# Methods of Interregional and Regional Analysis

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# 8. Applied general interregional equilibrium

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## 8.0 Introduction

As noted in chapter 3, national, regional and interregional models have stemmed from the pioneering work of Wassily Leontief. Leontief's thinking in turn was greatly influenced by the purely conceptual Walrasian general equilibrium system.[1] In order to obtain a very desirable operational character for the portrayal of this system, which Leontief termed an input-output model, he introduced, among others, assumptions of constant production, consumption and trade coefficients. These assumptions, however, have meant that market prices do not affect production, consumption and trade; and that in the systems his models portray, supply always adjusts to demand — falling if demand drops and increasing if demand grows. These assumptions then imply that unemployment of labor and capital have no effect on these systems, and in fact are not present. Equally significant, they imply no resource constraints; always, it is implicitly assumed that there exist unused facilities and resources for the expansion of an economy.

In chapter 4 dealing with regional, interregional and spatial econometrics, the existence of markets for factors of production and commodities along with prices was introduced into our analytical framework. But in doing so the very desirable requirement of supply and demand equality (market equilibrium) was by and large ignored; and to a significant degree in the past, extensive sets of data were employed for identification of relevant structural relationships.

In chapter 5 on programming and industrial and urban complex analysis, resource constraints were introduced as major factors affecting development. However, the play of prices and the market were only touched upon. Fixed final product prices were by and large assumed, the effect of unemployment of resources upon resource prices scarcely discussed, an excess supply often taken to imply a zero price. (There was, however, at the end of section 5.2, an indication of how nonlinear programming can be extended to cover price formation at the market.)

In the discussion of gravity and spatial interaction models of chapter 6, little attention was given to the formation of prices and the role of the market. In large part this results from the fact that gravity models pertain to masses and mass behavior. They average out the effects in reality of the numerous microelements — the behavior of individual units. For some spatial phenomena, the individual effects can be ignored; but they cannot be ignored for many other extremely significant phenomena, especially those associated with markets and monetary systems.

Like input-output, the SAM analysis of chapter 7, taken to embrace input-output, also assumes constant production and consumption coefficients. In effect it fails to capture the impact of changing markets and prices as major elements in a study, for example when policies of government are formulated and evaluated.

We now come to a chapter in which market equilibrium is a basic element, where prices are free to vary at least relatively and where they fully impact production, consumption, trade, and spatial interaction in general. Both linear and nonlinear functions will be involved. To reiterate, we now can obtain solutions to models extensively employing linear and nonlinear functions to characterize situations of reality, which until recently has not been possible — a result of the tremendous increase in computer capability. However, the reader is warned that this advance in analytical capability is achieved at a cost. Generally speaking, the magnitudes to be yielded by the models envisaged are not to be viewed as precise values. Rather these magnitudes have basic use only in indicating direction of change as change in exogenous inputs, such as policies, tastes and technology, are introduced. This is the result of the fact that the inputs of data that are currently employed to approximate parameters of a number of nonlinear functions is of lesser quality than the data in the models presented in the previous chapters. Moreover, most of these magnitudes will be cranked out by rather complex programs designed for high speed computers, and we will not be able fully, and often even partially, to follow how the play of the variables generated these magnitudes.

In the materials to follow we proceed step-by-step to develop the basic scaffolding for applied general interregional equilibrium (AGIE) models.

As far as the author is aware at the time of writing, there has not yet been developed in regional science such a model generally applicable for a space-economy although important forward steps have been taken. (These steps will be noted in section 8.5.) Distance, transport inputs and transport costs have been at best most inadequately treated. While, as a consequence, it would seem inappropriate to discuss in an introductory text for graduate and advanced undergraduate students a comprehensive method where no successful demonstration of its applicability can be pointed to, we do so for at least two reasons. First the method to be presented is highly likely soon to experience very rapid development and subsequently a fruitful application. Second, in the subsequent chapter major new research directions will be sketched which will involve the fruitful use of applied general interregional equilibrium, especially one involving a deeper probe with one or more methods of regional science — directions which graduate students may wish and are encouraged to pursue in dissertation work.

In section 8.1 we introduce some basic relationships of a relevant applied general interregional equilibrium analysis. We employ a highly simplified model. The model treats a two commodity, two resource, two region system with an external source of a raw material, a model intended to highlight distance and transport inputs to overcome the resistance of distance to movement. The next section considers trade and location within a two country world, each country comprising a single region. We do so in order to obtain a meaningful presentation of the impact of varying distance on specialization of production and the export/import pattern of a country. By so doing it avoids the pitfalls of the 'wonderland-of-no-dimensions' trade models so characteristic of many computable and applied general equilibrium analyses by economists. It also brings into consideration variables significant for treating a region engaged in trade outside its own nation.

Section 8.3 further approaches reality by considering a two country/three region world. One country comprises two regions, each of which possesses two resources. The aim of this section is to develop the scaffolding of a social accounting matrix to serve as a core for applied general interregional equilibrium analysis (AGIE) in which there is consumption, production, transport, government, investment and possibly financial activity in a market system — all in a many commodity, many country, many region world.

In section 8.4 we examine the problems and questionable character of the basic assumptions of standard applied general equilibrium (AGE) models, partly to be aware of them should we carry them over to interregional (AGIE) analysis and partly to make clear the need to reformulate some and discard others for effective AGIE analysis.

Section 8.5 notes some of the seminal contributions that regional scientists have already made for the yet-to-be realized effective AGIE model, and section 8.6 contains some concluding remarks.

## 8.1 A highly simplified model introducing distance and transport inputs as basic variables

We begin with a highly simplified interregional model covering production, consumption and trade using both nonlinear and linear functions with space explicitly included. Such inclusion implies that distance, transport inputs (in say ton miles) and transport costs must be able to be treated as *endogenous* variables.[2]

Accordingly, consider (for pedagogical purposes only) a model of the following *hypothetical* situation. There are two regions *A* and *B* and a location *Z* of a raw material, say coal. At *A* an agricultural commodity is produced by sector #1. At *B* a manufactured commodity is produced by sector #2, the production of which requires as input the raw material coal. In each region, consumption is an aggregate concentrated along with production at a single point. Region *A* possesses a fixed stock of the resource capital, 100 percent mobile, all the capital used in production in the two regions being owned by the set of consumers in *A*. Region *B* has a fixed amount of the resource labor completely mobile, i.e., the costs of commuting from region *A* to *B* are zero. All the labor required in production in the two regions is provided by the set of consumers in region *B*. Also, the supply of coal at *Z* is unlimited and available at its location at zero cost.

To obtain an equilibrium solution to this highly simplified situation, we need to specify the stocks of resources (as exogenous elements) that are available for production of the goods — which serve as both intermediate inputs into production and final products for household consumption. Let K be the fixed stock of capital, and L, the fixed stock of labor. Also let $r$ represent the price of capital, and $w$, the price of labor.

Since a Walrasian-type general equilibrium framework can determine only relative prices, we are free to set the price of one commodity or resource as unity. We do so for the price of labor, which then serves as a numeraire for this particular exercise. Within a programming framework, a price of labor equal to unity would be a constraint.

*8.1.1 The consumption subsystem*

Under the assumption of pure competition among producers in each sector, profits are taken to be zero, Accordingly, the respective incomes for regions *A* and *B* will be the earnings from only their resources. Thus

$$Y^A = rK \text{ and } Y^B = wL \tag{8-1}$$

We take the consumption of each region to be given by the linear homogeneous Cobb Douglas function

$$U = (C_1^i)^{0.5} (C_2^i)^{0.5} \qquad i = A,B \tag{8-2}$$

If consumers (as an aggregate in each region) are taken to maximize utility, it follows that[3]

$$C_1^A = \frac{Y^A}{2P_1^A} \text{ and } C_2^A = \frac{Y^A}{2P_2^A}$$ consumption of commodities #1 and #2, respectively in region *A* (8-3)

$$C_1^B = \frac{Y^B}{2P_1^B} \text{ and } C_2^B = \frac{Y^B}{2P_2^B}$$ consumption of commodities #1 and #2, respectively in region *B* (8-4)

where $P_1^A$, $P_2^A$, $P_1^B$ and $P_2^B$ are prices of commodities #1 and #2 in regions *A* and *B*, respectively.

Note that here and later, both in the text and endnotes, we assume that functions to be optimized satisfy second-order conditions for equilibrium. For example, we assume utility functions are concave or quasi-concave at all points.

*8.1.2 The production subsystem*

Production of each commodity is restricted by known technology. For sectors #1 and #2 we assume the respective linear homogeneous Cobb-Douglas production functions

$$X_1^A = K_1^{0.25} L_1^{0.75} \tag{8-5}$$

and

$$X_2^B = K_2^{0.5} L_2^{0.5} \tag{8-6}$$

Given these two functions, maximization of profits leads to the following relations[4]

$k_1 = (w/3r)^{0.75}$, the capital input per unit of good #1 (8-7)

$\ell_1 = (3r/w)^{0.25}$, the labor input per unit of good #1 (8-8)

$k_2 = (w/r)^{0.5}$, the capital input per unit of good #2 (8-9)

$\ell_2 = (r/w)^{0.5}$, the labor input per unit of good #2 (8-10)

The intermediate inputs required are to be calculated from the exogenously specified input-output coefficients, $a_{11}$, $a_{12}$, $a_{21}$, and $a_{22}$, the respective requirements of good #1 per unit output of sectors #1 and of #2, and of good #2 per unit output of sectors #1 and #2. Thus we obtain the familiar demand-driven supply equal demand equations of input-output:

$$X_1 = a_{11}X_1 + a_{12}X_2 + C_1^A + C_1^B \tag{8-11}$$

and

$$X_2 = a_{21}X_1 + a_{22}X_2 + C_2^A + C_2^B \tag{8-12}$$

whose solutions, after rearranging terms using the familiar inverse **A**, are given by

$$X_1 = [(1 - a_{22})(C_1^A + C_1^B) + a_{12}(C_2^A + C_2^B)]/\mathbf{A} \tag{8-13}$$

and

$$X_2 = [(1 - a_{11})(C_2^A + C_2^B) + a_{21}(C_1^A + C_1^B)]/\mathbf{A} \tag{8-14}$$

The input of coal per unit output of good #2, namely $a_{02}$, must also be exogenously specified.

### *8.1.3 The transport subsystem*

The basic activity of the transport subsystem is the production of transport inputs required for the shipment of goods. In the movement of a unit of a commodity, a transport input, is defined as the product of the *weight* of that unit and the *distance* over which it is to be moved. Thus a unit transport input can be defined as a shipment of one ton over one mile, that is a shipment of one ton-mile, which is of course equivalent to the movement of 100 lbs. of a commodity 20 miles, or 200 lbs. over 10 miles, etc. Use of other definitions is possible such as one kilogram-kilometer; in this book, however, we employ the term ton-mile.

In this highly simplified model, a Leontief production function is taken to characterize each of three transport activities: (1) shipment of coal from location $Z$ to production sector #2 in region $B$ (taken to be the activity of sector #3 in $B$); (2) shipment of commodity #1 produced in region $A$ to meet final demand consumption in $B$ as well as provide intermediate inputs of good #1 for the production of good #2 in $B$ (designated the activity of sector #4 in $A$); and (3) shipment of commodity #2 produced in region $B$ to meet final demand consumption in $A$ as well as provide intermediate inputs of good #2 for production of #1 in $A$ (designated the activity of sector #5 in $B$). Thus there are for sectors #3, #4, and #5 the respective constant production coefficients (capital and labor inputs per unit output)

$$k_3^B,\ \ell_3^B,\ k_4^A,\ \ell_4^A,\ k_5^B,\ \ell_5^B$$

where a unit output of each sector is a transport input. These coefficients need to be exogenously specified.

Where we let $\alpha_{32}$ represent the ton-miles (transport inputs) provided by the transport sector #3 in $B$ to ship one unit of coal from $Z$ to sector #2 in $B$, $\alpha_{41}$ represent the ton-miles (transport inputs) provided by transport sector #4 in $A$ to ship one unit of good #1 to $B$, and $\alpha_{52}$ represent the ton-miles (transport inputs) provided by transport sector #5 in $B$ to ship one unit of good #2 to $A$, the outputs of the transport sectors are:

$$X_3^B = \alpha_{32}\, a_{02}\, X_2^B \tag{8-15}$$

$$X_4^A = \alpha_{41}\, a_{12}\, X_2^B + \alpha_{41}\, C_1^B \tag{8-16}$$

$$X_5^A = \alpha_{52}\, a_{21}\, X_1^A + \alpha_{52}\, C_2^A \tag{8-17}$$

where $\alpha_{02}$ is the requirement of coal per unit output of sector #2 in B.

### *8.1.4 The market subsystem*

Under the assumption of pure competition in each sector, the price of a good will come to be equal to its unit costs. Therefore, for sectors #3, #4, and #5 (in whose production only labor and capital are required as inputs), prices of a unit of output are:

$$P_3^B = rk_3^B + w\ell_3^B \tag{8-18}$$

$$P_4^A = rk_4^A + w\ell_4^A \tag{8-19}$$

and

$$P_5^B = rk_5^B + wk_5^B \tag{8-20}$$

Also, two prices are realized as delivered costs, namely

$$P_2^A = P_2^B + \alpha_{52}P_5^B \tag{8-21}$$

and

$$P_1^B = P_1^A + \alpha_{41} P_4^A \tag{8-22}$$

Once, prices $P_1^A$ and $P_2^B$ are determined, $P_2^A$ and $P_1^B$ can be directly calculated since $P_4^A$ and $P_5^B$ are given by equations (8-19) and (8-20) and $\alpha_{41}$ and $\alpha_{52}$ are determinable once the distances and weights are specified. However, $P_1^A$ and $P_2^B$ are not directly obtainable because intermediate inputs are required in the production of goods #1 and #2. Since pure competition insures zero profits for producers in sectors #1 and #2, we can obtain from this condition the relations

$$P_1^A = rk_1 + w\ell_1 + a_{11}P_1^A + a_{21}P_2^B + a_{21}\alpha_{52}P_5^B \tag{8-23}$$

and

$$P_2^B = rk_2 + w\ell_2 + a_{02}\alpha_{32}P_3^B + a_{22}P_2^B + a_{12}P_1^A + a_{12}\alpha_{41}P_4^A \tag{8-24}$$

Here equation 8-23 states that the price of good #1 in *A* equals the unit cost of capital *plus* the unit cost of labor *plus* the cost of using good #1 as an intermediate good *plus* the cost of using good #2 as an intermediate good. The last cost is indicated by two terms: the first is the cost at the site of production in region *B* of the intermediate input of good #2 ($a_{21}P_2^B$); the second is the transport cost on that amount of intermediate input ($a_{21}$) from region *B* to *A*. That transport cost is equal to the $a_{21}$ units of good #2 times the transport cost of shipping one unit of good #2 over the distance from *B* to *A*, namely $\alpha_{52}P_5^B$, where $\alpha_{52}$ is the transport input involved per unit of good #2 (namely the weight of a unit of good #2 times the distance $d^{B\rightarrow A}$) and $P_5^B$ is the cost (the price) of a unit of that transport input. In other words, $a_{21}$ units of good #2 need to be shipped; thus $\alpha_{52}a_{21}$ is the total transport inputs for the required shipment. Multiplying this total by the price $P_5^B$ of a required transport input yields $a_{21}\alpha_{52}P_5^B$, the transport cost for delivering the intermediate input of good #2 per unit output of good #1.

In a similar manner equation 8-24 states that the price $P_2^B$ of good #2 in region *B* is equal to the sum of (1) the unit cost of capital, (2) the unit cost of labor, (3) the transport cost of the coal input which is $a_{02}\alpha_{32}P_5^B$, namely the amount of coal, $a_{02}$, required per unit of output of good #2 times $\alpha_{32}$ (the weight of a unit of coal times the distance $d^{Z\rightarrow B}$ to be overcome) times the price $P_3^B$ of a transport input, (4) the cost of using good #2 as an intermediate input, (5) the cost of using good #1 as an intermediate input (namely the amount used times the price of #1 in region *A*), and (6) the transport cost in shipping the amount of good #1 used as an intermediate input, $\alpha_{41}$, being the weight of good #1, and $d^{A\rightarrow B}$, the distance that good needs to be moved.

Using equations (8-18), (8-19) and (8-20) to eliminate $P_3^B$, $P_4^A$ and $P_5^B$, we obtain $P_1^A$ and $P_2^B$ in terms of $r$, $w$, the given values of the exogenous input-output coefficients and the $\alpha$'s.[5]

The market system also requires that supply and demand are equated for each factor. Thus

$$\bar{K} = \sum_i K_i \quad \text{where } K_i = k_i X_i \qquad i = 1, ..., 5 \qquad (8\text{-}25)$$

and

$$\bar{L} = \sum_i L_i \quad \text{where } L_i = \ell_i X_i \qquad i = 1, ..., 5 \qquad (8\text{-}26)$$

where $\bar{K}$ and $\bar{L}$ are the exogenously determined supply of capital and labor, respectively. In this highly simplified model we have 34 unknowns: $r$, $w$, $Y^A$, $Y^B$, $C_1^A$, $C_2^A$, $C_1^B$, $C_2^B$, $X_1$, $X_2$, $k_1$, $k_2$, $\ell_1$, $\ell_2$, $X_3$, $X_4$, $X_5$, $P_1^A$, $P_2^A$, $P_1^B$, $P_2^B$, $P_3^B$, $P_4^A$, $P_5^B$, $K_1$, $K_2$, $K_3$, $K_4$, $K_5$, $L_1$, $L_2$, $L_3$, $L_4$, and $L_5$. One of these, $w$, we take as numeraire and set at unity (recall, a Walrasian general equilibrium can determine only relative prices). Also, we have 34 equations,[6] of which only 33 are independent (if all but one market equation are independent, one becomes redundant). Thus we can solve our model, when we specify the values of all the exogenous variables, Setting these values as follows: $\bar{K} = 0.8$, $\bar{L} = 2.0$, $a_{11} = 0.05$, $a_{12} = 0.2$, $a_{21} = 0.15$, $a_{22} = 0.10$, $\alpha_{32} = 0.12$, $\alpha_{41} = 0.10$, $\alpha_{52} = 0.10$, $\ell_3 = 0.4$, $\ell_4 = 0.3$, $\ell_5 = 0.2$, $k_3 = 0.3$, $k_4 = 0.2$, and $k_5 = 0.5$, we present selected outcomes in row 6 of Table 8.1.

As noted in chapter 5, the solution to this problem can be obtained by programming where the nonlinear utility function is to be maximized subject to constraints. One of the constraints is that profits from production be nonpositive (which the assumption of pure competition requires). A second constraint is that profits be nonnegative (to insure that goods will be produced to satisfy consumption demands). Viewing the data of row 6 of Table 8-1 as the solution to a programming problem may be helpful to the reader in understanding the nature of general equilibrium analysis.

In order to make explicit the effect of distance, transport cost, and in general space upon the functioning of the economy depicted by our highly simplified model, we now let distances take different values. The results are recorded in Table 8-1.[7]

To see the impact of distance, we record in the first row the hypothetical case where the regions and $Z$ (the coal site) as points all coincide — where in effect distances are zero — the economist's wonderland of no dimension. In this case, a 'delivered cost' price such as $P_2^A$ is identical to the f.o.b. price $P_2^B$. In the second row, we show the results

**Table 8-1 Impacts of the distance and transport input variables (weights of units of grain, textiles, and coal are 20 lbs.)**

| | distances (miles) A - B B - A | Z - B | oc41 ; a 52 Transport inputs on (1) grain (2) textiles in ton-miles | a 32 Transport inputs on coal in ton-miles | Cf | c 2 A | C B | c 2 D | P* | P A | P? | P D | X f | x 2 | r | w | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0.239 | 0.185 | 0.387 | 0.300 | 2.58 | 3.32 | 2.58 | 3.32 | 0.800 | 0.672 | 1.542 | 1.0 | 0.80 | 2.0 |
| 2 | 10 | 0 | 0.1 | 0 | 0.238 | 0.182 | 0.377 | 0.299 | 2.59 | 3.38 | 2.65 | 3.35 | 0.787 | 0.666 | 1.540 | 1.0 | 0.80 | 2.0 |
| 3 | 100 | 0 | 1. | 0 | 0.229 | 0.159 | 0.308 | 0.289 | 2.65 | 3.81 | 3.25 | 3.46 | 0.693 | 0.613 | 1.513 | 1.0 | 0.80 | 2.0 |
| 4 | 300 | 0 | 3. | 0 | 0.209 | 0.123 | 0.220 | 0.272 | 2.77 | 4.71 | 4.54 | 3.68 | 0.564 | 0.532 | 1.445 | 1.0 | 0.80 | 2.0 |
| 5 | 0 | 12 | 0 | 0.12 | 0.238 | 0.179 | 0.384 | 0.289 | 2.61 | 3.46 | 2.61 | 3.46 | 0.791 | 0.652 | 1.55 | 1,0 | 0.80 | 2.0 |
| 6 | 10 | 12 | 0.10 | 0.12 | 0.237 | 0.177 | 0.374 | 0.288 | 2.61 | 3.51 | 2.68 | 3.48 | 0.779 | 0.646 | 1.55 | 1.0 | 0.80 | 2.0 |
| 7 | 300 | 36 | 3.0 | 0.36 | 0.200 | 0.166 | 0.216 | 0.245 | 2.85 | 5.13 | 4.64 | 4.08 | 0.551 | 0.498 | 1.49 | 1.0 | 0.80 | 2.0 |
| 8 | 300 | 300 | 3.0 | 3.0 | 0.210 | 0.086 | 0.182 | 0.135 | 3.50 | 8.56 | 5.51 | 7.41 | 0.481 | 0.326 | 1.84 | 1.0 | 0.80 | 2.0 |

when 10 miles is taken to separate regions *A* and *B*, but where the coal site *Z* still coincides with *B*. When the results are compared with those in the first row, outputs decline (since the transport requirement eats up capital and labor resources) and consumption of each good in each region declines, while prices rise. When we increase the distance between *A* and *B* to 100 miles and then to 300, there are, as to be expected, significant decreases in outputs and consumption, and large increases in prices. For example, comparison of results of rows 2 and 4 show that output of good #1 ($X_1^A$) falls from 0.787 to 0.564, consumption of good #1 in region *B* ($C_1^B$) falls from 0.377 to 0.220 and its delivered price ($P_1^B$) rises from 2.65 to 4.54.

Rows 5-8 of Table 8-1 inject changes in the distance variable $Z \rightarrow B$. When this distance is 12 miles and *A* and *B* coincide, we see, when comparison is made with row 1, small decreases in outputs and consumption and small increases in prices. When we further introduce a distance of 10 miles between *A* and *B*, these changes tend to be somewhat larger. When we consider the distance magnitudes of row 7 ($Z \rightarrow B = 36$ miles, with $A \rightarrow B$ and $B \rightarrow A$ = 300 miles) changes are much more pronounced; and when all distances are set at 300 miles the changes are enormous. In the latter case, for example, when compared with the wonderland case of row 1, output of good #1 falls from 0.800 to 0.481; of good #2 from 0.672 to 0.326; and except for good #1 in *A*, final consumption of goods is more than halved.

## 8.2 Transport inputs, location and trade in a two-country world

In the previous section we have considered the impact of the distance and transport input variables on consumption, production, prices and trade between two regions in a single nation. In regional science we also need to consider the impact of the distance and transport input variables upon trade (and location) among countries (political regions), especially since much of the exports of a given region within a country may go to another country or its regions. Again to do this, we have recourse to another highly simplified model to allow us to develop certain basic relations to be embodied in a core framework for a relevant applied general interregional equilibrium model.

To begin consider a simple two country model, each country having the same two production sectors but being endowed with different amounts of two resources. We choose to follow the framework embodied in the Heckscher-Ohlin model of trade between two differently endowed countries — a model which has been extensively discussed in the economic literature. Its important conclusion which has played a significant role in trade policy discussion for many years has been that a country's exports use intensively the country's abundant factor. For example, in a two-country situation involving the two scarce factors of capital and labor, the country having the greater stock of capital relative to labor will produce and export commodities that are capital intensive in production, and the country having the greater stock of labor relative to capital will produce and export commodities that are labor intensive in production. More significant for its effect on economic thought was the result that in the two countries factor prices will be the same when expressed in terms of the currency of either country. Hence, there will be no economic incentive or need for any factor migration. While in recent decades, many qualifications to the findings of the Heckscher-Ohlin model have been made from relaxing their assumptions in order to introduce more of reality into their analysis, the assumption of zero or negligible transport cost in trade has by and large not been relaxed. As a consequence, the development of applied general equilibrium (AGE) models which also have been concerned with trade, by and large have retained that transport cost assumption. This has led, as we will see, to serious shortcomings in the findings of these models, and also to inadequate analyses of the location problem for countries and regions of these countries.

Among others, the assumptions of Heckscher-Ohlin model are: (1) identical production functions in the two countries; (2) constant returns to scale in production and diminishing returns in the use of any given factor; (3) identical consumption patterns in the two countries at each relevant commodity price-ratio; (4) non-reversibility of factor intensities such that a given commodity is factor intensive in the use of labor (or capital) at all relevant factor price-ratios; and (5) zero or negligible transport costs in trade.[8] We now proceed to introduce the reality of transport inputs and transport costs into a simple Heckscher-Ohlin model.

Let two countries, *A* and *B*, have stocks of the two resources, labor and capital, *A* being labor abundant, and *B* capital abundant, relatively speaking. The factors are internationally immobile. Although different constant returns-to-scale production functions are taken to exist for the two goods #1 and #2, for any one good they are identical for the two countries.

### *8.2.1 The consumption subsystem*

With pure competition in each sector, profits are zero. Therefore, *incomes* in the two countries will be

$$Y^i = r^i K^i + w^i L^i \qquad i = A,B \qquad (8\text{-}27)$$

and, as in the previous section, their *consumption* of goods upon utility maximization will be

$$C^i_1 = Y^i/2P^i_1;\; C^i_2 = Y^i/2P^i_2 \qquad i = A,B \qquad (8\text{-}28)$$

### *8.2.2 The production system*

The production functions are

$$X^i_1 = (K^i_1)^{0.25} (L^i_1)^{0.75} \qquad i = A,B \qquad (8\text{-}29)$$

and

$$X^i_2 = (K^i_2)^{0.5} (L^i_2)^{0.5} \qquad i = A,B \qquad (8\text{-}30)$$

Thus, maximization of profits leads, as in the previous section, to

$$k^i_1 = (w^i/3r^i)^{0.75} \qquad \ell^i_1 = (3r^i/w^i)^{0.25}$$
$$k^i_2 = (w^i/r^i)^{0.5} \qquad \ell^i_2 = (r^i/w^i)^{0.5} \qquad i = A,B \qquad (8\text{-}31)$$

Note that the first production function is labor intensive, and the second, capital intensive. We therefore start off by assuming that the labor abundant nation *A* will tend to export commodity #1 to *B*, and that the capital abundant nation *B* will tend to export commodity #2 to *A*. Under conditions of pure competition, maximization of profits will lead, as noted in the previous section, to

$$P^A_1 = r^A k^A_1 + w^A \ell^A_1 \qquad (8\text{-}32)$$

and

$$P^B_2 = r^B k^B_2 + w^B \ell^B_2 \qquad (8\text{-}33)$$

(Prices $P^A_2$ and $P^B_1$ are to be specified below.)
Since no intermediate inputs are required in our Heckscher-Ohlin type model, we have

$$X^A_1 = C^A_1 + Ex^{A\to B}_1 \qquad X^A_2 = C^A_2 - Ex^{B\to A}_2 \qquad (8\text{-}34)$$

$$X^B_1 = C^B_1 - Ex^{A\to B}_1 \qquad X^B_2 = C^B_2 + Ex^{B\to A}_2 \qquad (8\text{-}35)$$

where $Ex_1^{A \to B}$, $Ex_2^{B \to A}$ represent exports (and imports), where for example $Ex_1^{A \to B}$ represents exports of good #1 from *A* to *B* (also imports of good #1 of *B* from *A*).

### *8.2.3 The transport subsystem*

Assuming *A* will export the labor intensive good #1 and *B* the capital intensive good #2, we have two *transport* activities. One is activity #4 in *A* which produces the transport inputs to ship good #1 from *A* to *B*, $\alpha_{41}$ being the amount of transport inputs required per unit of export of #1. The other is activity #5 in *B* which produces the transport inputs to ship good #2 from *B* to *A*, $\alpha_{52}$ being the amount of transport inputs required per unit of export of #2. Hence, we have

$$X_4^A = \alpha_{41} Ex_1^{A \to B} \qquad (8\text{-}36)$$

and

$$X_5^B = \alpha_{52} Ex_2^{B \to A} \qquad (8\text{-}37)$$

Both $X_4^A$ and $X_5^B$ are taken to be Leontief production functions whose only inputs (exogenously specified) are $k_4^A$ and $\ell_4^A$, and $k_5^B$ and $\ell_5^B$, respectively. Also under conditions of price competition

$$P_4^A = r^A k_4^A + w^A \ell_4^A \qquad (8\text{-}38)$$

$$P_5^B = r^B k_5^B + w^B \ell_5^B \qquad (8\text{-}39)$$

### *8.2.4 The market subsystem*

Wages ($w^A$ and $w^B$) and rents ($r^A$ and $r^B$) are determined, respectively, by the supply = demand equations for labor and capital. In endnote 9, sixteen equations on capital and labor requirements by each of the six production activities and their four subtotals are presented. They result in the four total resource demands $K^A$, $K^B$, $L^A$ and $L^B$ that enter the four market clearing equations

$$\begin{aligned} K^A &= \bar{K}^A \qquad & L^A &= \bar{L}^A \\ K^B &= \bar{K}^B \qquad & L^B &= \bar{L}^B \end{aligned} \qquad (8\text{-}40)$$

where $\bar{K}^A$, $\bar{L}^A$, $\bar{K}^B$ and $\bar{L}^B$ are the exogenous (given) supplies.

In addition, pure competition among traders (brokers)[10] leads to

$$P_2^A = fe(P_2^B + \alpha_{52} P_5^B) \qquad (8\text{-}41)$$

$$P_1^B = (1/fe)(P_1^A + \alpha_{41}P_4^A) \tag{8-42}$$

where $P_2^A$ and $P_1^B$ are in effect 'delivered cost' prices and equivalent to world prices and fe is the foreign exchange rate converting *B*'s currency into *A*'s.

Finally, the exchange rate variable is determined by the condition that the value of *B*'s imports at delivered cost prices (equal to the value of *A*'s exports) is equal to *A*'s imports at delivered cost prices (equal to the value of *B*'s exports). That is, where M represents imports, where for example superscript $A \rightarrow B$ stands for an import of *B* from *A*,

$$(1/fe)(P_1^A + \alpha_{41}P_4^A)M_1^{A\rightarrow B} - fe(P_2^B + \alpha_{52}P_5^B)M_2^{B\rightarrow A} = 0 \tag{8-43}$$

Altogether in this model, there are 49 unknowns and 49 equations to determine them. Recall that in a general equilibrium system, as is involved in this model, we can solve only for relative prices. However, we are free to set one price at unity to serve as numeraire. We thus set fe = 1, thereby insuring that the absolute prices in each country will be the same whether expressed in one currency (say dollars) or the other (say pounds). When we do set one price at unity, one of the equations can be taken to be redundant.[11] The prices are still relative to the magnitude set for the numeraire.

Taking initial resources to be $\bar{K}^A = 0.8$, $\bar{L}^A = 2.0$, $\bar{K}^B = 1.6$ and $\bar{L}^B = 1.8$ and setting as exogenous the values $\ell_4^A = \ell_5^B = 0.2$; $k_4^A = k_5^B = 0.25$; $d^{A\rightarrow B} = d^{B\rightarrow A} = 0$; and thus setting $\alpha_{41} = \alpha_{52} = 0$ (the dimensionless world of no transport costs and no space), we obtain with fe = 1 a solution where, in the first column (designated H/O) of Table 8-2, we list outcomes for selected variables. Since distance between *A* and *B* is set at zero, this column in effect yields Heckscher-Ohlin results given the above exogenous values. We particularly note that rents and wages in *A* and *B* are equalized.

We now introduce distance between *A* and *B*, for example 100 miles which for units of goods #1 and #2, each of which weighs 20 lbs., yield $\alpha_{41}$, $\alpha_{52} = 1$ (one ton-mile). In column 2 (headed I/A1) of Table 8-2 we record the changed magnitudes that result. Changes in other magnitudes not noted in Table 8-2 are presented in an endnote.[12] First note that the output of two transport activities $X_4^A$ and $X_5^B$ now become relevant, and are of course, positive. And likewise the respective prices, $P_4^A$ and $P_5^B$, for their unit outputs (one ton-mile of transport service). Because the production of transport inputs requires resources, the total of the regional outputs of each of the two goods has decreased [from 1.755 (1.432 + 0.323) to 1.744 (1.285 + 0.459) for #1, and from 1.560 (0.144 + 1.416) to 1.539 (0.260 + 1.279) for good #2]. Likewise, the total consumption of the two regions for

**Table 8-2 Impact of distance on trade among nations and the location problem**

| | H-O | I/A1 | I/A2 | I/A3 |
|---|---|---|---|---|
| $K^A$ | 0.8 | 0.800 | 0.6 | 0.783 |
| $L^A$ | 2.0 | 2.000 | 1.8 | 1.982 |
| $K^B$ | 1.6 | 1.594 | 1.584 | 1.419 |
| $L^B$ | 1.8 | 1.795 | 1.812 | 1.603 |
| $X_1^A$ | 1.432 | 1.285 | 1.711 | 1.285 |
| $X_2^A$ | 0.144 | 0.260 | 0.155 | 0.243 |
| $X_4^A$ | — | 0.485 | 0.485 | 0.485 |
| $X_1^B$ | 0.323 | 0.459 | 0.489 | 0.404 |
| $X_2^B$ | 1.416 | 1.279 | 1.256 | 1.151 |
| $X_5^B$ | — | 0.428 | 0.412 | 0.401 |
| $C_1^A$ | 0.797 | 0.800 | 0.686 | 0.800 |
| $C_2^A$ | 0.708 | 0.688 | 0.567 | 0.644 |
| $C_1^B$ | 0.958 | 0.944 | 0.974 | 0.889 |
| $C_2^B$ | 0.852 | 0.851 | 0.845 | 0.751 |

| | H-O | I/A1 | I/A2 | I/A3 |
|---|---|---|---|---|
| $Y^A$ | 4.689 | 13.816 | 11.466 | 13.816 |
| $Y^B$ | 5.640 | 16.719 | 16.719 | 15.754 |
| $P_1^A$ | 2.943 | 8.635 | 8.357 | 8.635 |
| $P_2^A$ | 3.312 | 10.045 | 10.118 | 10.728 |
| $P_4^A$ | — | 0.224 | 0.223 | 0.224 |
| $P_1^B$ | 2.943 | 8.859 | 8.580 | 8.859 |
| $P_2^B$ | 3.312 | 9.823 | 9.896 | 10.491 |
| $P_5^B$ | — | 0.222 | 0.222 | 0.237 |
| $r^A$ | 1.614 | 5.162 | 5.46 | 5.162 |
| $w^A$ | 1.699 | 4.843 | 4.55 | 4.843 |
| $r^B$ | 1.614 | 4.585 | 4.585 | 4.9 |
| $w^B$ | 1.699 | 5.213 | 5.213 | 5.48 |
| $Ex_2^{B-A}$ | 0.564 | 0.428 | 0.411 | 0.401 |
| $Ex_1^{A-B}$ | 0.635 | 0.485 | 0.485 | 0.485 |
| fe | 1.0 | 1.0 | 1.0 | 1.0 |

each good. However, notice that each region has become less specialized, producing less of the good in which it has comparative advantage and more of the good in which it has comparative disadvantage. Most important, no longer does trade equalize wages and rents over the two regions, the major conclusion of the Heckscher-Ohlin model extensively employed by trade analysts. As can be expected, rent in region *A* has increased *relative* to rent in *B* because indirectly through trade *A* has now diminished access (greater resistance or cost) to the relatively abundant supply of capital in region *B*. (Recall that the Heckscher-Ohlin model and our adaptation of it only solves for relative rents, wages and prices, so that no direct comparison of these items in the two models is possible.) And the wage in region *B* has increased *relative* to that in region *A* because indirectly through trade *B* has now diminished access (greater resistance or cost) to the relatively abundant supply of labor in region *A*.

Correspondingly, prices for each good in the two regions are no longer the same as they are in the Heckscher-Ohlin model. Under conditions of pure competition among producers and traders, prevailing prices are delivered costs which include the transport cost of a unit.

Having demonstrated the impact of the introduction of distance and transport inputs for one set of positive distances, we can proceed to do the same for other sets of distances, each of which would show the misleading results of a Heckscher-Ohlin model for the space-economy of reality.

### *8.2.5 The location problem in an applied general interregional equilibrium framework*

Before leaving the above modified Heckscher-Ohlin framework we wish to use it to introduce the general location problem in an applied general equilibrium setting. More specifically we wish to go beyond the partial classical comparative cost approach.[13]

Suppose a private multinational enterprise seeks to construct a new facility, and on the basis of cost alone to determine the better location, whether in *A* or *B*. Suppose also that it anticipates that 50% of its output will be marketed or aimed to service users in each country. Further, it is well aware of the fact that future fluctuations of the exchange rate are random phenomena; however, it assumes that the current rate (which we have set at unity in our model) is the best one to use. It anticipates that the f.o.b. price of the item to be produced will be the same for all consumers, whether the facility will be located in *A* or *B*. Moreover, it assumes that whether located in *A* or *B* the transport costs will be the same whether borne by it or users. It estimates that for the scale of operations it has in mind its input requirements (inclusive of those for transportation) will be 0.2 units of both capital and labor.

Using the classical comparative cost approach, the multinational looks at current prices as recorded in the second column of Table 8-2, and might conclude that *B* is the better location given that its total costs (0.2 [4.585 (rent) + 5.213 (wage)]) are 1.960, whereas in *A* they would be 2.001, relatively speaking. However, knowing from experience that a location of a major facility in any area tends to raise factor prices there, the enterprise wishes to estimate what costs will be when the facility has been constructed and has been in operation, subject to desirable constraints.[14] It therefore conducts an applied general interregional equilibrium model to do so. Were it to use the model of the second column of Table 8-2, it would conduct two runs. In the first, it would reduce the stock of each resource in *A* by 0.2 and thereby obtain the results of column 3 (I/A2). In the second run, it would leave unchanged the stock of *A*'s resources 0.8 (capital) and 2.0 (labor) and reduce the stock of each resource in *B* by 0.2. Thereby it would obtain the results of column 4 (I/A3). Based on the figures in the third column, the costs of a location in A would be 0.2 (5.46 + 4.55) = 1.001. Based on the figures in the fourth column, the costs of a location in *B* would be 0.2 (4.9 + 5.48) = 1.038. The unit might therefore choose to locate in *A* rather than in *B*, *B* being the classical location that comparative cost analysis would suggest. And as a consequence, the trade, production and consumption patterns in each of the two regions would be affected by the decision, as the data in the several columns indicate.

This simple exercise, of course, omits many diverse factors affecting location. But it clearly points up that a suitable applied general interregional equilibrium framework can provide a more probing locational analysis. This framework can serve as an extension, if not replacement, of the old-fashioned, classical partial equilibrium comparative cost approach in the consideration of the location of major facilities.

Parenthetically it should be observed that there disappears in thin air the relatively ancient controversy over whether a general trade theory incorporates location theory, or whether a general location theory incorporates trade theory. In reality there is only one general theory, a general theory of trade and location.

### *8.2.6 Some concluding remarks*

We conclude this section with the observation that recognition of distance as a basic variable and of the need for transport inputs for trade between two countries invalidate the Heckscher-Ohlin model for any but extreme situations where transport costs are zero or negligible. Obviously this conclusion holds for a many-country world. Also, for a many-country world, the location problem can be probed more deeply with an applied general interregional framework.

## 8.3 The scaffolding of a core social accounting matrix for an applied general interregional equilibrium (AGIE) model

### *8.3.1 Trade in a two country/three region world: the scaffolding of an interregional (international) input-output core*

Having shown that the injection of transport cost into the original Heckscher-Ohlin model invalidates its conclusions and those of many variants of that model developed over the last decades, we now wish to undertake the construction of a hypothetical model which, in as simple a manner as possible, captures the basic variables of a general interregional equilibrium framework for application — a framework that leaves the spaceless (dimensionless) world of the typical abstract economist.

We consider two nations, $Q$ and $Z$, each of spatial extent, engaged in trade. Nation $Q$ comprises two regions $A$ and $B$. To avoid unnecessary complication, nation $Z$ is taken to be a single region, having a different currency than $Q$. In both $A$ and $B$ the two final consumption commodities #1 and #2 are produced. However, in $Z$ only a single commodity #3, say coal, is produced which serves as an intermediate input in the production of #1 and #2. While $Z$ does not produce commodities #1 and #2, with income earned from coal production it imports #1 and #2 from $A$ and $B$ for final consumption. We assume that (1) the labor intensive region A producing the labor intensive commodity #1 ships it to region $B$ (the capital intensive region) as well as to $Z$; and (2) the capital intensive region $B$ producing the capital intensive commodity #2 ships it to region $A$ as well as to $Z$. (Under certain configurations of distances and resource stocks, shipment of #2 from $A$ to $Z$ and of #1 from $B$ to $Z$ may also occur; however to keep the presentation simple, we confine the analysis to the more likely situation where $A$ and $B$ each ship to $Z$ only one commodity, namely the one in which each has comparative advantage given their stocks of resources and existing technology.) Thus *A* engages in two transport activities, shipment of #1 to *B* (activity #4) and shipment of #1 to *Z* (activity #8). *B* also engages in two transport activities, shipment of #2 to *A* (activity #5) and shipment of #2 to *Z* (activity #9). *Z* in turn engages in two shipment activities, one of good #3 to *A* (activity #6), and the other of good #3 to *B* (activity #7). See Figure 8.1 which depicts the transport system.

We further simplify the model by allowing the use of the intermediate input coal (commodity #3) in the production of only good #1 in *A* (say a hot region requiring air conditioning in the plant manufacturing good #1) and in the production of only good #2 in *B* (say a cold region requiring heat for the production process of good #2). (We assume that the production process of #1 in *B* does not require heat and the production process #2 in *A* does not require air conditioning.) Again, see Figure 8.1. These simplifications do not interfere with the presentation of the basic interactions and interdependencies when more intermediate inputs and more possible transport activities are allowed.

When this highly simplified model is embodied in an applied general interregional equilibrium framework where labor and capital resources are taken to be immobile, ninety-one variables and corresponding equations become involved. To put all these variables and equations into the text would take an excessive amount of space, and is not necessary if the reader has digested the materials of the preceding sections. The equations and variables are placed in Appendix 8.1 of this chapter.

The equilibrium solution that results from the operation of this model using a GAMS program is presented in Tables 8-3 and 8-4. Table 8-3 records physical flows in the system; and above the body of the table are listed the equilibrium rents, wages, and prices. Table 8-4 records the corresponding money flows. Once again, a companion programming problem can be set up with consumers maximizing a nonlinear utility function under appropriate constraints that ensure equilibrium at commodity and resource markets. Its solution would yield the results of Tables 8-3 and 8-4.

Introduction of a third region (country *Z*) adds two final consumption variables $C_1^Z$ and $C_2^Z$ to the four for *A* and *B*. The six final consumption levels, in both physical and monetary terms, are presented, respectively, in the last columns of Tables 8-3 and 8-4, these levels having resulted from maximization of utility subject to income constraints. The three adjacent columns relate to: (1) production outputs; (2) exports (–) and imports (+) between the two regions *A* and *B* of country *Q;* and (3) exports and

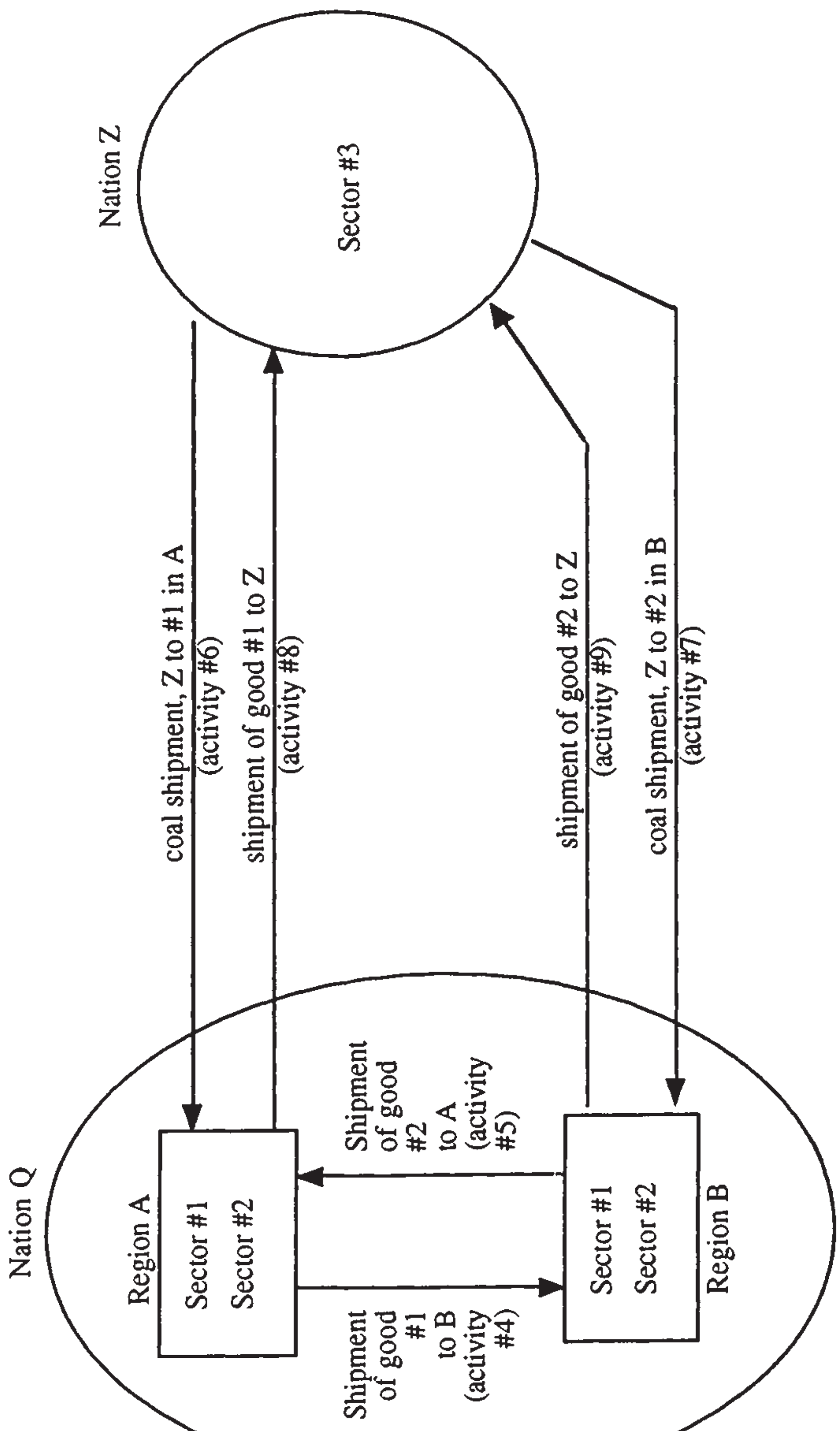


**Figure 8.1 The transport system in a two country/three region model**

## Table 8-3 Physical magnitudes (inputs, outputs and flows) and prices

$r^A=0.592$ $w^A=0.512$ $P_1^A=0.943$ $P_4^A=0.025$ $P_2^B=1.078$ $P_9^B=0.035$ $P_3^Z=0.148$

$r^B=0.478$ $w^B=0.579$ $P_2^A=1.102$ $P_8^A=0.036$ $P_3^B=0.222$ $P_1^Z=0.979$ $P_6^Z=0.074$

$r^Z=4.536$ $w^Z=0.290$ $P_3^A=0.222$ $P_1^B=0.968$ $P_5^B=0.024$ $P_2^Z=1.112$ $P_7^Z=0.074$

| | | Region A | | | | Region B | | | | Country Z | | | | Exports (−) Imports (+) | | Consumption |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | sec1 | sec2 | tr4 | tr8 | sec1 | sec2 | tr5r | tr9 | sec3 | tr6 | tr7 | Output | A→B B→A | Z→A,B A,B→Z | C |
| A | sec1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.180 | -0.363 | -0.023 | 0.794 |
| | sec2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.353 | 0.327 | 0 | 0.678 |
| | tr4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.363 | -0.363 | 0 | 0 |
| | tr8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.023 | 0 | -0.023 | 0 |
| B | sec1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.570 | 0.363 | 0 | 0.933 |
| | sec2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.185 | -0.327 | -0.020 | 0.838 |
| | tr5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.327 | -0.327 | 0 | 0 |
| | tr9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.020 | 0 | -0.020 | 0 |
| Z | sec1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.023 | 0.023 |
| | sec2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.020 | 0.020 |
| | sec3 | 0.059 | 0 | 0 | 0 | 0 | 0.141 | 0 | 0 | 0 | 0 | 0 | 0.200 | 0 | 0 | 0 |
| | tr6 | 0.059 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.059 | 0 | 0 | 0 |
| | tr7 | 0 | 0 | 0 | 0 | 0 | 0.141 | 0 | 0 | 0 | 0 | 0 | 0.141 | 0 | 0 | 0 |
| | L | 1.611 | 0.380 | 0.009 | 0.001 | 0.715 | 1.077 | 0.008 | 0.001 | 0.008 | 0.001 | 0.003 | | | | |
| | K | 0.464 | 0.328 | 0.007 | 0.001 | 0.289 | 1.304 | 0.007 | 0.001 | 0.006 | 0.001 | 0.002 | | | | |

Note: Discrepancies arise from rounding of numbers.

## Table 8-4 Money values (inputs, outputs and flows)

| | | Region A | | | | Region B | | | | Country Z | | | | Exports (–) Imports (+) | | Con-sump-tion |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | sec1 | sec2 | tr4 | tr8 | sec1 | sec2 | tr5 | tr9 | sec3 | tr6 | tr7 | Out-put | A → B / B → A | Z → A,B / A,B → Z | C |
| A | sec1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.113 | -0.342 | -0.021 | 0.750 |
| | sec2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.389 | 0.360 | 0 | -0.750 |
| | tr4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.009 | -0.009 | 0 | 0 |
| | tr8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.001 | 0 | -0.001 | 0 |
| B | sec1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.551 | 0.351 | 0 | 0.903 |
| | sec2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.277 | -0.353 | -0.021 | 0.903 |
| | tr5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.008 | -0.008 | 0 | 0 |
| | tr9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.001 | 0 | -0.001 | 0 |
| Z | sec1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.022 | 0.022 |
| | sec2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.022 | 0.022 |
| | sec3 | 0.009 | 0 | 0 | 0 | 0 | 0.021 | 0 | 0 | 0 | 0 | 0 | 0.030 | 0 | 0 | 0 |
| | tr6 | 0.004 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.004 | 0 | 0 | 0 |
| | tr7 | 0 | 0 | 0 | 0 | 0 | 0.010 | 0 | 0 | 0 | 0 | 0 | 0.010 | 0 | 0 | 0 |
| | L | 0.825 | 0.194 | 0.005 | 0.000 | 0.414 | 0.623 | 0.005 | 0.004 | 0.002 | 0.002 | 0.001 | | | | |
| | K | 0.275 | 0.194 | 0.004 | 0.000 | 0.138 | 0.623 | 0.003 | 0.000 | 0.028 | 0.004 | 0.010 | | | | |
| | Total Cost | 1.113 | 0.389 | 0.009 | 0.001 | 0.551 | 1.277 | 0.008 | 0.001 | 0.030 | 0.004 | 0.010 | | | | |

Note: Discrepancies arise from rounding of numbers.

imports between country $Z$ and regions $A$ and $B$. (Since two countries are involved, exports across countries must be distinguished from exports among regions of a given country.) Necessarily, for each production activity in each region and country the physical volume and money value of output must equal, respectively, the sum of the volumes and money values of net exports and consumption. Thus, for example, for activity #1 in $A$ its volume of production, namely 1.180 (as noted in the first row of Table 8-3) equals its export to $B$, namely 0.363, plus its exports to $Z$, namely 0.023, plus its final goods consumption, namely 0.794. (If the commodity #1 in $A$ were used as an intermediate input, the amount of that use would be indicated in the body of the table and would need to be added to the items just noted.) To take another example, for activity #2 in $B$ the money value [in some standard currency of its output, namely 1.277 (see row 6 of Table 8-4)] equals the value of its exports to $A$, namely 0.353, plus its exports to $Z$, namely 0.021, plus its final goods consumption, namely 0.903. Also, for each good imported into a country, the physical volume and money value of the sum of the country's production and net imports of that good (less the use of that good as an intermediate in production activities of that country) must equal its consumption. For example, in row 2 of Table 8-3 the production output of sector 2 in $A$, namely 0.353, plus its imports, namely 0.327, equals its consumption, namely 0.680.

In the case of transport activities, the direct demands for final consumption are zero, but there are the indirect demands for their services from the need to export final consumption goods. In this connection these transport services represent exports for final consumption. When transport services are required to export intermediate inputs (such as coal), they also represent services exported as intermediate inputs.

In our simplified model the use of coal as an intermediate input into production activities #1 in $A$ and #2 in $B$ is specified, respectively, by the constant coefficients $a_{31}$ (= 0.05) and $a_{32}$ (= 0.119). As an intermediate input, coal like any other export item is a subtraction from output.

In the production subsystem the six transport activities, tr4, tr5, tr6, tr7, tr8, and tr9, whose respective levels of production are $X_4^A$, $X_5^B$, $X_6^Z$, $X_7^Z$, $X_8^A$ and $X_9^B$, are listed in the rows and columns of the appropriate regions. While the unit labor and capital inputs for the production of commodities #1 and #2 in both regions are derived from maximization of profits subject to the production function constraints, the unit labor and capital inputs for the six transport activities are taken to be exogenously given as constant Leontief production coefficients. These constants are $\ell_4$, $\ell_5$, $\ell_6$, $\ell_7$, $\ell_8$ and $\ell_9$, and $k_4$, $k_5$, $k_6$, $k_7$, $k_8$ and $k_9$. Note, that here as previously, we treat the provision of transport services as production activity comparable to the treatment of the provision of other services — e.g., legal, health, educational and business. This is absolutely necessary in order to capture in full the reality of space and its implications for all socio-economic-political activity.

The addition of new production activities and their unit factor input requirements make new demands for the use of the stocks of resources. These unit factor inputs when multiplied by the aforementioned levels of output of the respective activities lead to the demands, by activity, for labor and capital, as recorded along the labor and capital rows at the bottom of Table 8-3. These rows cover *A*'s and *B*'s demands for resources as well as *Z*'s, namely $L^Z$ and $K^Z$. For each region, the resource requirements of each of its activities when summed are equal to its exogenously given immobile stocks of labor and capital.

The counterpart to Table 8-3 on physical magnitudes is Table 8-4 on monetary magnitudes (dollar values). There, money flows are derived by multiplying the physical flows by appropriate rents, wages and prices, as listed above the body of Table 8-3. In the fully interdependent framework of general equilibrium analysis, these prices are associated with balance (market clearing) equations. As already mentioned, demands in each region for each of its immobile resources must be equal to the exogenously given stock of the resource, which by up and down variation of rents and wages determines their equilibrium levels, namely $w^A$, $w^B$, $w^Z$, $r^A$, $r^B$, and $r^Z$. Except where imported, the prices of various goods in each region are determined by their unit production costs, the sum of the costs of the required factors (resources) and intermediate inputs. In the case of a finished consumption good that is imported in a region its price is the f.o.b. price in the country of export plus the cost of the required transport inputs. This relationship results from the assumption of pure competition among traders and producers. For example, if the price of a good produced in *A* based on unit factor (and intermediate input) costs were lower than the delivered cost price from *B*, then traders in *B* would find shipment of goods from *B* to *A* unprofitable and there would be no export from *B*. If the former price were higher than the latter, then there would be a positive gain (surplus) from export of the good but competition among traders would eliminate that gain.

Once we multiply the physical flows by the appropriate prices, we obtain the values recorded in Table 8-4. We observe that the value of output of each activity (listed in its row in the output column at the right of that table) is equal to its total production costs (factor plus intermediate input costs) listed at the bottom of its column below its labor and capital costs.

Next observe for each region the sum of payments by its activities for their labor and capital inputs in Table 8-4, alternatively viewed as a sum of value added by (or resource receipts from) its activities. We have defined this sum as regional income. For each region this sum also equals the sum of the values of the products made available for purchase as final consumption goods in each region, alternatively viewed as consumption expenditures. These purchases are listed in the final column of Table 8-4.

Further, if we now take for each region the sum of the value of outputs by its industries and subtract from it the region's *net* import (total imports from, less total exports to other regions), we have what we may define as its Gross Regional Product. Since in our highly simplified model we have not taken into account capital consumption and have implicitly set it at zero, this Gross Regional Product, from which capital consumption should be deducted to derive regional income, comes to equal regional income.

Since we are treating an international system involving two counties, *Z* and *Q* where *Q* comprises regions *A* and *B*, our equilibrium condition that the Balance of Payments among countries be zero requires that the value of imports of each country equals the value of its exports, namely that

$$P_3^A M_3^{Z\to A} + P_3^B M_3^{Z\to B} - \mathrm{fe}(P_1^Z Ex_1^{A\to Z} + P_2^Z Ex_2^{B\to Z}) = 0 \qquad (8\text{-}44)$$

where $P_3^A$, $P_3^B$, $\mathrm{fe}P_1^Z$ and $\mathrm{fe}P_2^Z$ are delivered cost prices in the currency of *Q*, which in our two nation model some analysts may wish to view as world prices $PW_3$, $PW_1$, and $PW_2$, respectively. However, generally speaking, when there are more than two countries engaged in trade in a number of commodities, the use of the concept of world prices can be highly misleading. Because there exist at least several sources of supply and at least several different markets there is no world price of coal, or steel or many other goods since the delivered costs to these markets can be significantly different. Additionally, the transfer of assets or equities from *A* to *B* may be calculated as the imports of *A* from *B* and *Z* less the exports of *A* to both *B* and *Z* that is:

$$\Delta \mathrm{Assets}^A = P_1^B Ex_1^{A\to B} - P_2^A M_2^{B\to A} + \mathrm{fe}(P_1^Z Ex_1^{A\to Z} - P_3^A Ex_3^{Z\to A}) \qquad (8\text{-}45)$$

When the last term is zero (that is when the value of *A*'s exports to *Z* equals the value of its imports from *Z*), then given the international balance of payments equation 8-44, *A*'s change in assets depends only on the difference between *A*'s exports to *B* and *A*'s imports from *B*. However, if *A*'s exports to *Z* are more than its imports from *Z*, then it follows from the international balance of payments equation 8-44, that *B*'s imports from *Z* are greater than its exports to *Z*. Therefore, the change in *A*'s assets is greater than from trade with *B* alone, and *B*'s change in assets is smaller than that trade.

We now have presented the basic scaffolding of a core interregional input-output transactions table for use in developing a core interregional SAM. One can easily add: (1) more production sectors in each region; (2) explicitly consider more resources and many more intermediate inputs; (3) add government, investment and other final demand sectors (as we will do in the next section). Then he/she would have a detailed interregional input-output table similar to the interregional tables discussed in Chapter 3. To present such a table here, however, would make it much more difficult for the reader to see how the various magnitudes are derived, and nothing important for understanding the basic analytical process would have been gained. One point to note, however, is that Table 8-4 requires that there exist a foreign exchange market (endogenous or exogenous) to establish the foreign exchange rate(s) when all regions are not in the same country.

### *8.3.2 Extensions to obtain the scaffolding of a core social accounting frame: a top-down approach*

The discussion of Social Accounting in Chapter 7 has made clear the need to introduce household groups into an applied general interregional equilibrium framework. We do so by deleting the last columns on consumption in Table 8-4 and adding two columns and two rows (representing urban and rural households) to both *A*'s and *B*'s block of transactors. To *Z*'s block we add only one row and column for its city population, assuming the rural sector conducts subsistence agriculture. See Table 8-5 which represents monetary magnitudes (outputs, inputs and flows). We could also add columns and rows to the body of Table 8-3 on physical magnitudes, and such would indeed be useful for certain purposes. However, as indicated in chapter 7, social accounting matrices and analyses have primarily focused on monetary magnitudes, as we shall do from here on.[15]

Once columns are added for household groups in each region, it becomes necessary to add two columns, one for each of the two factors, in order to record the income received by each household group whereby the expenditures in its column can be realized.[16] Parenthetically, in moving from Table 8-4 to Table 8-5 we assumed (1) that each type of household had the same utility function, namely that of equation 8-3, although in

**Table 8-5 Social accounting matrix**

| | | | | REGION A | | | | | | | | | | | | REGION B | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Production/Transport | | | | Factors | | | Institutions | | | | | Production/Transport | | | | Factors | | |
| | | | | Textile | Food | Tr: A-B | Tr: A-Z | Labor | Capital | Land | Urban Hds | Rural Hds | Companies | Govern-ment | Combined Capital | Textile | Food | Tr: B-A | Tr: B-Z | Labor | Capital | Land |
| | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| REGION A | Production Transport | Textile | 1 | 0.11129 | | | | | | | 0.27845 | 0.38224 | | 0.01350 | 0.07491 | | | | | | | |
| | | Food | 2 | | 0.03886 | | | | | | 0.14446 | 0.19832 | | 0.00700 | 0.03886 | | | | | | | |
| | | Tr: A-B | 3 | | | | | | | | | | | | | | | | | | | |
| | | Tr: A-Z | 4 | | | | | | | | | | | | | | | | | | | |
| | Factors | Labor | 5 | 0.82485 | 0.19432 | 0.00465 | 0.00040 | | | | | | | | | | | | | | | |
| | | Capital | 6 | 0.27495 | 0.19432 | 0.00430 | 0.00040 | | | | | | | | | | | | | | | |
| | | Land | 7 | | | | | | | | | | | | | | | | | | | |
| | Institutions | Urban Hds | 8 | | | | | 0.20484 | 0.42657 | | | | | | | | | | | | | |
| | | Rural Hds | 9 | | | | | 0.81938 | 0.04740 | | | | | | | | | | | | | |
| | | Companies | 10 | | | | | | | | | | | | | | | | | | | |
| | | Government | 11 | | | | | | | | 0.01137 | 0.0156 | | | | | | | | | | |
| | Combined Capital | | 12 | | | | | | | | 0.06314 | 0.08668 | | | | | | | | | | |
| REGION B | Production Transport | Textile | 13 | | | | | | | | | | | | | 0.05514 | | | | | | |
| | | Food | 14 | | | | | | | | 0.13107 | 0.17993 | | 0.00633 | 0.03526 | | 0.12768 | | | | | |
| | | Tr: B-Z | 15 | | | | | | | | 0.00292 | 0.00401 | | 0.00014 | 0.00079 | | | | | | | |
| | | Tr: B-Z | 16 | | | | | | | | | | | | | | | | | | | |
| | Factors | Labor | 17 | | | | | | | | | | | | | 0.41352 | 0.62279 | 0.00474 | 0.00041 | | | |
| | | Capital | 18 | | | | | | | | | | | | | 0.13784 | 0.62279 | 0.00312 | 0.00029 | | | |
| | | Land | 19 | | | | | | | | | | | | | | | | | | | |
| | Institutions | Urban Hds | 20 | | | | | | | | | | | | | | | | | 0.31244 | 0.58449 | |
| | | Rural Hds | 21 | | | | | | | | | | | | | | | | | 0.72902 | 0.10315 | |
| | | Companies | 22 | | | | | | | | | | | | | | | | | | | |
| | | Government | 23 | | | | | | | | | | | | | | | | | | 0.0764 | |
| | Combined Capital | | 24 | | | | | | | | | | | | | | | | | | | |
| COUNTRY Z | Production Transport | Coal | 25 | 0.00871 | | | | | | | | | | | | | 0.02082 | | | | | |
| | | Tr: Z-A | 26 | 0.00436 | | | | | | | | | | | | | | | | | | |
| | | Tr: Z-B | 27 | | | | | | | | | | | | | | 0.01041 | | | | | |
| | Factors | Labor | 28 | | | | | | | | | | | | | | | | | | | |
| | | Capital | 29 | | | | | | | | | | | | | | | | | | | |
| | | Land | 30 | | | | | | | | | | | | | | | | | | | |
| | Institutions | Urban Hds | 31 | | | | | | | | | | | | | | | | | | | |
| | | Rural Hds | 32 | | | | | | | | | | | | | | | | | | | |
| | | Companies | 33 | | | | | | | | | | | | | | | | | | | |
| | | Government | 34 | | | | | | | | | | | | | | | | | | | |
| | Combined Capital | | 35 | | | | | | | | | | | | | | | | | | | |
| ROW | | | 36 | | | | | | | | | | | | | | | | | | | |
| Total | | | 37 | 1.22416 | 0.42750 | 0.00895 | 0.00080 | 1.02422 | 0.47397 | 0 | 0.63141 | 0.86678 | 0 | 0.02697 | 0.14982 | 0.60651 | 1.40450 | 0.00786 | 0.00070 | 1.04146 | 0.76404 | 0 |

**Table 8-5 con't.**

| | | | | REGION B (continued) | | | | | COUNTRY - Z | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Institutions | | | | | Production/Transport | | | Factors | | | Institutions | | | | | | | |
| | | | | Urban Hds | Rural Hds | Companies | Govern-ment | Combined Capital | Coal | Tr: Z-A | Tr: Z-B | Labor | Capital | Land | Urban Hds | Rural Hds | Companies | Govern-ment | Combined Capital | ROW | Total |
| | | | | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 |
| REGION A | Production Transport | Textile | 1 | 0.1633 | 0.15783 | | 0.01449 | 0.00681 | | | | | | | 0.01921 | | | 0.00213 | | | 1.22416 |
| | | Food | 2 | | | | | | | | | | | | | | | | | | 0.42749 |
| | | Tr: A-B | 3 | 0.00427 | 0.00412 | | 0.00038 | 0.00018 | | | | | | | | | | | | | 0.00895 |
| | | Tr: A-Z | 4 | | | | | | | | | | | | 0.00072 | | | 0.00008 | | | 0.00080 |
| | Factors | Labor | 5 | | | | | | | | | | | | | | | | | | 1.02422 |
| | | Capital | 6 | | | | | | | | | | | | | | | | | | 0.47397 |
| | | Land | 7 | | | | | | | | | | | | | | | | | | 0 |
| | Institutions | Urban Hds | 8 | | | | | | | | | | | | | | | | | | 0.63141 |
| | | Rural Hds | 9 | | | | | | | | | | | | | | | | | | 0.86678 |
| | | Companies | 10 | | | | | | | | | | | | | | | | | | 0 |
| | | Government | 11 | | | | | | | | | | | | | | | | | | 0.02697 |
| | Combined Capital | | 12 | | | | | | | | | | | | | | | | | | 0.14982 |
| REGION B | Production Transport | Textile | 13 | 0.26294 | 0.25413 | | 0.02333 | 0.01096 | | | | | | | | | | | | | 0.60649 |
| | | Food | 14 | 0.43053 | 0.41609 | | 0.0382 | 0.01794 | | | | | | | 0.01931 | | | 0.00215 | | | 1.40450 |
| | | Tr: B-Z | 15 | | | | | | | | | | | | | | | | | | 0.00786 |
| | | Tr: B-Z | 16 | | | | | | | | | | | | 0.00063 | | | 0.00007 | | | 0.00070 |
| | Factors | Labor | 17 | | | | | | | | | | | | | | | | | | 1.04146 |
| | | Capital | 18 | | | | | | | | | | | | | | | | | | 0.76404 |
| | | Land | 19 | | | | | | | | | | | | | | | | | | 0 |
| | Institutions | Urban Hds | 20 | | | | | | | | | | | | | | | | | | 0.89693 |
| | | Rural Hds | 21 | | | | | | | | | | | | | | | | | | 0.83217 |
| | | Companies | 22 | | | | | | | | | | | | | | | | | | 0 |
| | | Government | 23 | | | | | | | | | | | | | | | | | | 0.07640 |
| | Combined Capital | | 24 | 0.03589 | | | | | | | | | | | | | | | | | 0.03589 |
| COUNTRY Z | Production Transport | Coal | 25 | | | | | | | | | | | | | | | | | | 0.02953 |
| | | Tr: Z-A | 26 | | | | | | | | | | | | | | | | | | 0.00436 |
| | | Tr: Z-B | 27 | | | | | | | | | | | | | | | | | | 0.01041 |
| | Factors | Labor | 28 | | | | | | 0.00232 | 0.00034 | 0.00083 | | | | | | | | | | 0.00349 |
| | | Capital | 29 | | | | | | 0.02721 | 0.00402 | 0.00958 | | | | | | | | | | 0.04081 |
| | | Land | 30 | | | | | | | | | | | | | | | | | | 0 |
| | Institutions | Urban Hds | 31 | | | | | | | | | 0.00314 | 0.03673 | | | | | | | | 0.03987 |
| | | Rural Hds | 32 | | | | | | | | | | | | | | | | | | 0 |
| | | Companies | 33 | | | | | | | | | | | | | | | | | | 0 |
| | | Government | 34 | | | | | | | | | 0.00035 | 0.00408 | | | | | | | | 0.00443 |
| | Combined Capital | | 35 | | | | | | | | | | | | | | | | | | 0 |
| ROW | | | 36 | | | | | | | | | | | | | | | | | | 0 |
| Total | | | 37 | 0.89693 | 0.83217 | 0 | 0.07640 | 0.03589 | 0.02953 | 0.00436 | 0.01041 | 0.00349 | 0.04081 | 0 | 0.03987 | 0 | 0 | 0.00443 | 0 | 0 | |

reality the utility functions would be different, and (2) that each shared its region's imports in proportion to its share of its region's income. These assumptions were made to facilitate the checking of magnitudes if the reader chooses to do so.[17]

Another basic extension in a core social accounting framework would involve the incorporation of the government sector.[18] This sector can produce commodities, such as postal services or services of railroads and diverse public utilities. In doing so, it would function much like a firm. It may confront a production function, purchase inputs for its operation, charge prices for its outputs and otherwise seek to maximize its profits, or minimize costs when a subsidy is provided from taxation or other revenue sources. When it does so, it can be treated much like any other production activity introduced into the model, whose level of operations would be derived from the operation of an AGIE model.

On the other hand, the government may function outside the market system to provide necessary services and goods to households and enterprises. In this type of functioning the government would need to make a decision on how much of each good and service to produce and distribute to consumers. Thus, where the level of these goods and services to be distributed to households in region $A$ is represented by $G^A$, the typical household utility function becomes

$$U^A = ((C_1^A)^{0.5}(C_2^A)^{0.5};\ G^A), \tag{8-46}$$

$G^A$ being an exogenous variable.

In its production, the government in $A$ will typically need to enter the factor markets to purchase capital and labor inputs, namely $L_g^A$ and $K_g^A$, which also are to be treated as exogenous variables. Thus, each of the balance equations for the respective factor markets will include an additional term. For example, the equation for the labor market in $A$ will be

$$L_1^A + L_2^A + L_4^A + L_8^A + L_g^A = \bar{L}^A. \tag{8-47}$$

In $A$ there will be government expenditures $H_g^A$ for factors and other goods. When no intermediate inputs and transport inputs are required we would have:

$$H_g^A = w^A L_g^A + r^A K_g^A \tag{8-48}$$

These expenditures would need to be met by government revenues or income $Y_g^A$. If the government in $A$ is not permitted to spend more than its income or to accumulate a surplus of funds, then we have an additional balance equation

$$Y_g^A - H_g^A = 0 \tag{8-49}$$

Revenues come from taxation. Typically, at least four sources are available: a tax ($t_x$) on production, a tax ($t_c$) on consumption, a tax ($t_y$) on factor earnings, and a tax ($t_m$) on tariff or imports. For example, if only the last three taxes are imposed in $A$, we would have

$$Y_g^A = t_c(C_1^A + C_2^A) + t_y(r^A K^A + w^A L^A) + t_m(M_2^{B \to A} + M_3^{Z \to A}) \tag{8-50}$$

The effective income equation of consumers in $A$ would change to be

$$Y^A = w^A L^A + r^A K^A - Y_g^A \tag{8-51}$$

However, $A$'s gross regional income (output) would be $Y^A + Y_g^A$. Also, the final goods price equations would change; for example, for commodity #1 in $A$ we would have:

$$P_1^A = w_1^A \ell_1^A + r_1^A k_1^A + t_c \tag{8-52}$$

Thus, if for $A$ only the income tax were to be endogenously determined, the above introduces five exogenous variables $G^A$, $L_g^A$, $K_g^A$, $t_c$, and $t_m$. There then exist for region $A$ only three new endogenous variables, namely $H_g^A$, $Y_g^A$ and the income tax rate $t_y$. These are matched by the three new equations (8-48, 8-49 and 8-50) to be added to arrive at a solution for the system if a government sector is introduced in region $A$ only. (If government sectors are introduced in other regions, then similar new variables and equations come to exist for each.) In effect, the appropriate level $t_y$ of the income tax in $A$ insures revenues adequate to meet the expenditures of government programs designed to provide diverse services deemed essential for its constituents, the final consumers. Much more sophisticated treatments of the government sector have been developed in the dimensionless AGE literature. It is left for the regional science researcher to effect the modifications of these sophisticated treatments for the reality of a space economy.

To keep at a minimum the number of rows and columns in the scaffolding of a core social accounting matrix we have added, in going from Table 8-4 to Table 8-5, only one row and one column for the government sector in each region. Along the row the various sources of revenues of a government are recorded. Along the column, the various expenditures of a government are listed. In the case of region $Z$, we assume the government obtains its revenue from a 10% tax on factor (labor and capital) earnings so that the incomes of urban households in $Z$, as listed at the bottom of its column, is only 90% of total payments for labor and capital in $Z$, all these payments going to the urban households in say a mining town. [Note that country $Z$ may be taken to have a rural household sector, but at the present time in most regions data on such a sector (in terms of equivalent money earnings and expenditures) are not available.[19]] In the case of region $A$, it is assumed that the government obtains its revenue from a personal (household) sales tax of 2 percent, imposed on the after-savings income of all households. To that extent, total household incomes available for expenditure are reduced. In the case of region $B$, it is posited that the government obtains its revenue from a 10% tax on the earnings of capital, no tax being levied on earnings from labor.

One more set of magnitudes to be entered into a core SAM framework are those on savings and investment. Savings might come from household income, unexpended revenue of government, undistributed profits of companies, foreign sources, etc. In a table for AGIE use there might be one row representing savings as a combined capital account or more than one row when specific types of savings need to be made explicit. Similarly, there could be one or more columns to represent investment. To illustrate these processes consider savings from household income only. Let there be an exogenously specified savings rate $s$ (say 0.1 or 10 percent) of income. Then each of the consumption equations would change; for example for good #1 in $A$ it would be

$$C_1^A = (1 - s)Y^A/2P_1^A \tag{8-53}$$

where total savings $S^A$ in region $A$ would be

$$S^A = sY^A \tag{8-54}$$

On the assumption that all savings would be absorbed by investment $I^A$ in $A$ when no credit is extended by banks, there would be the equation

$$I^A = S^A. \tag{8-55}$$

Finally, if the investment requires inputs of only labor and capital, namely $L_I^A$ and $K_I^A$ (assumed to be exogenously determined), there would be the equation

$$I^A = w^A L_I^A + r^A K_I^A. \tag{8-56}$$

Accordingly, the resource balance equations for $A$ change; for example, for labor it becomes

$$L^A = L_1^A + L_2^A + L_G^A + L_I^A. \tag{8-57}$$

Thus we have three new exogenous variables, $s$, $L_I^A$, $K_I^A$, and two new endogenous variables, $S^A$ and $I^A$, with the two additional equations 8-54 and 8-55 to determine them.

In a more realistic setting, we may have an interest rate (a price of savings) as another new variable to be involved in the determination of the level of savings by consumers, where also the interest rate is related to the level of profits (another new variable) of the companies engaged in production and investment whose inputs of labor and capital would be derived from maximization. Additional and extended equations related to conditions of maximization and market clearing would be involved.

Again, to keep the rows and column of Table 8-5 to a minimum we add to the blocks for regions *A* and *B* in Table 8-4 one row for savings (combined capital account) and one column for investment. In *A* we assume that before expending income on commodities and thus before incurring the consumption tax, all households save 10 percent of their money income and that this saving is available for investment. In *B*, we assume that only urban households save, and at the rate of 4 percent of their income. In nation *Z*, we assume that there is no saving and investment.

Once disposable income is obtained for each household group, we assume that the pattern of expenditure remains unchanged from that in Tables 8-3 and 8-4, only its size has been reduced from savings and/or taxation. To make it easy for the reader who desires to check out the figures in the SAM Table 8-5, we have assumed that government and investors expend their moneys over the same goods and in the same proportion as households. Hence the reader can easily work out the passage from an input-output table (whether in money or physical terms) to a SAM matrix.[20]

Finally in Table 8-5 we can easily introduce a second intermediate input, namely the use in each sector of its own product. We can do this without changing the price system and the reader-friendly production function that is assumed. To sectors 1 and 2 in both *A* and *B* we posit that this intermediate input can be represented by the constant production coefficient 0.1, that is amounts to 10 percent of each unit produced). Accordingly, output of each of these sectors is taken to expand by 10 percent. However, to keep prices unchanged, in effect to keep total demands and supplies at the commodity and factor markets the same, the capital, labor and coal input coefficients of sectors 1 and 2 in *A* and *B* are appropriately reduced, namely by dividing each by a factor of 1.1.[21]

### *8.3.3 Extensions and generalizations to a multi-region, multi-country world*

We now have covered, admittedly in a highly simplified manner, the basic scaffolding of a SAM core of an AGIE model wherein countries (political regions) as well as regions within a country are considered. We now extend the SAM framework to cover many production sectors, many commodities, many resources, many consuming household groups, many governments and government programs, many investment sectors, many regions, and many countries. Doing so in equation form or in a single table such as Table 8-5 would consume much space, way beyond the space allotted to a chapter in a regional science text. Consequently, we can only mention and briefly discuss some of the procedures involved in effecting these extensions.

First, consider the inclusion of more production sectors, $j = 1, \ldots, n$ and more commodities, $h = 1, \ldots, h'$. Each sector might produce one or more of these commodities, and each commodity might be produced by more than one sector. The framework for handling this type of situation is already discussed in the input-output chapter, pp. 98–101. To avoid complication, we shall continue to assume that only one commodity is produced in any given sector and that pure competition among many producers prevails in each sector. Hence, in a table like Table 8-5 there would be added a column for each new sector containing inputs consistent with maximization of profits given the production technology of that sector. It would be done just as an additional column was added for each region *A* and *B* when we moved from the case of only one sector in each region (section 8.1) to the case where each region had two sectors (section 8.2). In effect, the extended framework could incorporate many if not all the sectors of an input-output structural matrix when consistent production coefficients can be taken to characterize production in each of the sectors incorporated.

Moreover, in any sector the use, as intermediate inputs, of the products of other sectors would be common, intermediate inputs being recognized in a manner similar to that in standard input-output analysis. These inputs would be treated just as coal, transport inputs and a sector's own product were in the previous section.

The existence of many new sectors to represent more adequately a realistic space economy would in turn require recognition of more than just two resources. Land as a resource would need to be introduced as well as mineral deposits and other items. Equally important, each of the broad categories of resources would need to be disaggregated — for example, labor by type, land by quality, and minerals by composition. Hence, there would be many more rows than two for resources. The extended set of resources would be denoted by $g = 1,..., g'$. Generally speaking, for each new resource, the stocks by region would need to be exogenously specified.

Another highly desirable extension would involve a much finer disaggregation of the household sector. Such would recognize for each region its several ethnic, economic, and other groups of families, each with different incomes, tastes and other characteristics. The utility functions of these groups might differ significantly as well as their stocks of resources, thus leading to different consumption patterns under maximization of utility. Each group and its pattern would be represented by a separate column. See the discussion in the previous chapter on how this might be done and incorporated into a framework for AGIE use.

Another major extension would involve the disaggregation of government. This sector might be split up into major subsectors (civilian and military); in turn each of these major subsectors can be extensively disaggregated. For most of the resulting categories, the disaggregation would relate to specific government programs, but in others to general activity such as the judicial. In most cases, a category's level and required inputs would need to be specified exogenously. In some cases, however, the output of a government program can be a commodity such as the postal service, and that program can be operated much like production of a firm. The investment sector can also be disaggregated in ways similar to that for the government and household sectors. Infrastructure investment by type (transportation, recreational, medical, etc.) may be distinguished from R and D investment, investment to preserve the environment, and normal industry investment in plant and equipment.

More regions can be added, each in a way that another region was added to the two nation modified Heckscher-Ohlin model, to yield a more comprehensive interregional/international framework. For example, see the study by Whalley and Trela (1989). The establishment of relevant delivered cost prices to determine relevant balance of payments among nations, and also regions, becomes more complex, and is a significant area for research by regional scientists. Also, the general concept of world prices for commodities would need to be dropped.

### *8.3.4 Extensions with a bottoms-up approach to AGIE models*

As already noted, the previous discussion pertains primarily to a top-down approach in the development of a core interregional social accounting matrix appropriate for an AGIE. This reflects the fact that the construction of SAMs for individual countries has dominated past research activity in this field. But for many regional scientists including the authors, the most relevant approach to multi-region or more generally interregional analysis views the region as an active body in a multi-region system. It is subject to political and economic constraints imposed by an aggregate political unit — the nation. On the one hand, the region is not a replica of the national economy (or system). On the other, the magnitudes of the several relevant regions of a country (such as regions *A* and *B* in country *Q*) must add up to the national totals — totals that serve as constraints. Or, in the more general case which recognizes feedbacks from the regions that alter the constraints imposed by the nation, there must be consistency in that regional totals must add up to the changed national magnitudes (for example on employment).

With this perspective a scholar might design a new set of categories to handle transactions with units beyond a region's boundaries. This set would still keep explicit as a category each region such as *A*, *B*, *C*, .... However, when certain kinds of transactions are to be considered, it might often be necessary to introduce as meaningful units groups of regions (such as the first tier of regions surrounding each region, or each region's census region). For example, a scholar studying an urban region fully contained within a state would need to consider the state as an active unit for taxation purposes. This is so even though the scholar is primarily concerned with the interactions of that urban region's economy with the economies of other urban regions. Moreover, that urban region, as well as every other kind of region (for example the state itself), must also consider the nation as an aggregate unit (that affects say housing policy), and perhaps also an aggregate comprising a unit often designated the Rest of the Nation when concerned with the export market. Even more, the Rest of the World that is beyond the confines of a region's nation must be considered when again one is concerned with a region's export market. However, that Rest of the World might need to be disaggregated into meaningful categories (such as the European market, the Latin American market and the Asian market; this disaggregation can be extremely relevant for example when studying a Western U.S. region's export transactions). Even further, when foreign exchange transactions and thus the interest rate become key elements in a region's situation, it is not the Rest of the World, but the entire World which might be the relevant category. It should be noted that Rounds (1988, 1995) has taken a significant step in this direction with his notion of *supranational* units.

In short, the type of hand-me-down procedures for constructing regional SAMs coming from past SAM studies (biased by the perspective of national-oriented economists) may not be the most appropriate for AIGE models. Rather, the regional and interregional SAMs designed to serve as cores of AGIE models, must at the start be based upon an appropriate structure of active behaving units. It could be an hierarchical (nested) one, regular or irregular and overlapping — one that would replace the simple Rest of the Nation (regions *B*, *C*, ...) and Rest of the World categories noted in Figure 7.1. Admittedly, practical considerations stemming from data inadequacies and limited research resources may preclude the use of a framework that is ideal for constructing a SAM for an AGIE study. But it does not follow that the less-than-ideal structure of the SAM should be a standard one such as indicated in Figure 7.1. The required reduction in the scope of the SAM might be better achieved by eliminating and/or aggregating input-output and household categories than by reducing the number of categories of spatial and regional units of different orders and size. Or perhaps a reduction in both directions might be best.

### *8.3.5 The exploration of a Financial SAM and its fusion with a Real SAM*

In recent years much exploration has been conducted at the national level on developing a *Financial SAM*. At times, the objective is to fuse in an AGE model such a SAM covering financial accounts with a type of SAM we have been discussing. The latter type has been designated a *Real SAM* since its monetary magnitudes are based on physical magnitudes and relative prices, and not on physical magnitudes and nominal prices. (For example, see Thorbecke, 1992, ch. 4; Lewis, 1994; and Azis, 1996.) One typical set of financial accounts and items covered are recorded in Figure 8.2. To fuse these accounts with those in a Real SAM, additional equations are required as well as the disaggregation of (1) each combined capital account in the rows of Table 8-5 by sources of savings, and (2) each combined capital account column by type of investment. For example, following Fargeix and Sadoulet (1994), consider one possible set of equations that can be employed to determine for a nation interest rates charged to loans and paid to depositors. (To simplify, we assume these rates to be approximately the same.) The basic equation relates the demand for (BD) and supply of loanable funds. It is given by

$$\mathrm{BD} = \sum_{h} \mathrm{TD} + \mathrm{DD} - \mathrm{RR} + \mathrm{AS}_b \tag{8-58}$$

where, for given year $t$,

**Firms**

| *Assets* | *Liabilities* |
|---|---|
| Currency | Domestic borrowing |
| Demand deposits | Foreign borrowing |
| Stocks | Accumulated savings |
| Capital stock | Equity |

**Households**

| *Assets* | *Liabilities* |
|---|---|
| Currency | Accumulated savings (i.e., wealth) |
| Demand deposits | |
| Time deposits | |
| Foreign currency | |
| Equity | |

**Central Bank**

| *Assets* | *Liabilities* |
|---|---|
| Central Bank credit to government | Currency |
| Foreign currency reserves | Required reserves |
| Direct credit to private sector | Accumulated savings |

**Commercial Banks**

| *Assets* | *Liabilities* |
|---|---|
| Subsidized loans | Demand deposits |
| Unsubsidized loans | Time deposits |
| Required reserves | Accumulated savings |

**Government**

| *Assets* | *Liabilities* |
|---|---|
| Equity held | Central Bank credit to government |
| Capital stock | Domestic borrowing |
| Other stock | Foreign borrowing |
| | Accumulated savings |

**Rest of the World**

| *Assets* | *Liabilities* |
|---|---|
| Foreign loans | Foreign currency |
| | Accumulated savings |

**Figure 8.2 Balance sheet for a Financial SAM**

$\sum_h \text{TD}$ = the total of time deposits supplied by households
DD = demand deposits (of households, firms, governments and ROW)
RR = required reserves of commercial banks
$\text{AS}_b$ = the accumulated savings (undistributed profits) of banks.

In turn, $\text{AS}_b$ is given by

$$\text{AS}_b = \text{AS}_b^{t-1} + \text{S}_b \tag{8-59}$$

namely, the accumulated savings of the previous time period $(t - 1)$ plus savings $\text{S}_b$ from operations during the current year. $\text{S}_b$ is then given by

$$\text{S}_b = (1 - \tau_b - dr_b)\text{Y}_b \tag{8-60}$$

where

$\text{Y}_b$ = income of banks
$\tau_b$ = tax rate on income of banks
$dr_b$ = the fraction of income that is distributed

Income of banks $\text{Y}_b$ is given by

$$\text{Y}_b = r^{t-1}(\sum_{i,g} \text{BD}^{t-1}) - r^{t-1}(\sum \text{TD}_h^{t-1}) \tag{8-61}$$

where

$r^{t-1}$ = the nominal interest rate set by commercial banks in the previous time period $t - 1$
$\sum_{i,g} \text{BD}^{t-1}$ = outstanding domestic loans to firms $(i = 1,2\ldots)$ and government $(g = 1,2,\ldots)$ at $t - 1$
$\sum \text{TD}_h^{t-1}$ = outstanding time deposits of households at $t - 1$

All these variables come to affect and be affected by the numerous other variables in the real and financial systems as well as by changes in the exogenous (policy) magnitudes.

In addition to determining the interest rate set by a region's commercial banks, the financial accounts can be used to determine other variables that may be considered relevant, for example the interest rate a Central Bank charges on loans to commercial banks; and many of the equations in the Real SAM can be considered for revision to capture the effect of changes in relevant financial accounts.

It is to be noted that the recent exploration with the construction and use of a Financial SAM has not led to widespread acceptance of this analytical construct and to the formation of a standard classification of and tabular framework for the accounts covered. However, see Azis (1996) for one interesting and useful classification for fusing a *national* Financial SAM with a *national* Real SAM.

Moreover, the framework of a national Financial SAM is inappropriate for interregional analysis. Compared to the current inadequacies of much of *national* social accounting for use in *interregional analysis*, the inadequacies of current national Financial SAM frameworks for use in interregional analysis are much more serious. A more satisfactory framework (perhaps too disaggregated) for interregional analysis is suggested in the 1960 Methods book in the table on *An Interregional Flow of Funds Matrix* and in the background discussion relating to financial flows (Isard et al., pp. 100–115, 144–173 and 610–621).[22]

A number of serious problems have arisen in constructing a national Financial SAM to be fused with a Real SAM. One arises from the fact that such a Financial SAM has been basically expressed in terms of *stocks at a point in time*, whereas the Real SAM pertains to flows over a time period, usually a year. Thus a fusion of a Financial SAM and a Real SAM requires the estimation of financial flows to connect up with real flows. To do this in this approach, one needs to estimate stocks at two successive time-points (for example, of current assets of commercial banks), the difference being a financial flow (savings). Thus more data are required and furthermore the data on stocks (wealth of households and assets and liabilities of institutions) are often very scarce. (See the commendable attack by Thorbecke et al., 1992, ch. 4 on this problem.)

However, additional difficulties arise from the introduction of a much more extensive set of intangible factors such as expectations and speculation. As we have already mentioned in simulating the impact of any exogenous change on a nominal basis and even a real one, the models on foreign exchange rates and the parameters that are used are only adequate for the situations covered in the sample time period employed for estimating the parameters. When one considers a new situation, as generated by an exogenous change, the prediction of such a model is found to be no better than a random walk. The same can be said for interest rates and generally speaking for all phenomena affected by the random gyrations of currency, stock and other financial markets. As a consequence, there remains considerable skepticism about the possibility of effectively fusing Financial and Real SAMs in applied research. See Mercenier and Srinivasan, 1994, pp. 5–7; and Lora, 1994.

It should be noted, however, that while in the judgment of the authors much more research is required before an applicable AGIE model can be constructed, the problem in integrating appropriate Financial and Real SAMS for an interregional system confined to a single country is less formidable than for a multi-nation system. First, the play of the foreign exchange rate is typically of lesser importance than in multi-country studies, and likewise for the interest rate. Further, the role of transport inputs and transport cost based on real meaningful space and other types of available data on regions is generally useful in adding anchorage to a model of the real world.

## 8.4 Problems and questionable character of the basic assumptions of standard applied general equilibrium models

We have now presented a simplified social accounting framework for an applied general interregional equilibrium model. We have set down the equations and a consistent set of hypothetical data for that model and discussed how that framework and model can be conceptually extended to embrace a many region structure in physical space. In each region there can be many commodities, many consumers, many production sectors with many firms in each, many traders, a number of governments and other institutions, savings and investments of many types, and many markets at which demand for and supply of each of the many resources, commodities and money itself (in the form of more than one currency) determine the many prices. We have also considered some extensions that are necessitated for a bottoms-up approach rather than a top-down one, and seriously questioned whether the top-down is appropriate. Nonetheless, there are advantages, at least until appropriate procedures for a bottoms-up approach are developed, in pursuing a top-down approach. This is so since a top-down approach begins with a country's structure and procedures for modeling which may have already been extensively researched. However, the models of a country's structure have been predicated upon many strong and questionable assumptions. We now examine these assumptions in order to be better aware of the relevance and applicability of the country type of framework and model for regional and interregional study.

### *8.4.1 Problems in representing the consumption subsystem*

Underlying the conceived consumption system is the tenet that each consumer is motivated to maximize his/her utility, satisfaction or some equivalent notion. For an economist seeking quantitative measures of the desirability of policies within a market system, the relevant magnitudes are quantities of commodities and other elements when measurable and theoretically treatable. There are many ways in which utility for a behaving unit (a representative individual, or set of individuals, or household group in a given region) can be defined and has been. A most general way is:

$$U = U(C_1, C_2, C_3, ...; Z_k) \qquad (8\text{-}62)$$

where $Z_k$ represents quantitatively measurable characteristics of the given behaving unit. This general function, over many commodities and characteristics, is beyond specification, and stands in extreme contrast to the highly operational Cobb-Douglas function employed in the previous sections, namely

$$U = (C_1)^{\alpha_1}(C_2)^{\alpha_2} \qquad \Sigma\alpha_i = 1;\ i = 1,2 \qquad (8\text{-}63)$$

and where specifically $\alpha_1 = \alpha_2 = 0.5$. The widespread use of a Cobb-Douglas stems from the fact that it is conveniently embodied in and treated by an elaborate nonlinear model wherein commodity prices are allowed to vary. When this function covers two or more commodities, maximization of utility under budget constraint yields constant expenditure shares $\frac{\alpha_i Y}{P_i}$ with variation in income where $\Sigma\alpha_i = 1$, $i = 1, 2, 3, ...$[23] However, the resulting consumption pattern violates reality. Engle-type and many other consumption studies suggest other effects. For example, with increase in household income, they find positive but decreasing consumption of normal goods, positive and increasing consumption of luxury goods, and declining consumption of inferior goods.

Another widely used consumption framework, namely that of the unadjusted Leontief input-output model, directly assumes, as noted in Chapter 3, that per dollar of income, no matter what the level of income, the cents' worth of household expenditure on any commodity is constant — similar to the constant shares expenditure framework of the above Cobb-Douglas once equilibrium prices are given. An improvement on the Cobb-Douglas and Leontief frameworks is the Linear Expenditure System (LES). It derives from the Stone-Geary utility function where under utility maximization a household first spends its income on minimum subsistence goods, and then the remainder in fixed proportions on the commodities once subsistence is ensured. Hence we have

$$U = (C_1 - S_1)^{\alpha_1}(C_2 - S_2)^{\alpha_2}(C_3 - S_3)^{\alpha_3}, ...\Sigma\alpha_i = 1;\ i = 1, 2, 3, ... \qquad (8\text{-}64)$$

where $S_i$ ($i$ = *1*, *2*, *3*, ...) is the amount of good $i$ required for the subsistence of the behaving unit (that is of the individual, or individuals contained in the behaving unit).[24]

Another type of consumption function which does allow, under budget constraint, substitution among commodities, is the constant elasticity of substitution (CES) function. The mathematical statement of the function is rather complicated but it yields the fairly simple outcome, namely that for each commodity $i$, the elasticity $\sigma$ of the substitution in preferences between any pair of goods $i,j$ is such that $\Sigma\alpha_i^{1/\sigma} = 1$. However, the use of the same elasticity $\sigma$ of substitution regardless of the commodity $j$ is another result which substantially violates reality as we know it. This CES notion can be also combined with the LES function to yield a utility level above survival when $(C_i - S_i)$ replaces $C_i$ in the customary general utility function.

To facilitate calibration, that is the estimate of parameters for an applied model from existing data (often limited), a hierarchical (or nested) set of functions has been employed. The procedure is to construct at the top level of a hierarchy two or a few main (aggregate) categories of commodities, say normal (non-luxury goods) and luxuries, as in Figure 8.3, with an estimation of an appropriate elasticity $\sigma_a$ of substitution between the categories. At the next (a lower) level, non-luxuries might be disaggregated into food and housing with another appropriately estimated elasticity $\sigma_b$. At a still lower level, food might be disaggregated into another set of commodities, again with still another appropriately estimated $\sigma_c$ elasticity of substitution between them. Obviously, a more extensive and elaborate hierarchical structure might be employed.

There are a number of other procedures for deriving (estimating) consumption magnitudes such as the AIDS (Almost Ideal Demand System), GAIDS (Generalized Almost Ideal Demand System) which combine the LES and AIDS models, and so on. Clearly, from the standpoint of regional and interregional work, especially interregional, there is considerable advantage to retaining a Leontief structure when Engle-type findings are not available and when an interregional input-output table is available, even when such a table needs to be redeveloped for a less disaggregated set of household groups. Also when subsistence inputs are obtainable, this set of inputs augmented by an LES set of coefficients can replace the Leontief consumption coefficients.

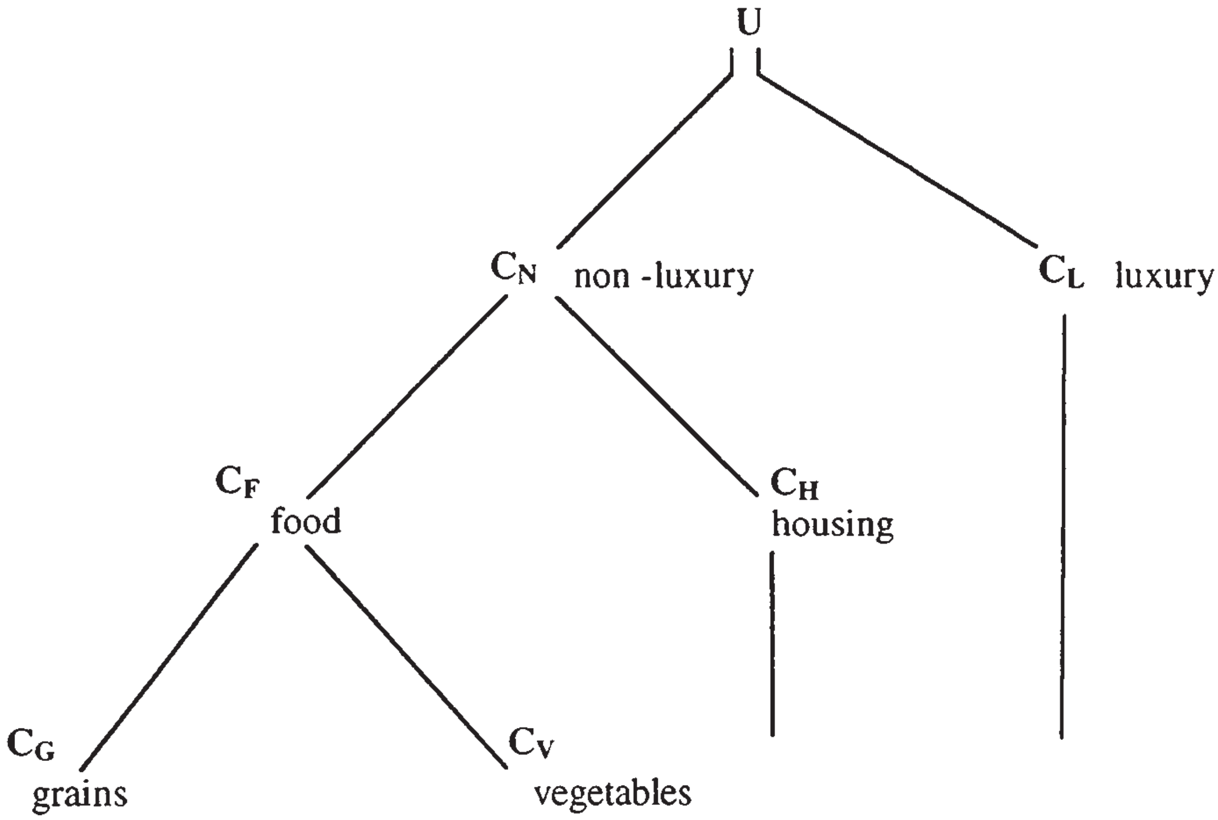


**Figure 8.3 A CES hierarchy re: consumer demands**

*8.4.2 Questionable character of the production subsystem*

With regard to the production subsystem, major problems need to be addressed for the construction of an AGIE model. Cobb-Douglas constant share production functions, such as 8-5 and 8-6, are widely employed in AGE studies. But clearly the fixed coefficients of labor and capital, which the use of such a function is often designed to yield for a set of factor prices endogenously determined, is not particularly helpful for considering realistic situations where substitution between these two basic primary factors occurs. Moreover, the use of Leontief-type constant coefficients for intermediate inputs that often accompanies the use of a Cobb-Douglas constant share function (as is done in the previous sections of this chapter), and which thus leads to constant unit cost of production, is also of questionable value in a general model. The employment of a CES production function which many analysts consider more desirable, also has its limitations. This is so even when one employs nesting operations like that depicted in Figure 8.3 to capture more substitutability among inputs in a mathematically tractable manner — for example: (1) between manufacturing and energy production, then between oil production and coal production as components of energy production, and so on; or (2) between a Value Added aggregate as a production element and intermediate goods as a second aggregate, and then on the one hand between capital and labor that comprise the Value Added aggregate and on the other hand an input-output breakdown of the intermediate aggregate by commodity input. Other production functions such as the generalized Leontief, the transcendental logarithmic, the generalized Cobb-Douglas, the generalized square root quadratic, etc. have their advantages but at the same time specific limitations, often failing to reflect market imperfections to be discussed below.

While at this time, one cannot state that there exists a satisfactory solution to the problem of representing adequately the production subsystem, the best approach may turn out to be the simultaneous employment of several different production functions for a regional economy, each function representing a set of a few basic activities. For example, one such set might comprise each of a number of oil refining and petrochemical production activities. As already indicated on p. 227, a careful survey and study of these activities found that based on the experiences of petrochemical and other chemical construction and operating companies, major economies of scale in the employment of both labor and capital could be reliably projected using the respective formulae

$$L = L_{ex}(O/O_{ex})^{0.22} \tag{8-65}$$

and

$$K = K_{ex}(O/O_{ex})^{0.7} \tag{8-66}$$

where L and K are labor and capital requirements, respectively, for a new plant with planned output O and where $K_{ex}$, $L_{ex}$ and $O_{ex}$ are magnitudes for an existing efficient plant.[25]

Other sets of activities, for example in steel and steel fabrication, or in food product manufacturing may also be found to be reliably projected by other formulae. In this way, a model might attain greater applicability.

### *8.4.3 Problems regarding scale economies in the transport and production subsystems, externalities, and market imperfections*

Economies of scale and increasing returns to production are ruled out by classical general equilibrium theory which generally requires convexity of production functions, let alone absence of externalities, for the existence of an equilibrium. Such economies and returns, however, are omnipresent in the transport subsystem and elsewhere. In fact, in modern life, economies of scale has practically always characterized and dominated transport systems, a result of the large fixed investments that are required. Hence, even when transport inputs and costs have been introduced into AGE models, let alone recognized and properly estimated, they have not been adequately incorporated. And of course, the constant cost function for the production of transport inputs employed in the discussion of the above sections in this chapter is indeed most inappropriate, it being introduced solely for explicit recognition of the reality of space and the distance variable. However, given that extensive data are available on transport activities, a more reliable transport services production function by each mode ought to be developed to replace the constant cost one.

Additionally, the use of a constant elasticity of transformation function (CET) for the allocation of production of a tradable commodity to export markets on the one hand and to the domestic on the other has serious shortcomings. It fails to reflect adequately the interplay of delivered cost pricing and market imperfections at the different levels and circumstances under which a system functions and often embodies the shortcomings in the use of a single world price instead of a set of delivered cost prices.

There have been attempts in AGE modeling to handle increasing returns to scale in production in general and treat diverse imperfections in the market when conditions of oligopoly and monopoly prevail. The use of markups to bring convexity into production functions (see Gunning and Keyzer, 1995), or the use of tariffs on imports, export subsidies, or a combination of them (see deMelo and Roland-Holst, 1994) leave much to be desired. So also does the assumption of monopolistic competition (see Krugman, 1993 and Bröcker, 1994) where in a free entry system: (1) each of a large group of producers puts out a differentiated product and is assumed to ignore the income effect of his/her choices on his/her perceived product demand and (2) the utility to the consumer of each product brand is independent of utility derived from consumption of other brands, his/her utility being a simple aggregate of utilities.

Also, there arises a technical question of whether the introduction of a device to handle one or more market imperfections leaves untouched the independence of the choice of a numeraire and thus the framework of relative prices (see Ginsburgh, 1994).

Then, when oligopoly and monopoly are permitted in 'imperfect' AGE models by various presumptions, there is the serious question regarding the nature of the behavior of oligopolies and monopolies, their interaction and games they play, and externalities in general. The results of such models can be highly sensitive to the different assumptions about the situational environment in which the behavior and interaction occurs.

### *8.4.4 Questionable character of intertemporal analysis*

Since AGE models are frequently employed to study the impact of alternative government policies or a possible change in policy, they typically need to be forward-looking and to have a dynamic aspect. Even a do-nothing policy requires recognition of on-going changes in economic structure and thus magnitudes of relevance. Included in such recognition would be the existing and likely changes in expectations of agents as they reach decisions on consumption, savings and investments. Typically, when parameters of savings, investment and other functions are estimated, they are oriented to current and past data and experience. They rarely, if ever, are, or can be, based on the future. Yet the desire and need to project the impact of changes and to project the future is central to the use of models, as we well know in regional science.

In previous chapters we have seen how comparative cost, input-output, econometric, gravity, programming, industrial complex and SAM-based studies have each conducted impact analysis to throw light upon the future, and have confronted similar problems. But these models have been much less complex than AGE ones and have involved fewer variables; and research associated with their use typically has needed to be less bold than that with AGE. They have also not been required to make, implicitly or explicitly, as many assumptions about expectations of behaving units. In some AGE models expectations are assumed to be consistent with the model's projections, when expectations are excluded as an endogenous variable. These models imply that each of the many behaving units possess an unrestricted information set in making their intertemporal decisions — that is 'take into account the complete sequence of signals they expect to face in the future, and that the expectations they form on endogenous variables will be self-fulfilling for a given exogenous environment'

(Mercenier and Srinivasan, 1994, p. 10). Other models, in contrast, pay little or no attention to forward-looking expectations. Some pursue policy impact analysis positing that current period outcomes (for example, on prices, profits, sales) fully govern future decisions on investment and other activities.

Still others posit imperfect foresight of agents with restricted information sets which then lead to a sequence of temporary (static) equilibria (see Benjamin, 1994, Shoven and Whalley, 1992). Problems then arise regarding the choice of a numeraire, calibration of the base period and the several temporary equilibria (if they can be considered to be equilibria), and so forth. But which of the many possible sets of assumptions on forward-looking expectations can be taken to characterize the hypothesized present and derived temporary equilibria? In any case, the recognition and use of forward-looking expectations, it must be admitted, leads to fragile projections which become even more fragile when technological advance in the real world and the uncertainty with respect to it and many other to-be-realized phenomena and behavior are considered.

### *8.4.5 Problems of capturing behavior of governments, the Rest of the World and feedback sequences*

One of the major actors in the typical AGE study is the Government. Its behavior, however, is not as well captured as that of economic sectors. For example, the government, national, regional or state, may take an action, say to reduce its expenditures as a result of insufficient income from taxation. Its plan, perhaps based on a preliminary AGE study, may have in mind reductions in each of a set of programs and activities. Some of these programs involve infrastructure investments, such as education and construction of transportation facilities, whose benefits accrue over a fairly long period of time; the identification of these is currently beyond AGE capability. The evaluation of government plans involving such programs cannot be made with two-period counterfactual studies. Equally, and perhaps more important, whatever the plan, the government at least in a democratic or semi-democratic society involves negotiations and gaming among political actors and interest groups. The realized set of outcomes at the political market place is different from what might be the initial plan or expectation. On both these accounts, the behavior of government as a long-run investor and as an arena for the play of competitors seeking power and political influence, needs to be extensively researched.

An equally important need is research on feedbacks. While AGE models clearly distinguish between endogenous and exogenous sectors and take a policy or other exogenous event or action as given, *consistency with a general equilibrium philosophy* requires feedback of the endogenous world upon the exogenous and the subsequent rounds of action and reaction of both worlds. This is particularly important when both the exogenous sector ROW (Rest of the World) is considered and government is taken as an exogenous sector.[26] And it will be true for the RON (Rest of the Nation) when such a sector is introduced into an AGIE model and certainly whenever a Financial SAM is added.

Thus we must conclude that presently there remain serious questions about the relevance of any policy recommendations that may be put forth from the findings of AGE models although valuable insights about the coplay of factors may be obtained by the policy modellers themselves.

### *8.4.6 Non-economic factors and other structural shortcomings*

It is maintained by the structuralist school of AGE modellers that the functioning of the real world is not that of the neoclassical economist, and this contention may be extended to cover many regional scientists and spatial equilibrium theorists, even when their findings are qualified to take into account non-economic and other factors. The structuralists claim that some of the real-world elements are so significant that they must be explicitly considered at the very start.

According to Taylor (1983), a leading exponent of this school, there are five key features of a structuralist's approach. First, given the available data and resources for data collection, they identify, as in a thorough SAM study, the relevant set of households and institutions in terms of income flows and possession of wealth. Among household groups there would be:

> rentiers who receive distributed profits, interest, and other financial incomes, suffer capital losses and gains on their (often considerable) assets, and save more than they invest; workers who get income from wages nominally fixed in the short run, don't save very much, and battle with firms about how wage increases will respond to unemployment and price inflation; agriculturalists whose savings rates are often high but whose income fluctuates sharply, following flexible price movements; and urban and rural 'marginals' who pick up residual income flows, suffer from deprivation of basic needs, and save at low, often negative rates. The state in its fiscal, public investment, and central banking roles, the commercial banks, and foreigners also enter as partially

> independent and powerful actors. The degrees of freedom available to any actor depend on the institutions and history of the economy at hand incorporating them in convincing fashion is part of the model-building art (p. 4).

Second, structuralists eschew models in real terms with only relative prices. Rather they stick to the actuality of nominal prices and income in money terms. Third, they recognize that different prices are under varying degrees of control by distinct groups in the economy. For example, firms apply a fixed mark-up or rate to variable production costs, often reflecting an oligopolistic or monopolistic position in an industry. Unions and companies negotiate over wages. And so forth. In many markets the neoclassical pure competition is a myth.

Fourth, there needs to be considered in a situation to be modelled the amount of economic rationality and price-mediated substitution to be incorporated, whether one considers consumption behavior of households, production behavior of firms, trading behavior of brokers and asset holders, and so forth. Price responsiveness and optimizing behavior may need to be replaced by rules of thumb and other practices.

Fifth, and perhaps most important to the structuralist approach, is the need to capture in a model the key causal linkages. As Taylor (1983) notes:

> There are many ways in which the economy can adjust when it is perturbed — output levels or the income distribution across classes may change, interest rates may vary, positions of wealth may expand or erode. A model builder has to select *which* of these adjustment mechanisms to build into his or her equations — the qualitative nature of the solutions will depend upon the choice. In the jargon, a model's 'closure' has to be chosen and justified on the basis of empirical and institutional analysis of the economy at hand. Setting closure is impossible unless class structures and economic power relationships have already been defined. Searching for sites of power and macro causal links is the key to the structuralist approach (pp. 6–7).

In short, a structuralist would not begin with a neoclassical approach as we have done in this chapter, and then reevaluate behavior and other processes after a first framework is constructed or qualify a first set of results. He/she would start off with key imperfections of the market, e.g. an exogenous mark-up policy of firms, or key firms optimizing on market share, or an institutional rule of thumb governing investment behavior, or a cultural practice in household consumption, and so forth. Then with the embodiment of such elements firmly incorporated bring into play, as of secondary importance, estimation and use of elasticities and substitution operations in the several submarkets.

While the structuralist's position cannot be ignored and must be given considerable weight, the exact approach to be employed, whether neoclassical, structuralist or hybrid, is the decision of a researcher. In this chapter we have chosen the neoclassical approach primarily for ease of exposition and to point up the potential of this approach without claiming that it is the best in general.

## 8.5 Some seminal contributions by regional scientists to the development of an applied general interregional (AGIE) model for a pure space economy

From the previous discussion, it should not be inferred that regional scientists have not been involved in significant thinking on an applied general interregional equilibrium model. They have and have made seminal contributions. A first one to note is that of Bröcker (1995) in his conceptual article on spatial CGE modeling of a Chamberlinean type. He attempts to develop a multiregional, multisectoral computable general equilibrium model by bringing together different strands of theoretical reasoning — input-output analysis, gravity modeling, the theory of intraindustry trade and the theory of general equilibrium under conditions of monopolistic competition. However, as most others who conduct a regional or multiregional (interregional) study, he employs the questionable CES functions to define composite goods as inputs or products that households consume and uses other standard aspects of AGE models. His exploitation of the use of a monopolistic competition framework to recognize diversity (different brands) of a commodity, thus introducing an element of market imperfection into the model, is a significant step in the right direction. But clearly regional science research must come to build upon this step, to treat more generally scale economies which is such a major factor governing the behavior of firms, particularly transport ones. To mention a second forward step, among other commendable ones, he proposes to use, in calibration when relevant data are lacking, a doubly constrained gravity model to estimate an effective distance function.

However, he employs a Samuelson device for treating the transport factor by assuming a certain percentage of every transported commodity to be used up during transportation. He thus avoids the complexity that would be involved were the production of transport inputs directly to use resources as inputs. This Samuelson device is unacceptable. As stated at the beginning of this chapter, the production of transportation services as a resource-using activity, whether to provide intermediate inputs or furnish final consumption goods, must be introduced at the very start in treating a regional or interregional space economy.

With regard to empirical implementation, another bold attempt at an interregional AGE is that of Jones and Whalley (1989) and Walley and Trela (1989). They attempt to trace the impact of diverse governmental policies upon the six basic Canadian regions functioning as an interregionally interactive economy.

In this research, they do embody a transportation industry, they do set up transport margins (fixed transport input coefficients) for shipping a unit of good between each pair of regions, they do attempt to capture scale economies on an industry basis (but fail to embody major scale economies that each company in the highly oligopolistic transport industry does experience), and in other ways provide a rich set of ideas for the development of an applicable interregional space economy that remains to be constructed by regional scientists. However, they do use questionable CES production functions and other relationships inherited from dimensionless international trade studies. And in particular when it comes to labor migration, they set up a very strange model, starting with f.o.b. prices on goods that are the same in the six regions and ending up with an extremely odd set of utility functions while failing to exploit the many fertile ideas covered in interregional migration studies. While clearly the authors' work represents the best that could have been done to answer the pressing political questions that motivated their research, and does make very important contributions, in no way can their model be claimed to provide findings that capture or approximate the real operations of the Canadian interregional system as a space economy.

One study that presents an excellent coverage of transport costs, spatial equilibrium pricing and the role of transportation in a space economy is that of Elbers (1992). Unfortunately, his model was oriented to the task of developing an applied model for the small economy of Nepal, one for which there was a dearth of data. As a consequence he developed a useful *network general equilibrium model* wherein trade pools play a major role. [Within a given trade pool, trade flows between one region (country) and a central node, say a national (world) market, is taken to be independent of other regions' (countries') trade.] The introduction of the trade pool concept (which considerably lessens data requirements and computational needs) to be combined with standard spatial equilibrium analysis resulted, however, in a highly complicated model, albeit useful for a unique situation such as Nepal's, but not generally applicable as a framework for general use in interregional analysis.

Among other numerous studies that represent one or more forward steps for the development of a useful applied general interregional model are those of Ando and Shibata (1997), and Ando (1996). These authors ambitiously attempt to construct a multiregion AGE model for China. They recognize from the start the significance of transportation for the development of the several regions and the role of transport costs in determining prices in these regions. Employing a doubly constrained gravity model based on time distances, they estimate regional exports and imports against a background of existing and implied regional imbalances. With national production coefficients serving as regional coefficients, they then go on to determine net final demand, value added and other magnitudes typically computed in an AGE model. While their efforts are plagued by serious deficiencies in regional data, they nonetheless conduct important exploratory work on unearthing the play of space within huge areas composed of highly diverse regions.

Another initially valuable work is that by Harrigan and McGregor (1988, 1989) employing the Malaysian two-region SAM. They carefully construct a standard AGE model for East Malaysia, West Malaysia and the Rest of the World (ROW). They are motivated to demonstrate how different macroeconomic visions of regional systems may be captured and yield different findings. They interestingly do so utilizing a model which may be viewed as a beginning at an applied general interregional equilibrium model — a model which because of the sophisticated way in which it is constructed and its results interpreted has much of value for those who wish to design interregional models, especially with respect to migration phenomena. However, since the production and demand in the third region (ROW) is taken as exogenous, only the two Malaysian regions are involved in basic interactions and feedback. Further, while they do not introduce transport costs into the model, they state that transport margins (presumably consistent ones) can be easily introduced in their essentially two-region system. This indeed is the case, but such margins would not capture the variation from interaction and feedback within a system of more than two regions, nor reflect major scale economies in transportation which must be incorporated in a truly spatial AGIE.

An insightful examination of the potentials for interregional AGE, but one which again is largely influenced by standard dimensionless studies, is that of Spencer (1988). However, it is one that does recognize the importance of transport costs.

One more insightful and extremely useful study is that of Kilkenney (1995) who develops a rural-urban AGE using a bi-regional SAM, but one which 'due to the nature of the regional delineation ... accounts for market segmentation but not distance' (p. 165). Skillfully employing questionable CES and CET functions in nested structures and other standard AGE concepts and carefully constructing an appropriate bi-region SAM, she paves the way for advanced rural-urban analysis of a more general nature and able to cover more than two interacting regional markets. The structure of her analysis is such that distance and transport costs can be explicitly introduced and can come to impact interregional trade and prices.

Among still other valuable studies, which space limitations preclude discussing, are Harris (1984), Harrison, Rutherford and Wooton (1995), Li and Rose (1995), Peter, Han, Meagher and Naqvi (1996),[27] Haddad (1998)[28] and ongoing research at the University of Strathclyde.

## 8.6 Concluding remarks

Applied general equilibrium analysis is a relatively new approach in economics and regional science which has only become available as a result of the recent computer revolution. There thus remain many important questions to attack in this approach. And this is especially so in its use for problems addressed in regional science since, to reiterate, this approach has been developed by policy-oriented economists who for the most part have dwelt in a world of no dimensions. For interregional study, insufficient general advances have been made by them and by regional scientists who have followed in their path.

Accordingly, in this chapter we have not been able to present a demonstration of an effective AGIE study that has general applicability in terms of the basic relationships covered and usefulness of findings. A number of seminal contributions that are of great value as steps for the attainment of such a demonstration have been made by regional scientists and others (only some of whom have been mentioned in the previous section because of space limitations); and these scholars are to be applauded. Because of these valuable steps that have been taken, that demonstration should be forthcoming in the near future.

The promise of an effective AGIE model is, to repeat, a much more comprehensive coverage of forces interacting within a space economy. We have indicated how an AGIE can effectively attack the location problem, embrace input-output, industrial complex and social accounting (SAM) analyses, extend the realm of optimization and scope of programming analysis by capturing the process of price formation at the market, and embody spatial equilibrium pricing when distance, transport costs and transport inputs are effectively incorporated at the start. Yet at the same time on a number of occasions, a comprehensive AGIE model may not be the best approach in regional and interregional studies given limited availability of research resources and relevant data. A less comprehensive AGIE involving the synthesis of a smaller combination of regional science methods discussed in previous chapters with perhaps an intensive use of one or more methods may constitute a more fruitful way to attack a problem. Or a non-general equilibrium framework involving another synthesis of methods may be more useful. In chapter 10, we examine the possibilities of such syntheses.

## Appendix 8.1 The basic functions, exogenous magnitudes, variables and equations of the two nation/three region model

### Basic functions

1. $X_1 = K_1^{0.25} L_1^{0.75}$ technology in producing commodity #1 in both *A* and *B*
2. $X_2 = K_2^{0.5} L_2^{0.5}$ technology in producing commodity #2 in both *A* and *B*
3. $U = C_1^{0.5} C_2^{0.5}$ utility in consumption, the same for all household groups
4. Constant input-output coefficient functions for producing commodity #3 and each type of transport service (transport input).

### Exogenous magnitudes

1. $\bar{K}^A = 0.8$ $\bar{K}^B = 1.6$ $\bar{K}^Z = 0.009$ (capital stocks)
2. $\bar{L}^A = 2.0$ $\bar{L}^B = 1.8$ $\bar{L}^Z = 0.012$ (labor endowments)
3. weight of a unit of each commodity = 20 lbs
4. distances $A \rightarrow B$, $B \rightarrow A$, $Z \rightarrow A$, $Z \rightarrow B$, $A \rightarrow Z$, $B \rightarrow Z$ = 100 miles

5. $\alpha_{41}, \alpha_{52}, \alpha_{63}, \alpha_{73}, \alpha_{81}, \alpha_{92} = 1$ ton mile
6. $k_4^A, k_5^B = 0.02$; $k_6^Z, k_7^Z = 0.015$; $k_8^A, k_9^B = 0.03$ (unit capital requirements in transport activities)
7. $\ell_4^A, \ell_5^B = 0.025$; $\ell_6^Z, \ell_7^Z = 0.020$; $\ell_8^A, \ell_9^A = 0.035$ (unit labor requirements in transport activities)
8. $k_3^Z = 0.03$; $\ell_3^Z = 0.04$ (unit factor requirements in producing coal)
9. $a_{31} = 0.05$; $a_{32} = 0.119$ (respective coal inputs per unit of output of #1 and #2)

**Variables**

| | |
|---|---|
| 1. $C_1^A, C_2^A, C_1^B, C_2^B, C_1^Z, C_2^Z$ | consumption of final goods |
| 2. $X_1^A, X_2^A, X_1^B, X_2^B, X_3^Z$ | outputs of commodities (excluding transport services) |
| 3. $X_4^A, X_8^A, X_5^B, X_9^B, X_6^Z, X_7^Z$ | outputs of transport inputs (transport services) |
| 4. $P_1^A, P_2^A, P_3^A, P_1^B, P_2^B, P_3^B, P_1^Z, P_2^Z, P_3^Z$ | prices of commodities |
| 5. $P_4^A, P_8^A, P_5^B, P_9^B, P_6^Z, P_7^Z$ | rates (prices) per ton mile of transport services (units of transport inputs) |
| 6. $r^A, r^B, r^Z, w^A, w^B, w^Z$ | rents and wages |
| 7. $k_1^A, k_2^A, k_1^B, k_2^B$ | unit capital requirements in production |
| 8. $\ell_1^A, \ell_2^A, \ell_1^B, \ell_2^B$ | unit labor requirements in production |
| 9. $K_1^A, K_2^A, K_4^A, K_8^A$ | capital demands of $A$'s activities |
| 10. $K_1^B, K_2^B, K_5^B, K_9^B$ | capital demands of $B$'s activities |
| 11. $K_3^Z, K_6^Z, K_7^Z$ | capital demands of $Z$'s activities |
| 12. $L_1^A, L_2^A, L_4^A, L_8^A$ | labor demands of $A$'s activities |
| 13. $L_1^B, L_2^B, L_5^B, L_9^B$ | labor demands of $B$'s activities |
| 14. $L_3^Z, L_6^Z, L_7^Z$ | labor demands of $Z$'s activities |
| 15. $K^A, K^B, K^Z$ | total regional demands of capital |
| 16. $L^A, L^B, L^Z$ | total regional demands of labor |
| 17. $Y^A, Y^B, Y^Z$ | regional incomes |

18. $M_2^{B\to A}$, $M_3^{Z\to A}$, $M_5^{B\to A}$, $M_6^{Z\to A}$, $M_1^{A\to B}$, $M_4^{A\to B}$, $M_3^{Z\to B}$, $M_7^{Z\to B}$, $M_1^{A\to Z}$, $M_8^{A\to Z}$, $M_2^{B\to Z}$, $M_9^{B\to Z}$ — imports by commodity (including transport services)

19. fe — foreign exchange rate (the number of units of *Z*'s currency for one of *Q*'s)

20. $\Delta\text{Assets}^A$ — balance of payments between *A* and *B* (change in assets of *A*)

**Equations**

1. $C_h^i = Y^i/2P_h$ $\quad i = A, B, Z;\ h = 1, 2$
2. $X_1^A = C_1^A + Ex_1^{A\to B} + Ex_1^{A\to Z}$; $\quad X_2^A = C_2^A - Ex_2^{B\to A}$
3. $X_1^B = C_1^B - Ex_1^{A\to B}$; $\quad X_2^B = C_2^B + Ex_2^{B\to A} + Ex_2^{B\to Z}$
4. $X_3^Z = Ex_3^{Z\to A} + Ex_3^{Z\to B}$; $\quad X_4^A = \alpha_{41} Ex_4^{A\to B}$; $\quad X_5^A = \alpha_{52} Ex_5^{B\to A}$
5. $X_6^Z = \alpha_{63} Ex_6^{Z\to A}$; $X_7^Z = \alpha_{73} Ex_7^{Z\to B}$; $X_8^A = \alpha_{81} Ex_8^{A\to Z}$; $X_9^B = \alpha_{92} Ex_9^{B\to Z}$
6. $k_1^i = (w^i/3r^i)^{0.75}$ $\quad i = A, B$ $\quad k_2^i = (w^i/r^i)^{0.5}$ $\quad i = A, B$
7. $\ell_1^i = (3r^i/w^i)^{0.25}$ $\quad i = A, B$ $\quad \ell_2^i = (r^i/w^i)^{0.5}$ $\quad i = A, B$
8. $K_g^A = k_g^A X_g^A$ $\quad g = 1, 2, 4, 8$ $\quad K_g^B = k_g^B X_g^B$ $\quad g = 1, 2, 5, 9$
9. $K_g^Z = k_g^Z X_g^Z$ $\quad g = 3, 6, 7$
10. $L_g^A = \ell_g^A X_g^A$ $\quad g = 1, 2, 4, 8$ $\quad L_g^B = \ell_g^B X_g^B$ $\quad g = 1, 2, 5, 9$
11. $L_g^Z = \ell_g^Z X_g^Z$ $\quad g = 3, 6, 7$
12. $K^A = \Sigma K_g^A$ $\quad g = 1, 2, 4, 8$ $\quad K^B = \Sigma K_g^B$ $\quad g = 1, 2, 5, 9$
13. $K^Z = \Sigma K_g^Z$ $\quad g = 3, 6, 7$
14. $L^A = \Sigma L_g^A$ $\quad g = 1, 2, 4, 8$ $\quad L^B = \Sigma L_g^B$ $\quad g = 1, 2, 5, 9$
15. $L^Z = \Sigma L_g^Z$ $\quad g = 3, 6, 7$
16. $K^A = \bar{K}^A \quad K^B = \bar{K}^B \quad K^Z = \bar{K}^Z \quad L^A = \bar{L}^A \quad L^B = \bar{L}^B \quad L^Z = \bar{L}^Z$
17. $P_1^A = r^A k_1^A + w^A \ell_1^A + a_{31} P_3^A$ $\quad P_2^B = r^B k_2^B + w^B \ell_2^B + a_{32} P_3^B$
18. $r^A k_2^A + w^A \ell_2^A = P_2^B + \alpha_{52} P_5^B \equiv P_2^A$
19. $r^B k_1^B + w^B \ell_1^B = P_1^A + \alpha_{41} P_4^A \equiv P_1^B$
20. $P_3^A = fe(P_3^Z + \alpha_{63} P_6^Z)$; $\quad P_3^B = fe(P_3^Z + \alpha_{73} P_7^Z)$
21. $P_1^Z = (1/fe)(P_1^A + \alpha_{81} P_8^A)$; $\quad P_2^Z = (1/fe)(P_2^B + \alpha_{92} P_9^B)$
22. $P_3^Z = r^Z k_3^Z + w^Z \ell_3^Z$; $\quad P_4^A = r^A k_4^A + w^A \ell_4^A$

23. $P_5^B = r^B k_5^B + w^B \ell_5^B$; $\quad P_6^Z = r^Z k_6^Z + w^Z \ell_6^Z$

24. $P_7^Z = r^Z k_7^Z + w^Z \ell_7^Z$; $\quad P_8^A = r^A k_8^A + w^A \ell_8^A$

25. $P_9^B = r^B k_9^B + w^B \ell_9^B$

26. $Y^i = r^i K^i + w^i L^i \qquad i = A, B, Z$

27. $P_1^A Ex_1^{A\to Z} + \alpha_{81} P_8^A Ex_8^{A\to Z} + P_2^B Ex_2^{B\to Z} + \alpha_{92} P_9^B Ex_9^{B\to Z}$

$$= fe(P_3^Z Ex_3^{Z\to A} + \alpha_{63} P_6^Z Ex_6^{Z\to A} + P_3^Z Ex_3^{Z\to B} + \alpha_{73} P_7^Z Ex_7^{Z\to B}$$

28. $\Delta Assets^A = P_1^B Ex_1^{A\to B} - P_2^A M_2^{B\to A} + fe(P_1^Z Ex_1^{A\to Z} - P_3^A M_3^{Z\to A})$

29. $M_1^{A\to B} = Ex_1^{A\to B} \qquad M_1^{A\to Z} = Ex_1^{A\to Z} \qquad M_2^{B\to A} = Ex_2^{B\to A}$

$M_2^{B\to Z} = Ex_2^{B\to Z} \qquad M_3^{Z\to A} = Ex_3^{Z\to A} \qquad M_3^{Z\to B} = Ex_3^{Z\to B}$

$M_4^{A\to B} = Ex_4^{A\to B} \qquad M_5^{B\to A} = Ex_5^{B\to A} \qquad M_6^{Z\to A} = Ex_6^{Z\to A}$

$M_7^{Z\to B} = Ex_7^{Z\to B} \qquad M_8^{A\to Z} = Ex_8^{A\to Z} \qquad M_9^{A\to Z} = Ex_9^{A\to Z}$

These equations together with four production functions less two redundant price (or import equal export) equations are 93 in number.

## Endnotes

1 An interregional general equilibrium system under conditions of pure competition generally involves $U$ regions ($J, N = A, \ldots, U$) and $\ell$ commodities ($h = 1, \ldots, \ell$) with prices $P_h^J$ in each region $J$. In each region $J$ there are also

1. $m$ consumers ($i = 1, \ldots, m$), each of whom buys $b_{h,i}^J$ amount (in reality often zero) of each good $h$;
2. $n$ producers ($j = 1, \ldots, n$), each of whom is involved with $y_{h,i}^J$ amount (in reality often zero) of each good $h$, this amount being negative when $h$ is an input and positive when $h$ is an output; and
3. $\bar{f}$ exporters ($f = 1, \ldots, \bar{f}$), each of whom ships $s_{h,f}^{J\to N}$ amount (in reality often zero) of good $h$ to each region $N$ ($N \neq J$).

The unknowns are

1. the $Um\ell$ purchases $b_{h,i}^J$,
2. the $Un\ell$ inputs and outputs $y_{h,j}^J$,
3. The $U(U-1)\bar{f}L$ unknown shipments $s_{h,f}^{J\to N}$,
4. the $U\ell - 1$ prices $P_h^J$, and
5. the $U - 1$ balance-of-trade positions of regions.

To determine these unknowns there are:

1. $Um\ell$ budget balance and utility-maximizing conditions for consumers;
2. $Un\ell$ transformation constraints and profit-maximizing conditions for producers;
3. $U(U-1)\bar{f}\ell$ 'no profit from trade' conditions associated with traders trying to maximize gains from trade;
4. $U\ell - 1$ demand-equals-supply conditions; and
5. $U - 1$ balance-of-trade relations.

For detailed discussion, see Isard et al. (1969, chapter 11).

2 At the time of writing most computable general equilibrium models have bypassed such treatment of space. Many of the models developed by economists have treated a dimensionless world. And models by regional scientists and others that may have been designated as interregional have been anemic. Either they have treated each region as a point, with transport cost fixed or ignored entirely; or if they have introduced transport cost as a variable they have presented a structure too highly restricted in coverage or insufficiently comprehensive.

3 For example, to derive the first part of equation 8-3, we specify the budget constraint

$$Y^A = P_1^A C_1^A + P_2^A C_2^A \qquad \text{(8-3a)}$$

that consumer $A$ is taken to face. We then construct the Lagrangian

$$\mathscr{L} = (C_1^A)^{0.5}(C_2^A)^{0.5} + \lambda(P_1^A C_1^A + P_2^A C_2^A - Y^A). \qquad \text{(8-3b)}$$

To maximize utility we set

$$\partial\mathscr{L}/\partial C_1^A = 0.5(C_1^A)^{-0.5}(C_2^A)^{0.5} + \lambda P_1^A = 0 \qquad \text{(8-3c)}$$

$$\partial\mathscr{L}/\partial C_2^A = 0.5(C_1^A)^{0.5}(C_2^A)^{-0.5} + \lambda P_2^A = 0 \qquad \text{(8-3d)}$$

$$\partial\mathscr{L}/\partial\lambda = P_1^A C_1^A + P_2^A C_2^A - Y = 0 \qquad \text{(8-3e)}$$

Eliminating $\lambda$ in 8-3c and 8-3d yields

$$\begin{aligned} P_2^A &= P_1^A[0.5(C_1^A)^{0.5}(C_2^A)^{-0.5})/0.5(C_1^A)^{-0.5}(C_2^A)^{0.5}] \\ &= P_1^A C_1^A / C_2^A \end{aligned} \qquad \text{(8-3f)}$$

Replacing $P_2^A$ in 8-3a with its expression in 8-3f we obtain

$$C_1^A = \frac{Y^A}{2P_1^A} \tag{8-3g}$$

4 For example, given the linear homogeneous Cobb-Douglas production function (equation 8-5, wherein exponents of $K_1$ and $L_1$ add to unity), constant returns to scale obtains. It follows that total costs (TC) of production can be stated as

$$TC = rK_1 + wL_1 \tag{8-5a}$$

After multiplying the expression of equation 8-5 by 4/3, we obtain

$$L_1 = (X_1^A)^{4/3}/(K_1)^{1/3}. \tag{8-5b}$$

Substituting the expression for $L_1$ into equation 8-5a, we have as one condition for total cost minimization (which under pure competition implies profit maximization)

$$\partial TC/\partial K_1 = 0 = r - (1/3)w(X_1/K_1)^{4/3} = r - (1/3)w/(k_1)^{4/3} \tag{8-5c}$$

or

$$k_1 = (w/3r)^{3/4} \qquad (8\text{-}5d) \equiv (8\text{-}7)$$

In similar fashion, the expressions for $\ell_1$, $k_2$, and $\ell_2$ are derived.

5 Rearranging terms in equations 8-23 and 8-24 and using equations 8-18, 8-19 and 8-20, we obtain

$$(1 - a_{11})P_1^A - a_{21}P_2^B = EC \tag{8-24a}$$

where $EC = rk_1 + w\ell_1 + a_{21}\alpha_{52}P_5^B$, and

$$-a_{12}P_1^A + (1 - a_{22})P_2^B = ED \tag{8-24b}$$

where $ED = rk_2^B + w\ell_2^B + a_{02}\alpha_{32}P_3^B + a_{12}\alpha_{41}P_4^A$.
We can use the input-output inverse **A** to obtain

$$P_1^A[(1 - a_{22})EC + a_{21}(ED)]/\mathbf{A} \tag{8-24c}$$

and

$$P_2^B[(1 - a_{11})ED + a_{12}(EC)]/\mathbf{A} \tag{8-24d}$$

where $\mathbf{A} = (1 - a_{11})(1 - a_{22}) - a_{12}a_{21}$

6 The 34 equations are 8-1 (two income equations), 8-3 and 8-4 (four consumption equations), 8-7 to 8-10 (four unit factor input equations), 8-13 to 8-17 (five output equations), 8-18 to 8-24 (seven price equations), 8-25 (constituting six equations, namely the capital market equation, instrumental in determining the price of capital and five capital requirements equations, $i = 1, 2, 3, 4, 5$), and 8-26 (constituting six equations, namely the total labor market equation, instrumental in determining the price of labor and five labor requirements equations, $i = 1, 2, 3, 4, 5$).

7 The data of Table 8-1 were obtained by an iterative process and the use of a spreadsheet; therefore they are only approximations. More advanced computation programs, such as GAMS (and its likely successors), can yield more precise results, but unfortunately results more difficult to check for the presence of errors.

8 See Takayama and Judge (1976), Bhagwati (1983), chapters 59–61.

9 The sixteen equations determing the total demands for each of the four factors are:

$$K^A = K_1^A + K_2^A + K_4^A \qquad L^A = L_1^A + L_2^A + L_4^A \qquad (8\text{-}40a)$$

$$K^B = K_1^B + K_2^B + K_5^B \qquad L^B = L_1^B + L_2^B + L_5^B$$

where

$$K_h^A = k_h^A X_h^A \qquad L_h^A = \ell_h^A X_h^A \qquad h = 1, 2, 4$$

$$K_g^B = k_g^B X_g^B \qquad L_g^B = \ell_g^B X_g^B \qquad g = 1, 2, 5$$

10 Unlike the conceptual framework depicted in endnote 1, we do not explicitly treat traders in this model in order to avoid excessive notation. They are, however, implied and operate at zero gains from trade.

11 The unknowns are: $Y^A$, $Y^B$, $C_1^A$, $C_2^A$, $C_1^B$, $C_2^B$, $X_1^A$, $X_2^A$, $X_1^B$, $X_2^B$, $k_1^A$, $\ell_1^A$, $k_2^A$, $\ell_2^A$, $k_1^B$, $k_2^B$, $\ell_1^B$, $\ell_2^B$, $P_1^A$, $P_2^B$, $P_2^A$, $P_1^B$, $Exp_1^{A \to B}$, $Exp_2^{B \to A}$, $X_4^A$, $X_5^B$, $P_4^A$, $P_5^B$, $r^A$, $r^B$, $w^A$, $w^B$, $K_1^A$, $K_2^A$, $K_4^A$, $L_1^A$, $L_2^A$, $L_4^A$, $K_1^B$, $K_2^B$, $K_5^B$, $L_1^B$, $L_2^B$, $L_5^B$, $K^A$, $L^A$, $K^B$, $L^B$ and fe. Necessarily, in this two country model, exports of *A* correspond to imports of *B* ($M_1^{A \to B}$) and exports of *B* correspond

to imports of *A* ($M_2^{B\to A}$). For more than two country (region) models, there will not be this correspondence.

The corresponding equations are 8.27 (2 income equations), 8.28 (four consumption equations), 8.29 and 8.30 (four production functions), 8.31 (eight unit factor input equations), 8.32 and 8.33 (two price equations), 8.34 and 8.35 (four commodity supply = demand equations, instrumental in determining market prices and exports), 8.36 and 8.37 (two transport production equations), 8.38 and 8.39 (two transport input price equations), 8.40-8.43 (four factor supply = demand equations, instrumental in determining factor prices), 8.40a (sixteen factor requirements equations, see endnote 9), and 8.43 (the equation instrumental in determining the foreign exchange rate).

Also note that the Balance of Payments of each nation is zero, namely

$$P_2^A M_2^{B\to A} - fe P_1^B Ex_1^{A\to B} = 0 \qquad (8\text{-}43a)$$

and

$$P_1^B M_1^{A\to B} - (1/fe) P_2^A Ex_2^{B\to A} = 0 \qquad (8\text{-}43b)$$

where $P_2^A M_2^{B\to A}$ and $P_1^B M_1^{A\to B}$ are the value of imports (at delivered prices) of *A* and *B*, respectively, and $P_1^B Ex_1^{A\to B}$ and $P_2^A Ex_2^{B\to A}$ are the value of exports (at delivered prices) of *A* and *B*, respectively.

12 The changes in magnitudes are for: $K_1^A$, from 0.653 to 0.537; $K_2^A$, from 0.147 to 0.252; $K_1^B$, from 0.147 to 0.222; $K_2^B$, from 1.453 to 1.364; $L_1^A$, from 1.860 to 1.718; $L_2^A$, from 0.140 to 0.268; $L_1^B$, from 0.420 to 0.585; $L_2^B$, from 1.380 to 1.199; $k_1^A$, from 0.456 to 0.418; $k_2^A$, from 1.026 to 0.969; $k_1^B$, from 0.456 to 0.483; $k_2^B$, from 1.026 to 1.066; $\ell_1^A$ from 1.299 to 1.337; $\ell_2^A$ from 0.975 to 1.032; $\ell_1^B$ from 1.299 to 1.275; and $\ell_2^B$ from 0.975 to 0.938. New magnitudes are: $K_4^A = 0.010$, $K_5^B = 0.009$; $L_4^A = 0.012$ and $L_5^B = 0.011$.

13 See pp. 8–21 and Isard et al. (1960), pp. 233–245 for discussion of this approach.

14 Here, for example, we impose the constraint that the exports of an essential good, namely #1 from *A*, remain unchanged.

15 Of course for a central planning system which might correspond to that of the Soviet Union before its collapse, an enlarged table on physical magnitudes would be the one that would be most relevant.

16 In an enlarged table on physical magnitudes, the consumption of different goods by each type of household would need to be determined outside the market subsystem.

In the case of region *A*, we allocated: (1) 80 percent of its labor income to rural households and 20 percent to urban; and (2) 10 percent of its rent income to rural households and 90 percent to urban. For region *B*, the corresponding figures are 70, 30, 15, and 85. For *Z*, where the rural households are taken to be 100 percent self-subsistent, all income goes to urban households.

17 In terms of variables, this specification of household group in each region requires that the six consumption variables whose magnitudes are listed in the last column of Table 8-4 be replaced by twelve new consumption variables, the three utility functions 8-2 be replaced by five new ones, the three regional income equations by five new household income equations and the six consumption level equations by 12 new ones. See Appendix 8.1.

18 See Isard and Liossatos (1979), pp. 93–105, for one approach to a comprehensive treatment of the government sector. Also see Isard et al. (1970).

19 Only in rare instances has such a sector been included in SAM tables, although logically they should be, especially when these tables are employed to study the impact of government policies on redistribution in real terms, directly or indirectly.

20 Implicit in these statements and the continued use of the utility function 8-2, whose maximization yields the consumption levels of Tables 8-3 and 8-4, is that the provision of government services (outputs such as security) and outcomes from investment (for example, higher quality goods) be such that at all possible consumption patterns the utility derived would be increased by the same amount as the utility foregone from the corresponding reductions in consumption levels resulting from savings and taxation.

21 Note that if we were to add to each sector inputs from other sectors, production functions would need to be respecified and prices reestimated.

22 In this connection it is useful to refer back to the extensive thinking and insights of regional economists in the 1950s and their considerable research on interregional financial flows and social accounts. See Isard et al. (1960), chapters 4, 5, and pp. 611–621.

23 See Shoven and Whalley (1992) for a bird's eye presentation of this and other functions discussed here.

24 See the discussion in Li and Rose (1995), who in their one-region Pennsylvania study reject the use of Cobb-Douglas, CES and Leontief functions because of their undesirable properties, such as pre-determinate restrictions on substitution elasticities.

25 See Isard, Schooler, and Vietorisz (1959) for details.

26 While a government as an exogenous sector may tax exports and impose a tariff on imports, for many studies it cannot be considered to have a negligible impact on the ROW. There is very likely to be retaliation which then can lead to sequences of action and reactions among the sectors and nations in a true general equilibrium framework. When the sequence of feedbacks is ignored, the model should be more truthfully designated ATGE, applied truncated general equilibrium.

27 A draft of a study which to some scholars may seem to be an AGIE type, but clearly is not, is by Peter, Han, Meagher and Naqvi (1996) entitled *MONASH–MRF: A Multiregional Model of the Australian Economy.* It is indeed an excellent study in terms of the ingenious ways the authors exploit many of the devices employed by regional scientists in input-output and related studies when the dearth of regional data requires the use of national and other sources of data. For example, they split the columns and rows of a national input-output table to obtain after a series of steps estimates of interregional flows. However, this study essentially ignores space as a variable and its changing configuration as reflected in transport costs and effective distance. For interregional analysis, two unacceptable and basic procedures that are used are: (1) the Armington constant-elasticity-of-substitution mechanism for projecting change in interregional trade (a hand-me-

down from the dimensionless world of the typical international trade economist) and (2) the use of margins as a composite of transportation and communications on an aggregate basis, which in no way captures the basic transport cost variable. Nonetheless, the study remains extremely valuable for overcoming regional data deficiencies.

28 As this book goes to press, a study just completed by Haddad (1998) represents a major step forward. In his three region model of Brazil, which follows closely the multiregional structure of the MONASH–MRF study, Haddad does introduce transport cost as a basic variable affecting commodity trade among regions. However, he still retains the Armington constant-elasticity-of-substitution concept in depicting the structure of trade among regions and other traditional CGE procedures, for example expressing effects in terms of percentage changes. His work, which presents a good summary of the literature, still does not start with a proper scaffolding.

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