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Bayesian estimation of labor demand by age: theoretical consistency and an application to an input–output model

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ABSTRACT
Extended input–output models require careful estimation of disaggregated consumption by households and comparable sources of labor income by sector. The latter components most often have to be estimated. The primary focus of this paper is to produce labor demand disaggregated by workers' age. The results are evaluated through considerations of its consistency with a static labor demand model restricted with theoretical requirements. A Bayesian approach is used for more straightforward imposition of regularity conditions. The Bayesian model confirms elastic labor demand for youth workers, which is consistent with what past studies find. Additionally, to explore the effects of changes in age structure on a regional economy, the estimated age-group-specific labor demand model is integrated into a regional input–output model. The integrated model suggests that *ceteris paribus* ageing population contributes to lowering aggregate economic multipliers due to the rapidly growing number of elderly workers who earn less than younger workers.

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1. Introduction

The ageing of the population in most developed economies is beginning to exert a significant impact on labor markets generating a concomitant need to measure this impact from both the supply side (e.g. skills and age of the labor inputs) and demand side (consumption demand by households of different ages). Demographic change is also transforming the industry mix since each industry reveals a unique pattern of workers' age distribution. Different productivity and skill sets associated with different age-group workers change total labor income as well as the income distribution by age group. Demographic changes also bring about changes in labor demand for workers of different age groups across the board.

As noted by Diewert and Wales (1987), when it comes to measuring labor demand empirically, dealing with the violation of theoretical requirements occasionally occurring in the estimated demand function is 'one of the most vexing problems'. In practice, a number of studies on static demand models often exclude a validity check after estimation or

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proceed without referring to regularity conditions (O'Donnell and Coelli, 2005). Without evaluating theoretical requirements, it may be inappropriate to argue that the outcomes are intuitively correct or empirically consistent with past findings even if they are seemingly so. Therefore, the recent literature on static demand models strongly argues for the imposition of theoretical properties if necessary, following critical assessment (Sauer et al., 2006).

This paper first examines the theoretical and empirical consistencies of a static labor demand model. Then, to explore the regional economic impacts of changing age distribution, an application of the estimated labor demand into a regional econometric input–output model is conducted.

First, disaggregated labor demand, particularly by age group, is investigated. Examining the recent American Community Survey (ACS) data for the US states, we find that a labor demand model imposed with regularity conditions using a Bayesian estimation approach yields empirically coherent wage elasticities of labor demand. The Bayesian approach, proposed by Griffiths et al. (2000), is implemented since regularity conditions can be more easily imposed than conventional constrained optimization approaches. The estimation results confirm elastic labor demand for youth workers (aged 16–24) as past studies consistently find (Hamermesh and Grant, 1979). In addition, labor demand for elderly workers (aged 65 and over) is found to be elastic, varying little across sectors, in contrast to higher sectoral variability in labor demand elasticities for youth workers.

Thereafter, the estimated age-group-specific labor demand model for the state of Illinois is integrated into a regional input–output model for Chicago. The integration presents an enhancement to the representative agent-based input–output model by providing additional capability to conduct impact studies on heterogeneous agents. The new model implies that, other things being equal, an ageing population may result in lower aggregate economic multipliers mainly due to the rapidly growing number of elderly workers who earn less than younger workers.

This paper contributes to the literature on static labor demand modeling in several aspects. First, a representative example is presented in which monotonicity is highly likely to be violated due to very small factor cost shares, in this case, for labor cost shares of youth and elderly workers.[1] The empirical evidence suggests that monotonicity requires extra scrutiny especially when one or more factor cost shares are exceptionally small. Second, the model considers workers beyond the average retirement age.[2] In a number of studies on workers in different age groups, older workers are generally those prior to retirement age and are often too broadly grouped together with other age-group workers in their 20s to 50s. Third, by using highly disaggregated geographic and industrial units of observations, the model reduces concerns about the aggregation problem since a model using aggregate data is subject to aggregation bias.[3]

[1] Under a translog cost function, monotonicity implies nonnegative factor cost shares. In the presence of very small or large input cost shares, estimated shares are likely to deviate from the 0–1 range unless the range of predicted values is imposed *a priori*.

[2] Munnell (2011) calculates the recent average retirement age for men and women to be 64 and 62, respectively. She argues that the retirement age will continue to rise. The surveys in Hamermesh and Grant (1979) and Hamermesh (1996) cover studies on labor demand by age that had been published until the early 1990s. Among the papers in the surveys, Ferguson (1986) is the only study that includes workers aged 65 and over. We could not find any papers on labor demand for the elderly group henceforth. A most recent survey on demand for aggregate and heterogeneous (mostly by skill level) labor, including empirical studies released from 1980 to 2012, can be found in Lichter et al. (2014).

[3] For example, Lee et al. (1990) find statistically significant aggregation bias when a disaggregate employment model with 41 industries is compared with an aggregate employment model for the UK.

The paper also adds to the recent efforts on regional model integration. Integrating different types of regional models has been actively embraced particularly in computable general equilibrium (CGE) models whose main objective is often policy simulation based on a representative agent assumption (Colombo, 2010). In addition, similar applications have been explored in both traditional and econometric input–output models into which consumer and labor demand models are integrated (see, e.g. Kim et al., 2015; Mongelli et al., 2010; Maier et al., 2015).

This paper is organized as follows. Section 2 describes a static model of labor demand and discusses theoretical properties of a cost function. Thereafter, a Bayesian approach is described as an alternative to conventional methods. In Section 3, data and exploratory analyses are presented. Section 4 shows estimation results for the Bayesian and non-Bayesian models, followed by an investigation of regularity conditions and labor demand elasticity estimates. Section 5 describes an application of the labor demand model within an input–output model framework. Section 6 concludes with major findings and policy implications.

2. The model

2.1. A translog labor cost function

To account for sector-specific firm behavior in the demand for labor by age, the labor demand model includes age-group-specific trends that vary by sector. A twice-differentiable strictly quasi-concave production function with four types of aggregate inputs is used. Among the inputs, labor comprises G subtypes of workers of different age. By duality, a master cost function can be written as

$$C = C(P_{L_1}, \ldots, P_{L_G}, P_K, P_E, P_M, Y) \tag{1}$$

where L, K, E, M, and Y indicate labor, capital, energy, non-energy intermediate materials, and gross output, respectively; P_i is the price of factor i ($i = K, L, E, M$); P_{L_g} is the real wages for age group g. Assuming weak separability between labor and other factors, i.e. substitution between labor subgroups is independent of output and prices of the other inputs, Equation 1 can be rewritten as[4]

$$C = C[P_L(w_1, \ldots, w_G), P_K, P_E, P_M, Y] \tag{2}$$

where the price of a unit of labor P_L is assumed to be linearly homogeneous; $w_g \equiv P_{L_g}$ for $g = 1, \ldots, G$.

A translog cost function proposed by Christensen et al. (1971) is chosen for the unit labor cost function with G types of labor $P_L(w_1, \ldots, w_G)$ because it is a generalization of any arbitrary cost functions by a second-order approximation. It is also convenient for

[4] In practice, many empirical studies on factor demand assume separability due to data availability (Atkinson and Manning, 1995). However, separability is essentially an empirical issue that requires statistical testing. If labor is not separable from other factors, the estimates of labor–labor substitution are biased when other factors are omitted in the model. Since this paper focuses on the regional level (i.e. the US states) where data on prices and quantities of other factors, especially capital among others, are usually not available, measurement errors due to constructing estimates for capital might be more problematic (Hamermesh and Grant, 1979). Furthermore, regional models are often developed upon a single-input (usually labor) assumption that inputs other than labor can be approximated by local employment (Glaeser et al. 1992; Bishop and Gripaios, 2010; Felipe and McCombie, 2012).

empirical estimation and interpretation due to the linearity in parameters in the derived factor shares equations. The translog unit labor cost function is given by

$$\log(P_L) = \alpha_0 + \sum\nolimits_g \alpha_g \log(w_g) + \frac{1}{2}\sum\nolimits_g \sum\nolimits_h \beta_{gh} \log(w_g)\log(w_h) \tag{3}$$

This unit labor cost function is generally estimated by industry (see, e.g. Jorgenson et al., 2013; Kratena et al., 2013).

Based upon this form, a time trend, interactions with the time and group-specific wage, and region (state) fixed effects are added to capture changes in the characteristics of labor over time as well as regional variations. Hence, the final specification is written as

$$\begin{aligned}\log(W_t^r) = \alpha_0 + \mu^r + \theta t + \sum\nolimits_g \alpha_g \log(w_{g,t}^r) + \frac{1}{2}\sum\nolimits_g \sum\nolimits_h \beta_{gh} \log(w_{g,t}^r)\log(w_{h,t}^r) \\ + \sum\nolimits_g \gamma_g [\log(w_{g,t}^r)]t \end{aligned} \tag{4}$$

where the subscript i for industry is omitted for convenience; r is a region (state); t is time; W is the mean of annual wages and salaries that approximate the unit labor cost per year; μ^r is the region fixed effect. Applying Shepherd's lemma yields a set of G labor cost share equations for each sector as follows:

$$s_{g,t}^r = \frac{\partial \log(W_t^r)}{\partial \log(w_{g,t}^r)} = \alpha_g + \sum\nolimits_h \beta_{gh} \log(w_{h,t}^r) + \gamma_g t, \quad g = 1, \ldots, G \tag{5}$$

The unit labor cost function takes into account the characteristics of labor by sector and age group as well as the cost structure by region, while the derived labor cost shares imply that labor demand depends on sector and workers' age group. First, the common time trend in Equation 4 approximates the *industry-specific* overall labor quality over time (analogous to using a time trend as a proxy for technology progress over time in production).[5] Labor quality may include knowledge, intelligence, and strength of workers to which age and years of schooling contributes (Fuchs, 1964). Second, the region fixed effects, μ^r, account for *region-specific* cost differentials such as a fixed cost of labor that may vary by region. Third, the γ_g's in Equation 5 represent *age-group-specific* characteristics, such as rising or falling labor group input share due to the ageing of the population and an increase in labor force participation of the oldest group, holding the wage fixed.

It is worth noting that identification of the unit labor cost and labor cost shares is based on the assumption that the labor supply is perfectly elastic so that changes in relative wages determine changes in labor demand. This assumption can be justified in studies with small units and the unit of observations (i.e. region-specific 45 sectors) as 'relatively small' enough to reduce concern about wages being exogenous.[6]

With parameter estimates and predicted factor shares, partial own- and cross-price elasticities of labor demand for an age group, holding the wages of the other age-group workers

[5] Although it is not explored here because of a relatively short time series data (13 years), a time varying trend, which can be estimated using the Kalman filter, might be a more sensible choice (Jorgenson et al., 2013).

[6] Similar identification assumptions can be found, for example, in Slaughter (2001).

constant, are given as follows:

$$\eta_{gg} = \frac{\beta_{gg}}{s_g} + s_g - 1 \quad \text{for } g = 1, \ldots, G$$

$$\eta_{gh} = \frac{\beta_{gh}}{s_g} + s_h \quad \text{for } g, h = 1, \ldots, G; g \neq h$$

Note that the labor demand elasticities here are *gross price elasticities* that measure substitution along the utilized labor isoquant holding the total labor input L (i.e. 'output' for the labor cost sub-model) constant. Another commonly used measure for labor demand elasticities is *net price elasticities* where output Y is held constant. Given L, for example, an increase in the wage of age group g, w_g, will lead to a decrease in demand for labor in the same group, L_g (gross substitution). Following a resulting rise in the total price of labor P_L, aggregate labor L will decline and thus the L isoquant will shift inward (*net substitution*) at the new equilibrium.[7] Thus the net price elasticities tend to be more negative than the gross price elasticities (Hamermesh, 1996).

2.2. Regularity conditions

A regularity check is necessary because a failure to comply with certain regularity conditions would result in biased elasticity estimates. Particularly, to assess cost and production efficiencies for a sector or an individual firm, estimated cost and production functions must satisfy theoretical conditions. Otherwise, efficiency measures cannot be correctly interpreted since irregular shapes of these functions could result in over- or underestimated efficiency measures (Henningsen and Henning, 2009; Sauer et al., 2006). Among all the theoretical properties, monotonicity and concavity require special attention since these conditions are rather complex to implement in practice and violation of the two conditions could result in theoretically and empirically inconsistent parameter estimates. In what follows, the requirements are reviewed that ensure that a cost function satisfies theoretical properties.

As a result of the cost minimization, a cost function should be non-decreasing, linearly homogenous, concave, and continuous in input prices (Varian, 1992). By Young's theorem, the twice continuously differentiable cost function requires a symmetric Hessian matrix as well. Homogeneity in prices and the symmetry of the second-order derivative matrix can be imposed on Equations 4 and 5 as

$$\sum\nolimits_g \alpha_g = 1, \quad \sum\nolimits_h \beta_{gh} = 0, \quad \sum\nolimits_g \gamma_g = 0; \beta_{gh} = \beta_{hg}, \quad g \neq h \tag{6}$$

Monotonicity, i.e. non-decreasing in prices, requires non-negative labor cost shares in Equation 5 since:

$$\frac{\partial c_{L,t}}{\partial w^r_{g,t}} = \frac{c_{L,t}}{w_{g,t}} \frac{\partial \log(c_{L,t})}{\partial \log(w^r_{g,t})} = \frac{c_{L,t}}{w_{g,t}} \left[\alpha_g + \sum\nolimits_h \beta_{gh} \log(w^r_{h,t}) + \gamma_g t \right] > 0$$

where $c_{L,t}$ is the unit labor cost, W^r_t. Concavity is satisfied if the Hessian matrix of the cost function is negative semi-definite at the optimal point. Diewert and Wales (1987) prove

[7] See Berndt and Wood (1979) for a geometric interpretation of differences between gross and net price elasticities.

that the negative semi-definiteness of the Hessian is assured if and only if, given the non-negative shares, the matrix M with the following entries is negative semi-definite:

$$m_{gh} = \beta_{gh} + s_g s_h - s_g \delta_{gh} \quad \text{for} \quad g, h = 1, \ldots, G$$

where $M = \{m_{gh}\}$; β is a parameter in the cost function; s is a labor cost share; $\delta_{gh} = 1$ if $g = h$ and 0 otherwise. The eigenvalues of the M matrix are used to determine concavity because a matrix is negative semi-definite if and only if its largest eigenvalue is less than or equal to zero.

Each condition has important implications on estimation procedures and elasticity estimates. First, notice that due to homogeneity and symmetry, the number of parameters is reduced by the number of restrictions, i.e. $(G^2 + G)/2 + 2$. Second, if monotonicity is violated, negative signs of estimated cost shares would lead to seriously biased elasticity estimates in terms of signs. There is a high chance that monotonicity will be violated particularly when shares for one or more factors are very small relative to those for the rest of the factors. Third, concavity essentially means negative own-price elasticities, provided the shares are non-negative. Negative semi-definiteness requires the first-order principal minors of the M matrix, i.e. the diagonal entries equivalent to own-price elasticities, $\left(\frac{\beta_{gg}}{s_g} + s_g - 1 \quad \text{for} \quad g = 1, \ldots, G\right)$, to be non-positive.

2.3. *Estimation: a Bayesian SUR model*

After a brief review of conventional estimation methods and their limitations, we show that a Bayesian approach offers a more convenient way of estimation to restrict the labor demand model with the properties of monotonicity and concavity.

The labor cost function and the share equations are initially estimated with homogeneity and symmetry imposed using the seemingly unrelated regression (SUR) model of Zellner (1962).[8] Joint estimation of the cost function and the share equations yield more efficient estimates than the OLS estimation of the cost function alone (Christensen and Greene, 1976). Additionally, the joint estimation ensures that the cost function and the share equations are consistent with each other. For example, if the share equations (Equation 5) are estimated alone, it is not possible to recover the region fixed effects and the time trend in the cost function (Equation 4) by the integration of the share equations.

The maximum likelihood (ML) method does not allow for the imposition of monotonicity or concavity (Griffiths et al., 2000). Constrained maximization of the likelihood function is rather complex and the algorithms used for the optimization frequently have convergence problems (Henningsen and Henning, 2009). Furthermore, linear programming is appropriate for linear inequality constraints like monotonicity, but is not implementable with non-linear inequality constraints like concavity (O'Donnell and Coelli, 2005). A strand of recent literature on stochastic frontier analysis, whose main objective is to measure production/cost efficiency of firms, addresses the regularity problem using a Bayesian estimation (Griffin and Steel, 2007; O'Donnell and Coelli, 2005) or a multiple step estimation procedure (Henningsen and Henning, 2009).

[8] One of the share equations is dropped due to the singularity of the covariance matrix. In addition, we use the maximum likelihood (ML) method to ensure that estimates are invariant to the choice of the omitted equation.

As an alternative to the ML method, following Griffiths et al., (2000), a Bayesian SUR model is used to simultaneously estimate the translog unit labor cost function and the share equations retaining the requirements of homogeneity, symmetry, monotonicity, and concavity. Monotonicity is imposed at every data point (locally) whereas homogeneity and symmetry are restricted at any arbitrary point (globally). When imposed *globally*, it is known that concavity compromises the second-order flexibility of the translog function (Diewert and Wales, 1987). As a result, concavity is generally imposed only locally at a single or multiple reference points, which may result in concavity holding at many points while still maintaining the flexibility (Ryan and Wales, 2000). Therefore, following Ryan and Wales (2000), concavity is imposed at a single point where labor demand elasticities are measured, i.e. a mean vector of predicted labor cost shares. Later, concavity is checked *ex post* for every data point.

To obtain a sequence of sample parameter vectors, the Metropolis–Hastings (MH) algorithm is used because it can be computationally more efficient in the Bayesian SUR model than other popular algorithms such as the Gibbs sampling (Griffiths et al., 2000). The procedure of the MH algorithm is described below:

Step 1: Set initial values for a parameter vector $\boldsymbol{\lambda}^{(0)} = [\boldsymbol{\alpha}^{(0)}, \boldsymbol{\beta}^{(0)}, \boldsymbol{\gamma}^{(0)}, \boldsymbol{\mu}^{(0)}, \theta^{(0)}]^t$ where $\boldsymbol{\alpha}$ is a vector of parameter α's and the same applies to $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, and $\boldsymbol{\mu}$; θ is a parameter on the time trend. The values are chosen so as to satisfy homogeneity, symmetry, monotonicity, and concavity. Set $n = 1$.

Step 2: Set $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{(n-1)}$

Step 3: Draw a candidate $\tilde{\boldsymbol{\lambda}}$ from a *proposal* density $N(\boldsymbol{\lambda}, c\boldsymbol{\Omega})$ where c is a constant and $\boldsymbol{\Omega}$ is a variance–covariance matrix estimated from the ML method with homogeneity and symmetry imposed.[9]

Step 4: Evaluate monotonicity at every data points and concavity at the mean of fitted labor cost shares using $\tilde{\lambda}$. If either monotonicity or concavity is violated, update $\boldsymbol{\lambda}^{(n)} = \boldsymbol{\lambda}$, set $n = n + 1$ and go to Step 2. Otherwise, proceed to Step 5.

Step 5: Calculate $\alpha = \min\left[\frac{f(\tilde{\boldsymbol{\lambda}}|\mathbf{y})}{f(\boldsymbol{\lambda}|\mathbf{y})}, 1\right]$ where $\boldsymbol{y}$ is a vector of observations on a dependent variable; $f(\boldsymbol{\lambda}|\mathbf{y})$ is a *target* density from which we want to draw samples from for inference.[10]

Step 6: Accept $\tilde{\boldsymbol{\lambda}}$ with probability α and set $\boldsymbol{\lambda}^{(n)} = \tilde{\boldsymbol{\lambda}}$. Set $n = n + 1$ and go to Step 2.

The tuning parameter c in Step 2 is set to 0.01 or 0.05, determined by trial and error so that the acceptance rate for $\tilde{\boldsymbol{\lambda}}$ ranges from 10% to 40%.[11] Depending on the speed of convergence, 100,000–400,000 samples are drawn for each sector and the first 10,000, 20,000, or

[9] The objective of the Bayesian method is to obtain samples for statistical inference from a target (posterior) distribution. However, since a target density is often analytically intractable, the MH algorithm generates a sequence of samples from a proposal density instead. In the limit, these samples follow the target density. See Chib and Greenberg (1996) for more details on the MH algorithm.

[10] When conventional non-informative prior distributions (e.g. the inverted-Wishart distribution for the variance–covariance matrix) are assumed, the target density $f(\lambda|y)$ is proportionate to the determinant of the variance–covariance matrix for the errors in the SUR model (Griffiths, 2003).

[11] Setting a proper value for the tuning parameter ensures a candidate parameter vector drawn from a proposal density to move around the parameter space more efficiently. For example, we first set $c = 0.05$ that is the same value used in Griffiths et al. (2000). When we found the acceptance rate too low, say less than 10%, we then reset $c = 0.01$ for candidate parameter vectors not to traverse too far from the target density.

30,000 samples are discarded for a burn-in. Then, every 100th, 200th, or 300th observation is redrawn in the remaining samples (thinning).

3. Data

3.1. The American Community Survey

We use the ACS Public Use Microdata (PUMS) compiled by the Census Bureau because it is the most comprehensive publicly available data. In the ACS, an individual generally represents 100 people while in another popular survey, the March Current Population Survey (CPS) by the Bureau of Labor Statistics (BLS), a sample represents more than 1000–1500 individuals.

Based on the 2000–2013 ACS PUMS, the number of employees, mean annual pre-tax wages, and salary per employee are aggregated by sector and state (excluding Alaska and Hawaii) for each survey year.[12] An individual is mostly a 1-in-100 random sample except for a 1-in-240 sample from 2001 to 2004. Table 1 presents 45 sectors reclassified from the 3-digit North American Industry Classification System (NAICS). An employed person is grouped by their age: (1) a youth worker aged 16–24 who participates the labor market at an early stage; (2) a worker aged 25–44 as the most actively working group; (3) a worker aged 45–64 who is at around the peak of their career and subsequently preparing for retirement; and (4) an elderly worker aged 65 and over who continues to work or reattaches to the labor market after retirement. The final samples used for analysis include only private wage and salary workers with non-zero labor income: Armed Forces, state, local and federal government employees, and self-employed workers are excluded.[13]

Table 1. Sector description.

1	Livestock & other agri. prod.	16	Stone, clay, & glass prod.	31	Pipeline trans.
2	Agriculture, forestry. & fisheries	17	Primary metals prod.	32	Information
3	Mining	18	Fabricated metal prod.	33	Motion picture & sound recording
4	Utilities	19	Industrial machinery & equip.	34	Finance & insurance
5	Construction	20	Computer & other electric prod.	35	Real estate
6	Food & kindred prod.	21	Trans. equip. manuf.	36	Professional & management serv.
7	Tobacco prod.	22	Furniture & related product	37	Educational serv.
8	Apparel & textile prod.	23	Misc. manuf.	38	Health care
9	Leather & leather prod.	24	Wholesale trade	39	Social assistance
10	Lumber & wood prod.	25	Retail trade	40	Arts, entertainment, & recreation
11	Paper & allied prod.	26	Air trans.	41	Accommodation serv.
12	Printing & publishing	27	Railroad trans. & trans. serv.	42	Food serv.
13	Petroleum & coal prod.	28	Water trans.	43	Repair & maintenance
14	Chemicals & allied prod.	29	Truck trans. & warehousing	44	Personal & laundry serv.
15	Rubber & misc. plastics prod.	30	Transit & ground passenger trans.	45	Membership org. & households serv.

Notes: Aggregate sectors are classified as follows: Resources 1–3; Construction 5; Non-durables goods 6–9 & 11–15; Durables goods 10 & 16–23; TCU (transportations, communications, and utilities) 4 & 26–32; Trade 24–25; FIRE (finance, insurance, and real estate) 34–35; Services 33 & 36–45.

[12] The data can be downloaded from the IPUMS USA, the Minnesota Population Center, University of Minnesota (https://usa.ipums.org/usa/; Ruggles et al., 2015). According to the Employment Cost Trends (ECT) compiled by the BLS, wages and salaries make up around 70% of employee compensation costs and the remaining 30% is comprised of benefits such as health insurance, paid leave, legally required benefits, and retirement and savings. However, neither the ACS nor the ECT provides comprehensive benefits data by worker's age.

[13] Self-employment in the ACS includes both the unincorporated (a dominant type) and incorporated self-employed while the CPS treats the incorporated self-employed as wage and salary workers.

Figure 1. Labor costs, employment, and wages by age in the US (2000–2013).

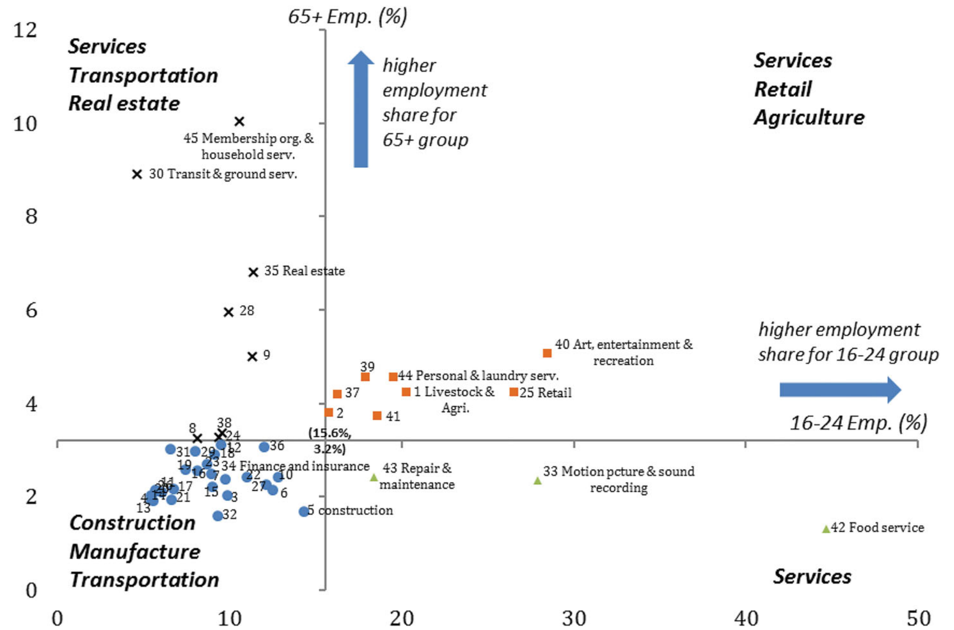
Note: Self-employed, Armed Forces, and government employees are excluded.
Source: Authors' calculation based on the 2000–2013 ACS.

3.2. Characteristics of labor cost by age

Figure 1 shows that the labor cost shares for the 45–64 and 65+ age groups in the US have been constantly rising since 2000. The share of labor costs for the two oldest groups rose to 50% in 2013, from 39% in 2000. This rise is attributed to the increase in employment and wages for the two groups. First, as baby boomers age, employment for the 45–64 and 65+ workers increased 40% and 70% since 2000 to reach 38.1 and 4.5 million, respectively, in 2013. However, employment for the remaining younger workers declined 1% over the same period. Second, the oldest workers' real labor income, in particular, shows a large gain of 41% between 2000 and 2013.[14] Total annual wages for the 45–65 group rose 3% while wages for the 25–44 and 16–24 groups fell 14% and 4%, respectively. According to the ACS data, a rise (decline) in labor costs share for the 65+ (16–24) age group can be explained by the following: the rapidly rising wages for the elderly workers can be characterized by

[14] In the CPS, median inflation-adjusted weekly earnings for wage and salary workers show similar trends over the 2000–2013 periods.

Figure 2. The US employment shares by sector for the youngest and oldest age-group employees: 2000–2013 average.



Notes: (1) *x*-axis and *y*-axis represent the employment shares for the 16–24 and 65+ age groups, respectively. (2) The origin represents mean shares. (3) The aggregate sectors in bold represent the ones appearing most in the corresponding quadrants. (4) The numbers on each symbol represent the sectors in Table 1. (5) Each symbol is specific to each quadrant.
Source: Authors' calculations based on the 2000–2013 ACS.

a rise in labor force participation of high-skilled full-time workers aged 65 and over. The youth workers, on the other hand, experienced falling wages as a result of a rising share of part-time workers, combined with a decline in labor force participation, possibly to pursue higher education.

Figure 2 shows that the youngest and oldest age groups tend to work predominantly in service sectors.[15] It also indicates that they are less likely to work in physically demanding industries such as construction and manufacturing than the middle age groups. Food services show the highest employment share for the youngest group (45%) while membership organizations and private household services have the largest employment share for the oldest group (11%).

4. Results

Throughout this section, homogeneity and symmetry are globally imposed so that these properties hold at *any* input prices. For the Bayesian models, monotonicity is imposed at every data point while concavity is restricted only at the mean of predicted labor cost shares where wage elasticities of labor demand are evaluated.

[15] See Appendix A (supplementary material) for labor cost shares, employment and wages by sector for all age groups.

4.1. Parameter estimates[16]

Table 2 presents non-Bayesian and Bayesian parameter estimates for the translog cost function and share equations in one of the 45 sectors. Membership organizations and household services (sector 45) are chosen for illustration purposes, and this sector has the highest employment share for elderly workers.[17] The results show that including share equations and imposing monotonicity generally incur considerable changes in parameter estimates. Examining columns 1 through 6 in Table 2, note that the first large changes in parameter estimates occur when the shares are included in the estimation. Once the share equations are present, estimates remain relatively unchanged regardless of whether the cost function is added (columns 2–4). The second large changes occur when monotonicity is imposed. It does not seem that imposing concavity in addition to monotonicity causes changes in estimates to any great extent.

Imposing theoretical requirements does not appear to incur significantly large losses of prediction errors in the Bayesian models. To compare *ex post* prediction performances between the Bayesian and non-Bayesian models, mean absolute errors (MAEs) are calculated. The Bayesian SUR estimates with monotonicity and concavity (column 6) generally show only slightly larger MAEs for predicted values of the cost and shares than the SUR model with no restriction (column 3). Meanwhile, in the Bayesian model with all theoretical requirements, every data point meets monotonicity and only 11% of total observations violate concavity while 2% violates monotonicity, and 57% fail to comply with concavity in the SUR model with no restrictions. Similar patterns are also found in the other sectors.

For sector 45, the non-Bayesian OLS and SUR models clearly violate monotonicity and concavity. Out of 685 observations, 8% violates monotonicity in the cost-only OLS model (column 1) and 1.5% in the SUR models (columns 2 and 3), mostly occurring in the fitted labor cost shares for youth and elderly workers. As for concavity, 49% of observations violate concavity in the share-only SUR model and 57% in the cost-share joint SUR model. Particularly, all data points predicted by the cost function alone violate concavity. The Bayesian SUR model with no restriction but homogeneity and symmetry (column 4) essentially feature the same estimates and the same number of observations that violate regularity conditions as the non-Bayesian SUR model (column 3).

Among the non-Bayesian models, the evaluation of regularity conditions and goodness-of-fit justifies the need for *simultaneous* estimation of a cost function and factor share equations, as discussed in Section 2.3. For sector 45, when the share equations are estimated together with the cost function, the percentage of observations in violation of monotonicity and concavity significantly declines compared to the cost-only model. This is also true for the majority of industries.

4.2. Evaluation of monotonicity and concavity

In Figure 3, monotonicity and concavity are evaluated at *every* data point for the cost-share joint models by sector. As a benchmark model, the SUR model is presented without *a*

16 State fixed effects were initially explored, but majority of sectors showed a fair amount of insignificant state fixed effects. Thus region fixed effects were scaled down to the four Census regions, i.e. Northeast, Midwest, South and West. Time dummy variables accounting for the recent financial crisis in the US (2008 and 2009) did not significantly change the results, and thus they were not included in the final specification.

17 The same sets of results for other sectors are available upon request.

Table 2. Parameter estimates for membership organizations and households services (sector 45).

	OLS[1]	SUR[1]		Bayesian SUR[1]		
	No monotonicity or concavity	No monotonicity or concavity		No restriction	Monotonicity[2]	Monotonicity & concavity[2,3]
	Cost only (1)	Share only (2)	Cost & share (3)	Cost & share (4)	Cost & share (5)	Cost & share (6)
α_0	−0.0503 (0.015)	–	0.0123 (0.003)	−0.0121 (0.003)	−0.0206 (0.004)	−0.0228 (0.003)
α_1	0.0538 (0.012)	0.0736 (0.002)	0.0721 (0.002)	0.0715 (0.002)	0.0568 (0.002)	0.0561 (0.002)
α_2	0.4966 (0.027)	0.4307 (0.007)	0.4347 (0.006)	0.4353 (0.006)	0.4473 (0.006)	0.4477 (0.006)
α_3	0.4127 (0.032)	0.4095 (0.007)	0.4119 (0.006)	0.4121 (0.006)	0.4178 (0.006)	0.4195 (0.006)
α_4	0.0369 (0.018)	0.0862 (0.003)	0.0813 (0.003)	0.0810 (0.003)	0.0781 (0.003)	0.0766 (0.003)
β_{11}	0.0428 (0.005)	0.0299 (0.001)	0.0308 (0.001)	0.0308 (0.001)	0.0127 (0.000)	0.0127 (0.000)
β_{12}	0.0240 (0.014)	−0.0098 (0.003)	−0.0118 (0.003)	−0.0120 (0.003)	0.0020 (0.002)	0.0030 (0.002)
β_{13}	−0.0386 (0.015)	−0.0178 (0.003)	−0.0142 (0.003)	−0.0137 (0.003)	−0.0124 (0.002)	−0.0129 (0.002)
β_{14}	−0.0283 (0.008)	−0.0023 (0.001)	−0.0049 (0.001)	−0.0051 (0.001)	−0.0023 (0.001)	−0.0028 (0.001)
β_{22}	0.0231 (0.045)	0.1530 (0.013)	0.1498 (0.012)	0.1498 (0.012)	0.1389 (0.012)	0.1430 (0.013)
β_{23}	−0.0442 (0.048)	−0.1412 (0.012)	−0.1377 (0.011)	−0.1378 (0.011)	−0.1420 (0.011)	−0.1461 (0.012)
β_{24}	−0.0030 (0.018)	−0.0020 (0.004)	−0.0003 (0.004)	0.0000 (0.004)	0.0011 (0.004)	0.0001 (0.005)
β_{33}	0.0951 (0.060)	0.2074 (0.013)	0.1991 (0.012)	0.1987 (0.012)	0.1946 (0.012)	0.1976 (0.013)
β_{34}	−0.0124 (0.022)	−0.0484 (0.004)	−0.0472 (0.004)	−0.0472 (0.004)	−0.0402 (0.004)	−0.0385 (0.004)
β_{44}	0.0437 (0.014)	0.0527 (0.003)	0.0524 (0.003)	0.0523 (0.002)	0.0413 (0.001)	0.0411 (0.001)
θ	0.0008 (0.001)	–	−0.0005 (0.000)	−0.0005 (0.000)	−0.0009 (0.000)	−0.0008 (0.000)
γ_1	0.0020 (0.001)	0.0002 (0.000)	0.0002 (0.000)	0.0002 (0.000)	−0.0002 (0.000)	−0.0002 (0.000)
γ_2	−0.0117 (0.002)	−0.0065 (0.001)	−0.0073 (0.001)	−0.0074 (0.001)	−0.0069 (0.001)	−0.0067 (0.001)
γ_3	0.0078 (0.002)	0.0044 (0.001)	0.0049 (0.001)	0.0050 (0.001)	0.0049 (0.001)	0.0046 (0.001)
γ_4	0.0019 (0.001)	0.0020 (0.000)	0.0021 (0.000)	0.0021 (0.000)	0.0023 (0.000)	0.0023 (0.000)
Region FE[4]	Yes	–	Yes	Yes	Yes	Yes
Observations	685	685	685	685	685	685
Violating conc. @ a single pt.	Yes	No	No	No	No	No
%violating mono.	8.2	1.5	1.5	1.5	0.0	0.0
%violating conc. @ all pts.	100.0	48.6	56.5	56.5	14.9	11.2
MAE[5] (cost)	0.0248	–	0.0256	0.0256	0.0265	0.0265
MAE[5] (avg. of shares)	0.0351	0.0286	0.0287	0.0287	0.0293	0.0292

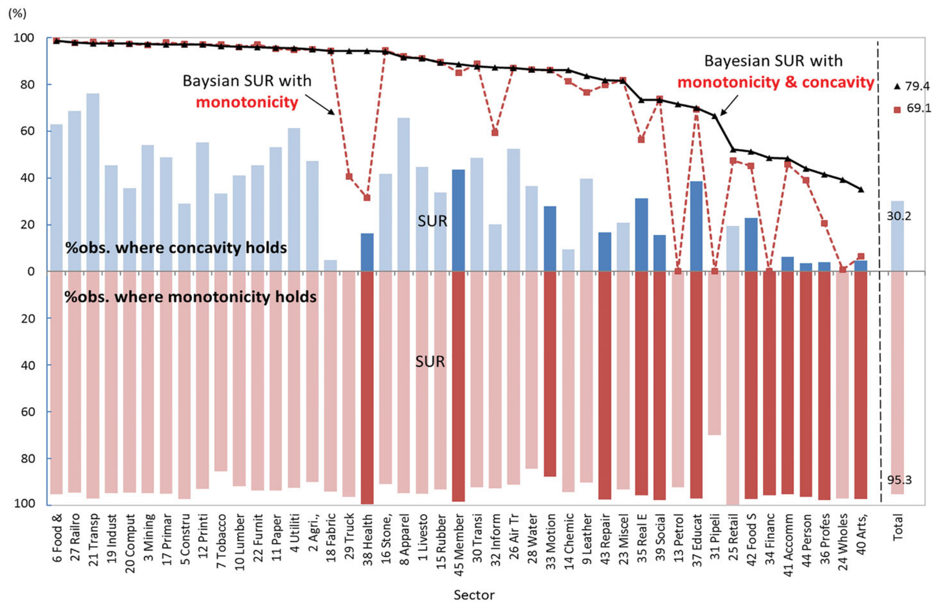
(*continued*).

Table 2. Continued.

	OLS[1]	SUR[1]		Bayesian SUR[1]		
	No monotonicity or concavity	No monotonicity or concavity		No restriction	Monotonicity[2]	Monotonicity & concavity[2,3]
	Cost only (1)	Share only (2)	Cost & share (3)	Cost & share (4)	Cost & share (5)	Cost & share (6)
(All sectors)						
Observations	22,216	22,216	22,216	22,216	22,216	22,216
%violating mono.	8.3	5.5	4.7	4.7	0.0	0.0
%violating conc. @ all pts.	84.6	68.8	69.8	69.8	30.9	20.6
Log-likelihood (mean)	−1643.0	−5839.8	−7928.9	–	–	–
MAE[5] (cost; mean)	0.0315	–	0.0333	0.0333	0.0340	0.0342
MAE[5] (shares; mean)	0.0444	0.0353	0.0356	0.0356	0.0364	0.0376

Notes: 1) Homogeneity and symmetry imposed. 2) Monotonicity are imposed at all data points and concavity is imposed at the mean of predicted labor cost shares. 3) Concavity is satisfied conditionally on monotonicity. 4) Four Census regions: West, Midwest, Northeast, and South. 5) Mean Absolute Error. 6) Figures in parentheses are standard errors for OLS and SUR, and standard deviations of the Metropolis–Hastings samples for the Bayesian SUR.

Figure 3. Monotonicity and concavity by sector: percentage of observations where these properties hold[1)].



Notes: (1) Cost function and labor cost shares are simultaneously estimated; homogeneity and symmetry are globally satisfied. (2) Concavity is imposed at a single reference point, i.e. means shares of predicted labor cost shares. (3) Concavity is satisfied conditionally on monotonicity. (4) Sectors are sorted in a descending order of the proportion of concavity-satisfying samples in the Bayesian SUR model with monotonicity and concavity imposed. (5) Total number of observations is 22,216. (6) The light- and dark-colored bars represent the primary & secondary and tertiary sectors, respectively.

priori monotonicity and concavity conditions in the form of bar graphs. Note that when the two conditions are not imposed, 95% of total samples meet monotonicity while concavity holds only in 30% of observations. It is commonly found in the literature on technology that concavity is more often violated than monotonicity (Barnett 2002).[18] Also recall that concavity is satisfied conditionally on monotonicity.

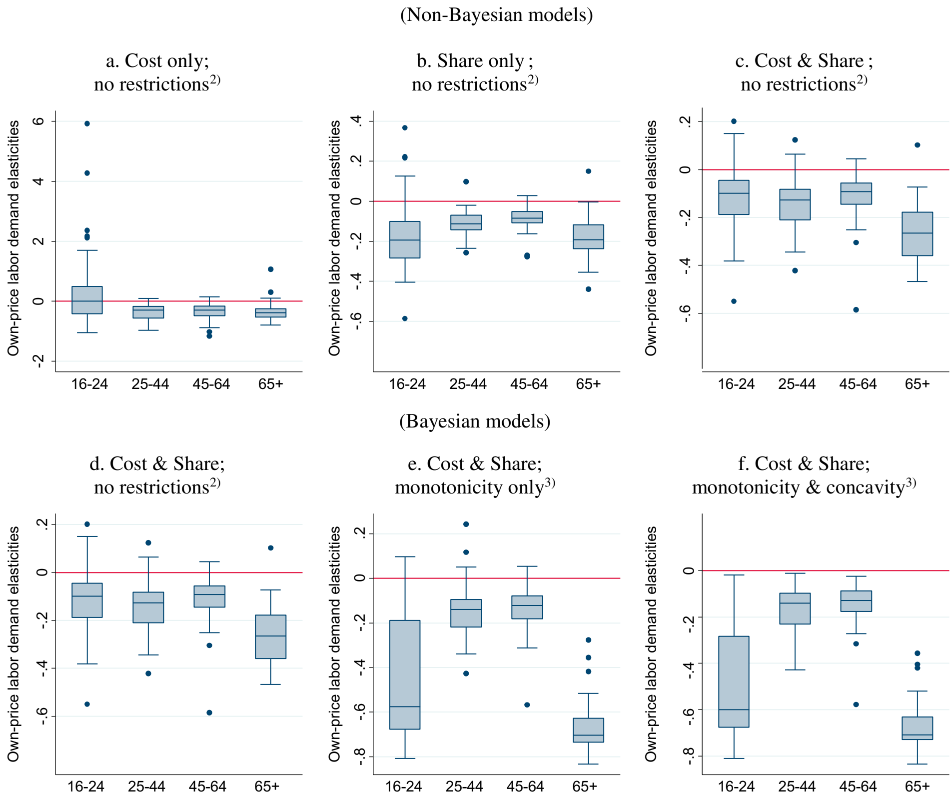
Figure 3 shows that imposing monotonicity results in an improvement in concavity to a great extent. Overall, when only monotonicity is imposed at every data point, the share of concavity-satisfying samples increases to 69%, up from 30% in the SUR model. Furthermore, imposing concavity in addition to monotonicity elevates an extra 10% of samples to satisfy concavity so that 79% of samples comply with concavity in the fully restricted Bayesian model.

Figure 3 also confirms that the imposition of concavity at a single point does improve concavity at other data points, as Ryan and Wales (2000) find.[19] When concavity is imposed only on the mean shares, the overall share of concavity-satisfying samples increases to 79% from 69%. However, the degree to which concavity improves, represented by the distance

[18] Barnett (2002) further explains that since monotonicity is less often violated, researchers commonly impose only curvature in practice.

[19] An empirical example in Ryan and Wales (2000) shows that choosing one concavity-restricted point could make all points satisfy concavity. Our finding suggests that the choice of a restriction point affects the degree to which concavity holds at other points.

Figure 4. Distributions of own-price labor demand elasticities for 45 sectors by estimation method: evaluated at mean predicted labor cost shares1).



Notes: (1) Homogeneity and symmetry are globally imposed. (2) One very large positive number in the 16–24 group is intentionally omitted for easier comparisons. (3) Monotonicity is imposed at all data points and concavity is imposed at the mean of predicted labor cost shares.

between a square marker and a bar in the graph, varies considerably by sector. For example, sector 38 (health care) is one of the few sectors that shows a great improvement in concavity while concavity imposition at a single point has modest or little effects on other points in many of the remaining sectors.

It is worth noticing that the primary and secondary sectors are more likely to satisfy concavity than the tertiary sectors in the most preferred Bayesian model with all restrictions. In other words, cost frontiers inferred from observed wages and employment in the agricultural and manufacturing sectors are more theoretically well behaved than those in the service sectors.

4.3. Labor demand elasticity estimates

Figure 4 presents the distributions of own-price labor demand elasticities for all sectors by estimation method. Wage elasticities of labor demand are evaluated at the mean of predicted labor cost shares. Complete sets of own- and cross-price elasticities for the fully restricted Bayesian model are reported in Appendix B (supplementary material).

Elasticity comparisons by method show that the negativity of own-price elasticities is guaranteed only if monotonicity and concavity are satisfied. Considering the fact that many empirical studies find these two conditions frequently violated, unrestricted models are likely to generate elasticity estimates that lack not only theoretical consistency but also empirical feasibility. As panel (a) shows, one cannot exclude the possibility of numerous large positive own-price elasticities without the imposition of the theoretical conditions.

Examining elasticity estimates reveals that the Bayesian model with all theoretical requirements in panel (f) predicts *elastic* labor demand for youth and elderly workers.[20] More specifically, labor demand for elderly workers is the most elastic, a median of −0.71, with small variation across sectors. Labor demand for youth workers is the second most elastic, −0.60, but with much larger variation by sector. For the remaining two mid-aged groups, labor demand elasticities are similar, −0.14 for the 25–44 group and −0.13 for the 45–64 group with smaller variation across sectors.

Elastic labor demand for youth and elderly workers estimated from the fully restricted Bayesian model is more empirically and theoretically coherent than the other approaches. Both non-Bayesian and Bayesian models consistently estimate elastic labor demand for the oldest group compared to other age groups. In particular, elastic labor demand for youth workers containing teenage workers has long been supported in past empirical studies but with little consensus for other age groups (Hamermesh and Grant, 1979). However, some of the models with no restrictions yield *inelastic* elasticity estimates for youth workers.

One can naturally ask why many past studies on labor demand for youth workers neglected regularity conditions other than homogeneity and symmetry. Given that the fact that labor cost share for youth workers was relatively large in 1980s through the early 2000s, although showing a downward trend from 15% to 9%,[21] one can suspect that monotonicity, in particular, was likely to be satisfied in labor demand studies using the data for those periods. Furthermore, smooth time series data with highly aggregated sectors might have reduced the probability of violating monotonicity and concavity.

In Figure 5, own-price labor demand elasticities for youth and elderly workers by sector are presented. The scatter plot shows that labor demand for youth and elderly workers tend to be inelastic in the service sectors where employment for these groups is concentrated. By contrast, labor demand for the same groups is elastic in the more physically demanding sectors such as construction and manufacturing.

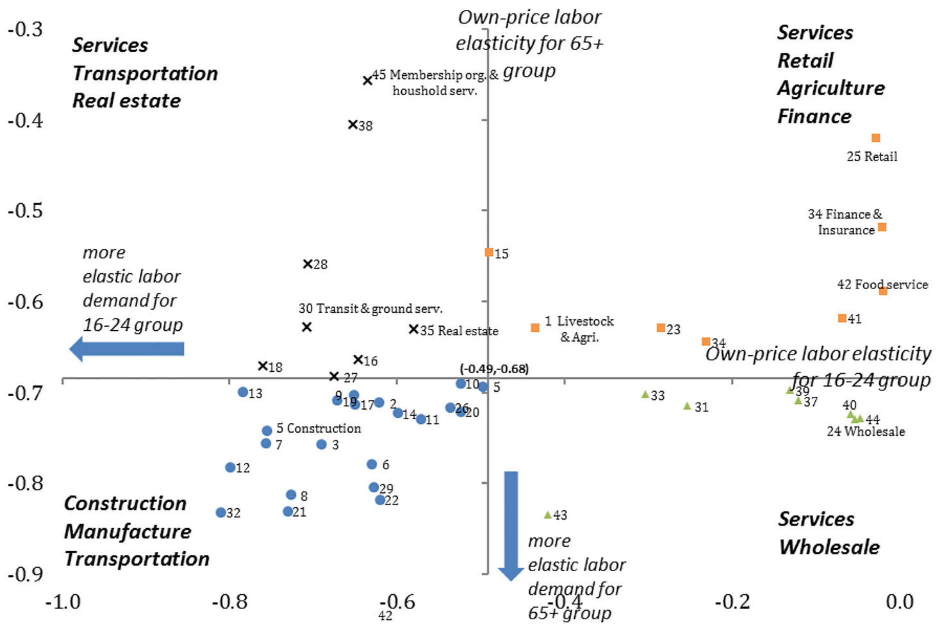
Figure 6 shows that all age-group employee pairs except for the youth–elderly pair are substitutes, i.e. positive cross-price elasticity.[22] According to the estimates, for example, wage subsidies for hiring applicants aged 65 and over, say, equivalent to the amount of 10% of market wage, would incentivize private employers to hire more of the age-group workers by 7% (a median of η_{44}'s), resulting in an increase in the employment of the youngest workers by 4% (η_{14}), while the 25–44 and 45–64 age-group workers would be substituted with the 65+ age-group workers by 3% (η_{24}) and 2% (η_{34}), respectively.

[20] From this subsection on, the term "elastic" is used in the context of relative comparison among the age groups for easier illustration despite the convention referring to the elasticity of demand greater than unity.

[21] These figures are based on the aggregate employment and wage at the US level in the CPS data.

[22] We are measuring the effects of input price on quantity demanded: two inputs are p-substitutes if $\eta_{gh} = \frac{\partial \log x_g}{\partial \log w_h} > 0$, p-compliments, otherwise. By contrast, q-substitute ($\in_{gh} < 0$) or q-compliment ($\in_{gh} > 0$) are based on the cross-demand elasticity of factor price $\in_{gh} = \frac{\partial \log w_g}{\partial \log x_g}$. In the case of three or more inputs, equal signs for η_{gh} and $\in_{gh}$ are not guaranteed (Hamermesh, 1996).

Figure 5. Own-price labor elasticities by sector for the youngest and oldest age-group employees: evaluated at fitted mean shares.



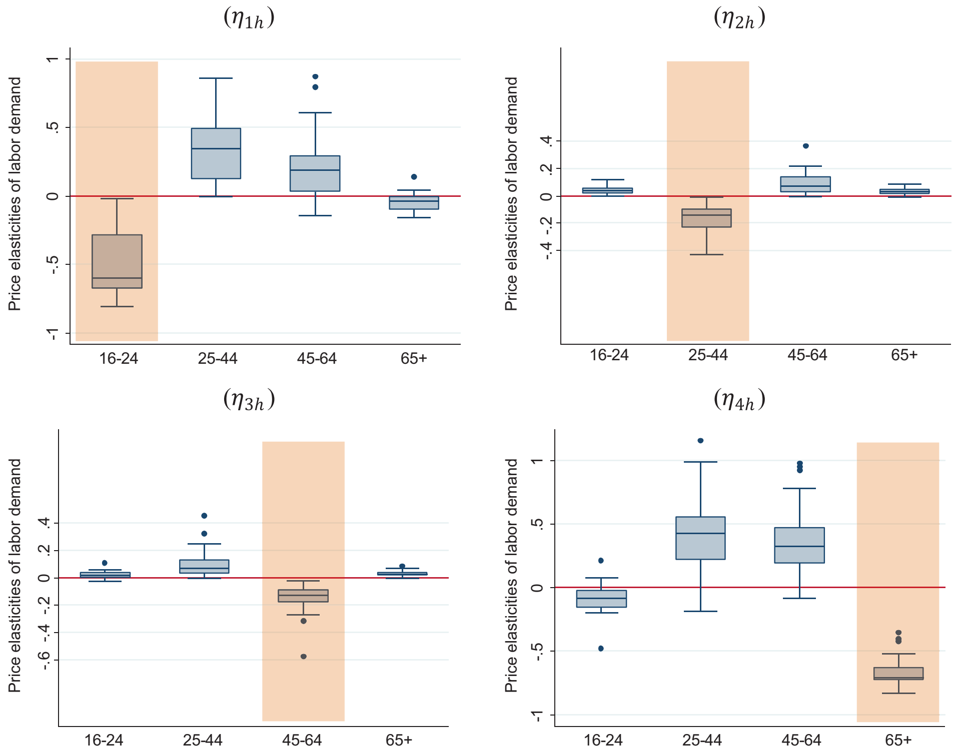
Notes: (1) *x*-axis and *y*-axis represent the own-price labor elasticities for the 16–24 and 65+ age groups, respectively. (2) The origin represents mean of elasticities. (3) The aggregate sectors in bold represent the ones appearing most in the corresponding quadrants. (4) The numbers on each symbol represent the sectors in Table 1. (5) Each symbol is specific to each quadrant.

To comprehensively evaluate the employment effects of wage decline in each age group, a simple simulation exercise was conducted by taking into account own- and cross-wage elasticities of labor demand; the results are shown in Figure 7. Each box plot represents a distribution of employment changes for 45 sectors in response to a negative wage shock of 10%. The simulation shows that real wage declines for the youngest and oldest workers lead to a net positive growth in total employment, resulting from a larger contribution from own-price labor demand than from cross-price demand while wage reduction for the two middle age groups induces job losses in total.

5. An application to a regional input–output model

Following an investigation in Section 4.3 on the impact of relative wage changes on employment, a question that naturally arises centers on the economy-wide impact of distributional changes in the heterogeneity of labor (or households more broadly). For an empirical exploration to this question, a modification of Miyazawa's extended input–output framework (Miyazawa, 1968) is adopted to account for heterogeneity in age of consumers and workers at the state level. Miyazawa's approach provides a simple yet very useful framework that facilitates analysis of endogenous, heterogeneous households once consumption and income data disaggregated by household characteristics become accessible. In this section, the sensitivity of the Chicago economy to changes in age structure is evaluated, represented

Figure 6. Distributions of cross-price labor demand elasticities for 45 sectors: the Bayesian SUR evaluated at fitted mean shares with monotonicity and concavity imposed.



Notes: (1) Homogeneity and symmetry are globally imposed. (2) Monotonicity are imposed at all data points and concavity is imposed at a single point, i.e. mean labor cost shares. (3) Shaded areas are own-price elasticities and the rests are cross-price elasticities. (4) η_{gh} = %Δ(labor demand of age group g)/%Δ(wage of age group h).

by economic multipliers, following a description of the Miyazawa's model and the data used.

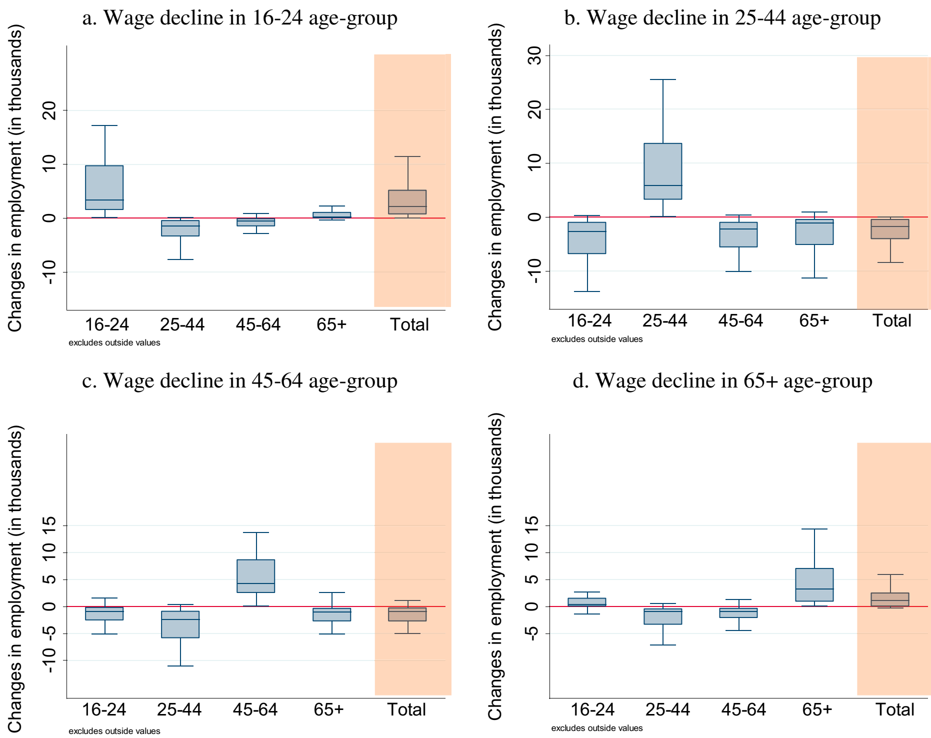
5.1. Miyazawa's extended input–output model

The input–output model in Miyazawa (1968) was originally constructed for three regions in Japan where the household sector in each region is endogenous. Miyazawa's approach is 'the most parsimonious' extended input–output formulation in that an extension of multiple household sectors is based solely on an input–output table rather than a social accounting matrix (SAM) (Hewings et al., 2001). As such, Pyatt (2001) claims that the Miyazawa multipliers should be interpreted as *factorial* income multipliers involved with wage and salary payments in an input–output table as distinguished from *institutional* income multipliers based on a SAM.

The Miyazawa system is specified as follows:

$$\begin{bmatrix} \mathbf{x}_{n\times 1} \\ \mathbf{y}_{q\times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{n\times n} & \mathbf{C}_{n\times q} \\ \mathbf{V}_{q\times n} & \mathbf{0}_{q\times q} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{n\times 1} \\ \mathbf{y}_{q\times 1} \end{bmatrix} + \begin{bmatrix} \mathbf{f}^{*}_{n\times 1} \\ \mathbf{g}_{q\times 1} \end{bmatrix} \quad (7)$$

Figure 7. The effects of wage decline on employment in 45 sectors: a 10% wage decline in each age group.



Notes: (1) Elasticities are calculated from the Bayesian SUR estimates with monotonicity and concavity imposed. (2) Calculation of changes in employment is based on the 2013 figures.

where n is the number of sectors; q is the number of household groups; $\mathbf{x}$ is a vector of output; $\mathbf{y}$ is a vector of total income; $\mathbf{A}$ is a direct requirement coefficient matrix; $\mathbf{V}$ is a labor income coefficient matrix; $\mathbf{C}$ is a consumption coefficient matrix; $\mathbf{f}^*$ is a vector of exogenous final demand; $\mathbf{g}$ is a vector of exogenous income.

Solving Equation 7 for $\mathbf{x}$ and $\mathbf{y}$ yields:[23]

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{B}(\mathbf{I}+\mathbf{CKVB}) & \mathbf{BCK} \\ \mathbf{KVB} & \mathbf{K} \end{bmatrix} \begin{bmatrix} \mathbf{f}^* \\ \mathbf{g} \end{bmatrix} \tag{8}$$

where $\mathbf{B}$ is a traditional Leontief inverse matrix, i.e. $\mathbf{B} = (\mathbf{I}\text{–}\mathbf{A})^{-1}$; $\mathbf{K} = (\mathbf{I}\text{–}\mathbf{A})^{-1}$ for $\mathbf{L} = \mathbf{VBC}$. The $\mathbf{K}$ matrix is the 'interrelational income multiplier' matrix, indicating how much income in one group is generated by a unit income increase in the other groups. The matrix of 'multi-sector income multipliers', $\mathbf{KVB}$, indicates how much income in one group is generated by a unit increase of final demand in each sector.

[23] More details on the intermediate steps to solve the system can be found in Miyazawa (1968, pp. 43–45).

5.2. Data construction for the Miyazawa analysis

The Miyazawa multipliers in Equation 8 consist of the coefficient matrices of direct requirement (**A**), labor income (**V**) and consumption (**C**). The **A** matrix is derived from the input–output table for Chicago (the 2009 base year) constructed by aggregating county-specific input–output tables obtained from the IMPLAN.[24] The sectors in the IMPLAN input–output tables are re-categorized to match with 45 sectors in Table 1. In the Chicago input–output table, there exist only aggregate labor income and consumption by sector. To obtain the **V** and **C** matrices disaggregated by age group, a consumer demand model as well as the labor demand model discussed in the previous sections are utilized to generate labor income and consumption shares by age group. In what follows, we describe how to disaggregate the **V** and **C** matrices by age using the labor and consumer demand models in more details.

First, to disaggregate total employee compensation in each sector by age group, we use the age-group-specific labor cost share equations for Illinois estimated in the preceding sections. Population and employment in the Chicago region account for 70% of total population and employment in Illinois. Since the estimated labor cost share equations are only state specific, we assume that the estimates for Illinois are good approximates for the Chicago region. The original 1×45 employee compensation vector from the input–output table is transformed into a 4×45 matrix where the (i,j)th entry shows compensation paid to workers in each age group i in sector j. In the final **V** matrix, labor income by age group is expressed as a share of output for each sector.

Next, to estimate the consumption coefficient matrix **C**, the original 45×1 column vector of household consumption is disaggregated into a 45×4 matrix. The (i,j)th entry of the 45×4 matrix represents consumption of households in age group j on good i. Following Kim et al. (2015), the Almost Ideal Demand System (AIDS; Deaton and Muellbauer, 1980) model is used to estimate age-group-specific consumer demand for Chicago.[25] Then, each entry in the 45×4 sector-by-age-group matrix is divided by the column sum to represent the consumption share of total expenditure. Finally, a consumption coefficient matrix **C** is generated by multiplying each column of shares by average propensity to consume of the corresponding age group, i.e. the ratio of total expenditure to total income.[26] Hence, each entry in the **C** matrix indicates the consumption share of *total income* ($\mathbf{y}$).

One might argue that a bias could occur due to a unit mismatch between an individual worker as a labor income earner and a household as a consumer. Unfortunately, data on expenditure by individual family members are not available for the Chicago region in the Consumer Expenditure Survey (CES), the data on which the consumer demand model is based. A bias occurs when two or more labor income earners in a household are in different age brackets. However, considering that the age brackets used in this paper are broad and

[24] The Chicago region in this study includes seven counties in Illinois: Cook, Du Page, Kane, Kendall, Lake, McHenry, and Will.

[25] More specifically, Kim et al. (2015) estimate the AIDS model for five nondurable goods and services using the data from the Consumer Expenditure Survey (CES). The five types of expenditures are then disaggregated into consumption in 45 sectors via a bridge matrix. Durable goods consumption is allocated across age groups, proportional to the number of households in each group.

[26] Average propensity to consume by age group is calculated from the 2009 CES for the US. It is worth mentioning that average propensity to consume significantly varies by age group: 1.11 for the under 25 group, 0.77 for the 25–44 group, 0.73 for the 45–64 group, and 0.93 for the 65+ group.

Table 3. The effects of age distribution changes on *interrelational* income multipliers (**K** matrix).

	Age group of income origin				
	16–24	25–44	45–64	65+	Total
Age group of income receipt: 2009					
16–24	1.056	0.037	0.036	0.047	1.175
25–44	0.409	1.281	0.274	0.360	2.325
45–64	0.385	0.268	1.265	0.361	2.279
65+	0.038	0.027	0.028	1.040	1.133
Total	1.888	1.614	1.602	1.808	6.912
Age group of income receipt: 2020					
16–24	1.044	0.029	0.028	0.037	1.138
25–44	0.348	1.238	0.232	0.303	2.122
45–64	0.445	0.308	1.304	0.413	2.470
65+	0.049	0.035	0.035	1.052	1.171
Total	1.886	1.611	1.599	1.804	6.901
Changes in indirect and induced impacts (%): 2020–2009					
16–24	−21.5	−21.6	−21.5	−20.5	−21.3
25–44	−14.9	−15.1	−15.3	−15.9	−15.3
45–64	15.6	14.9	14.6	14.3	14.9
65+	29.3	28.8	28.7	28.0	28.7
Total	−0.20	−0.43	−0.49	−0.49	−0.39

Notes: (1) The [*i,j*]th entry represents a direct increase of $1 in income to group *j* leads to *k* cents in income payments to group *i*. (2) It is assumed that technology and relative prices of goods and labor groups do not change from 2009 on.

that age difference between a head of family and his/her spouse is relatively small in most cases, the bias from the unit mismatch is not likely to be large.

5.3. Comparative statics: the effects of ageing population

This subsection presents the Miyazawa multipliers and assesses how an ageing population would affect these multipliers for 2020 compared to those for 2009. To identify the effects of the age distribution changes alone, we assume that production technology, the relative prices of goods, the relative wages of workers in different age groups, and in- and out-migration rates in Chicago are assumed to remain the same as their base year (2009) values. Therefore, the changes in the labor income coefficient matrix **V** can be attributed to rising or falling employment shares for age-group workers represented by the age-group-specific linear time trends γ's in the share equations in Equation 5. To project the consumption coefficient matrix **C** of 2020, we use a regional macroeconometric forecasting model for Chicago proposed by Kim et al. (2015) to generate aggregate personal income and the number of households by age group until 2020 under the assumption of constant average propensity to consume by age group.

The interrelational income multipliers **K** in Table 3 show, by column, that given a labor income shock, the 25–44 and 45–64 groups are expected to receive much larger induced income than the youth and elderly groups. In 2009, for example, a $1 increase of wage and salary income in the oldest group induces 5 cents in the youngest group, 36 cents in the two middle age groups, and 4 cents in the oldest group. This is simply due to the fact that the two middle age groups account for the largest employment shares.

A row direction indicates the income inducement generated by a $1 increase of wage and salary income in all groups (one should be subtracted to account for the principal diagonal elements). Higher induced income generated by the youngest and oldest groups

Table 4. The effects of age distribution changes on multi-sector income multiplier (**KVB** matrix).

	Sector of final demand origin								
	Resource	Const.	Non-dur.	Dur.	TCU	Trade	FIRE	Services	Total
Age group of income receipt: 2009									
16–24	0.037	0.062	0.042	0.046	0.042	0.055	0.026	0.069	0.377
25–44	0.220	0.486	0.378	0.404	0.370	0.409	0.279	0.452	2.997
45–64	0.214	0.419	0.385	0.416	0.393	0.389	0.241	0.419	2.877
65+	0.018	0.033	0.032	0.033	0.031	0.034	0.020	0.044	0.245
Total	0.489	0.999	0.837	0.898	0.836	0.887	0.566	0.983	6.496
Age group of income receipt: 2020									
16–24	0.028	0.037	0.033	0.034	0.032	0.044	0.015	0.056	0.279
25–44	0.187	0.411	0.309	0.326	0.309	0.337	0.239	0.393	2.510
45–64	0.250	0.502	0.452	0.492	0.454	0.460	0.282	0.476	3.368
65+	0.022	0.042	0.041	0.043	0.039	0.043	0.028	0.055	0.313
Total	0.487	0.992	0.834	0.895	0.834	0.884	0.564	0.980	6.470
Differences (%): 2020–2009									
16–24	−23.2	−40.0	−21.5	−25.5	−24.0	−20.2	−39.0	−19.0	−26.0
25–44	−15.0	−15.3	−18.3	−19.2	−16.7	−17.4	−14.6	−13.0	−16.2
45–64	16.7	20.1	17.5	18.5	15.6	18.1	17.4	14.0	17.3
65+	16.7	24.0	24.2	27.3	24.0	25.8	30.9	23.4	24.6
Total	−0.53	−0.71	−0.31	−0.36	−0.34	−0.34	−0.45	−0.33	−0.42

Notes: (1) The [*i,j*]th entry in the matrix represents the total (direct, indirect, and induced) income for group *i* resulting from a dollar increase in consumption in sector *j*. (2) It is assumed that technology and relative prices of goods and labor groups do not change from 2009 on. (3) TCU = transportation, communications, and utilities; FIRE = finance, insurance, and real estate.

results from higher propensities to consume for these two groups, characterized by ‘earn less and spend more’.

Table 3 also shows that ageing population increases the induced income that the 45–64 and 65+ groups receive in 2020 while the other younger groups experience a decline in induced income. This can be explained by the population projection that expects a large positive growth in the population of the 45–64 and 65+ groups.[27] It is, however, important to note that the Miyazawa analysis suggests that with an ageing population, the entire local economy could suffer from a decline in additional income generated by an income shock.

The multi-sector income multipliers **KVB** in Table 4 show that sectors with higher employment share for a specific age group (see Figure 2) tend to generate higher income inducement for that age group. For example, among the eight aggregate sectors, a $1 direct demand impact from the service sector generates the highest induced income for the youngest and oldest groups. It is the construction sector that generates the highest income inducement for the middle age groups.

Comparing multi-sector income multipliers between 2009 and 2020 suggests that increasing employment shares for older workers result in higher multipliers for the 45–64 and 65+ groups and smaller multipliers for the 16–24 and 25–44 groups. Recall that the linear time trends in labor share equations vary by age group and by sector. Therefore, the degree to which multipliers in each cell change depends on the corresponding time trend estimates for changes in age-specific employment by sector. The construction sector, for example, shows the largest decline (−0.71%) in total income inducement from 2009 to

[27] According to the Census Bureau, the Illinois population aged 45–64 and 65+ is projected to increase 14% and 61%, respectively, between 2000 and 2030 while the total population is expected to grow only 8% over the same period.

Table 5. The effects of age distribution changes on *output* multipliers[1,2].

	Sector of final demand origin								
	Res.	Const.	Non-dur.	Dur.	TCU	Trade	FIRE	Serv.	Avg.
Type I: Direct & indirect (2009)[1]	1.427	1.587	1.862	1.691	1.624	1.329	1.483	1.506	1.563
Type II: Direct, indirect, and induced (2009)[2]	2.002	2.754	2.833	2.734	2.594	2.366	2.140	2.660	2.511
Type II: Direct, indirect, and induced (2020)[2]	1.994	2.731	2.824	2.722	2.585	2.356	2.132	2.650	2.499
Changes in indirect & induced impacts (%): 2020–2009	−0.82	−1.30	−0.49	−0.65	−0.61	−0.78	−0.75	−0.61	−0.75

Notes: (1) Column sums of $\mathbf{B} = (\mathbf{I}–\mathbf{A})^{-1}$. (2) Column sums of $\mathbf{B}(\mathbf{I} + \mathbf{CKVB})$. (3) It is assumed that technology and relative prices of goods and labor groups do not change from 2009 on. (3) TCU = transportation, communications, and utilities; FIRE = finance, insurance, and real estate.

2020 since construction employment for young workers is predicted to fall more rapidly than employment of young workers in other sectors.

In Table 5, output multipliers are compared among sectors when a household sector is treated as either exogenous or endogenous. Type I multipliers are the column sums of the Leontief inverse $\mathbf{B} = (\mathbf{I}–\mathbf{A})^{-1}$ while type II multipliers are the column sums of $\mathbf{B}(\mathbf{I} + \mathbf{CKVB})$. Type II multipliers of 2009 imply that a \$1 increase in total household consumption generates \$1.511 of indirect and induced income on average, whereas type I multipliers show a dollar increase in demand generates only \$0.563. Note that output multipliers show larger declines than multi-sector income multipliers in percentage terms. These findings for the output multipliers continue the ‘hollowing-out’ trend noted by Hewings et al. (1998) that was attributed to the increasing spatial fragmentation of production in the US economy.

6. Conclusion

The main objective of this paper was to complete the link between income generation and consumption at a more disaggregated level than is customary in most input–output systems. Several alternatives were explored to estimate labor shares by sector disaggregated by age to enable evaluation of the impact of the ageing of the population on economic activity in a regional economy.

Using a Bayesian SUR model, wage elasticities of labor demand by age for the US have been estimated. This approach is relevant for a wide spectrum of demand analysis since it facilitates the imposition of regularity conditions implied by the economic theory. When applied to the ACS data, the Bayesian approach shows that the labor demand for youth workers is elastic. This finding is empirically consistent with past empirical studies that highlight elastic labor demand for youth workers. Labor demand for the elderly workers is also found to be elastic but with smaller sectoral variation, relative to large variation in wage elasticities of labor demand for youth workers across sectors.

An application of the estimated labor demand of Illinois used together with the consumer demand model proposed by Kim et al. (2015) to the Miyazawa extended input–output model facilitates an examination of changes in economic and demographic structures of Chicago over time. The results suggest that, *ceteris paribus*, an ageing population contributes to lowering aggregate economic multipliers of a regional economy mainly because the number of elderly workers, who earn less labor income than younger groups, is expected to grow more rapidly.

This paper provides an example where empirical consistency can be acquired by strengthening theoretical coherence without significantly incurring additional costs such as loss of prediction accuracy. Additional implications of main findings in this paper are as follows. Monotonicity and concavity must be checked and addressed particularly in the case where one or more factor shares are so small that monotonicity is in doubt. Moreover, it is desirable for a static factor demand model with a translog cost function to simultaneously estimate a cost function and factor shares. The share equations alone do not contain enough information to recover the corresponding cost structure.

One policy implication is that a labor policy that intends to influence the price of labor for youth workers needs to be differentiated by sector, while a labor policy targeting the oldest group's wages is expected to produce similar degrees of changes in labor demand across sectors. In addition, a simulation suggests that the effectiveness of wage policy in terms of total job creation varies depending on a target age group when own- and cross-wage elasticities of labor demand are taken into account.

A further extension to this study for future research is to include not only wage and salary but also benefits in the input prices since the employers' cost of providing retirement benefits and health insurance is much higher for older workers than for younger workers (Munnell and Sass, 2008). In addition, the inclusion of institutional income (factor income plus nonwage and salary income) might further alter the results. Embedding the results in a full econometric input–output model would provide important insights into the way changes in economic structure, demographic structure and the interactions between income generation and consumption affect forecasts, compared to those using a single representative household.

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