is, of course, because a utility function consisting of a sum of one-variable functions has implications for consumer behavior that are contradicted by observation, the most striking, perhaps, being the implication that the higher the income of a consumer, the more he will consume of every commodity separately, *i.e.*, that there are no inferior goods. Needless to say, Walras does not explore such implications, though he does record the corresponding implication that a demand curve for one commodity is always negatively sloped for given amounts possessed of other commodities (which is equivalent to given money income and other prices). However, he asserts this (on p. 91) prior to introducing his marginal utility analysis, giving little justification for it, apparently because he regarded it as obvious.

One must conclude, I think, that this part of Walras' book has interest almost solely for the student of economic thought. In so far as utility theory plays a role in modern economic analysis, it does so in a more sophisticated, albeit empirically emptier, form than in Walras, though it should perhaps be recorded that there is much current literature that has not advanced beyond Walras in its understanding of the meaning and role of the measurability of utility.

II. *The Theory of General Equilibrium*

Cournot writes in Chapter 11 of his *Researches,*

So far we have studied how, for each commodity by itself, the law of demand in connection with the conditions of production of that commodity, determines the price of it and regulates the incomes of its producers. We considered as given and invariable the prices of other commodities and the incomes of other producers; but in reality the economic system is a

whole of which all the parts are connected and react on each other. An increase in the income of the producers of commodity A will affect the demand for commodities B, C, etc., and the incomes of their producers, and, by its reaction, will involve a change in the demand for commodity A. It seems, therefore, as if, for a complete and rigorous solution of the problems relative to some parts of the economic system, it were indispensable to take the entire system into consideration. But this would surpass the powers of mathematical analysis and of our practical methods of calculation, even if the values of all the constants could be assigned to them numerically.[4]

It is Walras' great and living achievement to have constructed a mathematical system displaying in considerable detail precisely the interrelationships emphasized by Cournot. Did he thereby show Cournot to be wrong in supposing that the task surpassed the powers of mathematical analysis? I believe not. For there is a fundamental, if subtle, difference between the task Cournot outlined and the task Walras accomplished; an understanding of this difference is essential to an assessment of both the positive contribution of Walras and the limitations to that contribution; and failure to recognize the difference seems to me a primary source of methodological confusion in economics. It is clear from Cournot's references to "practical methods of calculation" and to the assignment of numerical values to constants that the "rigorous solution" he had in mind was not a solution " in principle," but a numerical solution to a specific problem. His goal was an analysis that would, given the relevant statistical material, yield specific answers to specific empirical questions, such as the effects of a specified tax on a specified product; answers that could be confronted by observation and confirmed or contradicted. And surely there can be little doubt that a "complete and rigorous solution" of this kind does "surpass the powers of mathematical analysis and of our practical methods of calculation" even today despite the enormous advances in methods of calculation. Cournot was quite right that *for his problem* a "complete and rigorous" solution was out of the question, that the thing to do was, "while maintaining a certain kind of approximation, . . . to carry on . . . a useful analysis."[5]

Walras solved a different, though no less important, problem. He emptied Cournot's problem of its empirical content and produced a "complete and rigorous" solution "in principle," making no pretense that it could be used directly in numerical calculations. His problem is the problem of form, not of content: of displaying an idealized picture of the economic system, not of constructing an engine for analyzing concrete problems.[6] His achievement

[4] Augustin Cournot, *Researches into the Mathematical Principles of the Theory of Wealth* (1838), transl. by Nathaniel T. Bacon (New York, 1897), p. 127.

[5] *Ibid.*, pp. 127-28.

[6] Walras comments that "when we pass from the realm of pure theory to that of applied theory or to actual practice, . . . the variations in the unknown quantities will be effects of either the first or the second order, that is to say, effects which need or need not be taken into consideration, according as they arise from variations in the special or the general data" (pp. 307-8; see also similar comment on p. 431). In a translator's note, Jaffé cites this sentence as evidence that I "drew too sharp a contrast between Marshall

cannot but impress the reader with its beauty, its grandeur, its architectonic structure; it would verge on the ludicrous to describe it as a demonstration how to calculate the numerical solution to a numerically specified set of equations. The difference is brought out clearly by the further developments along Walras' line that have been—and rightly—regarded as improvements in his system. These have all consisted in making the system still more general and elegant, in eliminating empirically specializing assumptions. The clearest example is, of course, in the theory of production: Walras assumed constant coefficients of production, recognizing that this was an "approximation" and in later editions suggesting the route to generalize the analysis. Pareto generalized Walras' solution to cover variable as well as constant coefficients of production. The recent reintroduction of the assumption of constant coefficients of production in connection with input-output analysis has not been a further development of Walras' pure theory. It has rather been an attempt— so far largely unsuccessful—to use Walrasian constructs in solving Cournot's problem.

Emphasis on pure form has an important role to play in economics in two rather different respects. One, the easier to specify, is the role of mathematics or pure logic in general, namely, to help us to avoid contradictory statements —to avoid mistakes in arithmetic, as it were. This role is immediately recognized and granted, and for that reason tends to be passed over rapidly; yet it deserves to be emphasized how many, how important, and sometimes how difficult to detect, are the fallacies in economics of this kind; fallacies that consist in the assertion that contradictory statements are simultaneously valid, that we can have our cake and eat it too. The ability to think clearly and exactly is a scarce resource for which, unfortunately, there seems no adequate

and Walras" in my article "The Marshallian Demand Curve," *Jour. Pol. Econ.* Dec., 1949, LVII, 463-95; reprinted in my *Essays in Positive Economics* (Chicago, 1953), pp. 47-99. He goes on to say, "There one gets the impression that Walras's sole preoccupation was the achievement of 'abstractness, generality and mathematical elegance' (p. 490), while Marshall sought 'an engine for the discovery of concrete truth.' A more valid and important distinction between Walras and Marshall resides in the fact that the former always took great care not to confuse pure theory with applied theory, while the latter gloried in fusing the two" (p. 542).

In his final sentence, Jaffé speaks like a true Walrasian in methodology. One first constructs a pure theory, somehow on purely formal considerations without introducing any empirical content; one then turns to the "real" world, fills in the empty boxes, assigns numerical values to constants, and neglects "second-order" effects at this stage. As I have argued extensively elsewhere [particularly in "The Methodology of Positive Economics" and "Lange on Price Flexibility and Employment: A Methodological Criticism," both in my *Essays,* pp. 3-43, 277-300, the latter reprinted from this *Review,* Sept., 1946, XXXVI, pp. 613-31], this seems to me a basically false view. Without denying the importance of what Jaffé and Walras call "pure theory" (see my comments below), I deny that it is the whole of "pure theory." More important in the present context, two largely parenthetical comments in the *Elements* to the effect that second-order effects will have to be or can be neglected in application seem a rather thin basis on which to claim that Walras was concerned with the construction of "an engine for the discovery of concrete truth." As I argue in the text, I remain of my original opinion; indeed, I am confirmed therein by the careful rereading of Walras to which I was led by the request to write this article, by Jaffé's critical comment, and by similar comments in reviews of my *Essays.*

substitute. Walras' discussion of bimetallism (Lessons 31 and 32) and of
Ricardo's and Mill's theories of rent and wages (Lessons 39 and 40) are
excellent examples, largely peripheral to his own general equilibrium theory,
of how useful emphasis on pure form can be. By translating vague statements
into symbolic form and using very elementary mathematics indeed, Walras is
able to clear away much irrelevant material, show that some widely accepted
statements are mutually contradictory, and specify the conditions under
which others are valid.

The other respect in which emphasis on pure form has an important role to
play is in providing a language, a classificatory scheme to use in organizing
materials—labels, as it were, for the compartments of our analytical filing box.
This is Walras' great contribution. His general equilibrium system gives a
bird's-eye view of the economic system as a whole, which has not only an
extraordinary aesthetic appeal as a beautifully articulated abstraction but
also a utilitarian appeal as providing relevant, meaningful, and mutually ex-
haustive categories. This bird's-eye view rests fundamentally on two dichot-
omies: between services and sources of services or between income and capi-
tal; and between the markets for consumer services or goods and for produc-
tive services or goods. A third dichotomy might almost be added: between
entrepreneurs and consumer units, though this seems somewhat less funda-
mental. Each consumer unit and entrepreneur is conceived as operating in
both markets: in terms of markets for services, a consumer unit sells produc-
tive services of the capital sources he owns in the resource market and buys
consumer services in the consumption market; an entrepreneur buys produc-
tive services in the resource market and sells consumer services in the con-
sumption market either directly or indirectly. The distinction between markets
thus leads naturally and directly to the distinction between demand and
supply.

This classificatory scheme is developed in considerable detail, with extra-
ordinary skill and ingenuity, great attention being devoted to showing, or
attempting to show, that it is internally consistent and exhaustive (*i.e.*, that
the system of equations has a solution that tends to be attained and main-
tained by the operation of market forces). I have described this analysis as
involving emphasis on pure form, which I think in a meaningful way it does.
Yet I do not mean thereby to imply either that it lacks importance for eco-
nomics as a substantive science, or that empirical considerations play no role
in its construction and use. Quite to the contrary. Walras' picture is not pure
mathematics but economics precisely because it was constructed to provide a
framework for organizing substantive material of an economic character; the
classifications it employs reflect a judgment about the empirically important
characteristics of the economic structure; the usefulness of the picture, though
not its logical coherence, depends on the extent to which this judgment is con-
firmed by experience. One cannot read Walras, it seems to me, without recog-
nizing that he was an economist first and a mathematician and logician
second; he accomplished what he did not because he was a mathematical
genius but despite inferior mathematical equipment—reading the *Elements*
gives no reason to doubt the fairness of the examiners who failed him twice in

the mathematics examination for entry to the École Polytechnique.[7] In some ways, indeed, "despite" might perhaps be replaced by "because." Walras' necessity to work things out rather cumbrously, from the simplest cases to the more complicated, must have forced him to give much more attention to the economic significance and meaning of his categories than he would have if he had been able to proceed on a still higher level of abstraction. I hasten to add that I do not mean to be urging that bad mathematics is better than good but only that each task requires its own tools. A hand spade may well be better than a modern steam shovel for some kinds of work; pure mathematicians are notoriously bad at simple arithmetic.

Though emphasis on form can and does play a vital role in economic analysis, it can also be mischievous if it is not illuminated by empirical judgment and understanding. An excellent example is Walras' utility analysis of savings. This analysis was first introduced into the fourth edition, which appeared in 1900, about a quarter of a century after the first edition. In this edition, Walras yielded to the temptation, which has claimed so many lesser men, of treating "savings" like a consumer good and simply carrying over mechanically the formal analysis applicable to consumer goods. So he defines a commodity (E) consisting of a perpetual net income stream, a unit of (E) being one unit of *numeraire* per unit of time indefinitely, writes down for each individual a marginal utility function for (E), and regards him as possessing a certain quantity of (E) and maximizing his utility subject to a budget constraint which includes expenditures on (E) along with expenditures on other commodities. He regards this process as yielding a demand function for (E) like other demand functions (pp. 274 and 275).

In symbols, this looks like a simple extension of Walras' general analysis and one is led to ask why it was that he did not discover this obvious yet important extension for a quarter of a century. But the moment one digs beneath the symbols and asks why, as economists, we regard it as important to distinguish savings from current consumption, it becomes clear that Walras' procedure is fallacious and involves precisely the kind of confusion between stocks and flows that Walras elsewhere so carefully avoids and indeed underlines. I can perhaps illustrate this best by Walras' utility function for (E) which, in deriving the demand curve, he writes as

$$\Phi_e(q_e + d_e),$$

where q_e is the initial quantity of (E) possessed, d_e, the quantity purchased or sold during the time unit in question. Now q_e and d_e are of different dimensions and cannot be added: q_e is the number of units of (E) that the individual possesses, *i.e.*, the number of units of *numeraire* per unit of time that the individual can receive indefinitely if he so chooses—for simplicity, let us say the number of dollars per year that is yielded by his existing stock of wealth; d_e is the number of dollars per year that he is going to add to this flow as income on the savings he accumulates during the time period in question (see p. 117), say a year, so that savings during that period are $p_e d_e$, where p_e is the price of

a dollar a year indefinitely. In other words, q_e is of the dimension of dollars per year; d_e, of the dimension of dollars per year per year. Let the time period in question be half a year instead of a year; the same numerical value of d_e means that he saves twice as large a fraction of his income. q_e and d_e simply cannot be added: an individual will not be indifferent, as Walras' equation implies that he is, between a situation in which he starts with an income of $10,000 a year and adds $100 a year income to it by saving $2,000 during the year in question, which means that the rate of interest is 5 per cent, and a situation in which he starts with an income of $9,700 a year and adds $400 a year to it by saving $8,000 during the year in question. Savings cannot be assimilated directly to current consumption, precisely because their whole function is to provide a stream of consumer services.

In the earlier editions of the *Elements*, Walras made no attempt to derive the demand for savings from utility analysis. He simply wrote down as an empirical datum an individual savings function, and noted, quite correctly, that in order to derive it from utility considerations it would be necessary "to consider utility under a new aspect, distinguishing present utility from future utility."[8] This was no oversight and the change in the fourth edition no belated discovery of a neglected truth. Surely, the explanation must be that when Walras made the change in the fourth edition, he no longer had his system and its meaning and its role in his bones the way he did when he developed it; he was taken in by considerations of pure form; the substance which the form was to represent was no longer part of him. It would be hard to find a better example of the nonsense to which even a great economist can be led by the divorce of form from substance.

III. *Conclusion*

Walras has done more than perhaps any other economist to give us a framework for organizing our ideas, a way of looking at the economic system and describing it that facilitates the avoidance of mistakes in logic. It is no derogation of this contribution to emphasize that it is not by itself enough for a fruitful and meaningful economic theory; division of labor is appropriate in economic theory too. Economics not only requires a framework for organizing our ideas, it requires also ideas to be organized. We need the right kind of language; we also need something to say. Substantive hypotheses about economic phenomena of the kind that were the goal of Cournot are an essential ingredient of a fruitful and meaningful economic theory. Walras has little to contribute in this direction; for this we must turn to other economists, notably, of course, to Alfred Marshall.

The large and substantial immediate rewards from Walras' concentration on form; the prestige and intellectual appeal of mathematics; the difficulty of making experiments in economics and the consequent laboriousness and seeming unproductiveness of substantive work devoted to filling in our analytical filing boxes—all these have combined to favor the Walrasian emphasis on form, to make it seem not only an essential part of a full-blown economic

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[8] Jaffé's Collation of Editions, note [h] to Lesson 23, p. 587; my translation from Jaffé's quotation in French.

theory, but that economic theory itself. This conception—or misconception—of economic theory has helped to produce an economics that is far better equipped in respect of form than of substance. In consequence, the major work that needs now to be done is Marshallian rather than Walrasian in character—itself a tribute to Walras' impact.

I am tempted, in concluding this rather discursive commentary, to paraphrase Mill's comment that "A person is not likely to be a good economist who is nothing else."[9] A person is not likely to be a good economist who does not have a firm command of Walrasian economics; equally, he is not likely to be a good economist if he knows nothing else.

[9] From *On Comte*, p. 82, as quoted by Alfred Marshall, *Principles of Economics*, 8th ed., (London, 1920), p. 771.