NEREUS

Núcleo de Economia Regional e Urbana
da Universidade de São Paulo
The University of São Paulo
Regional and Urban Economics Lab

OCP POLICY CENTER
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Lecture 1: Input-Output Models

"Multi-regional Economic Modeling: Applications for Morocco"

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Input-output analysis

Idea developed by Wassily Leontief (awarded Nobel Prize in Economics in 1973)

Employed in all countries – no matter what their political sentiments

Part of **N**ational **I**ncome and **P**roduct **A**ccounts

Extend ideas of the economic base model by disaggregating production into a set of sectors

Can be extended to explore issues of income distribution, tax policy, development strategies etc.

Input-output analysis

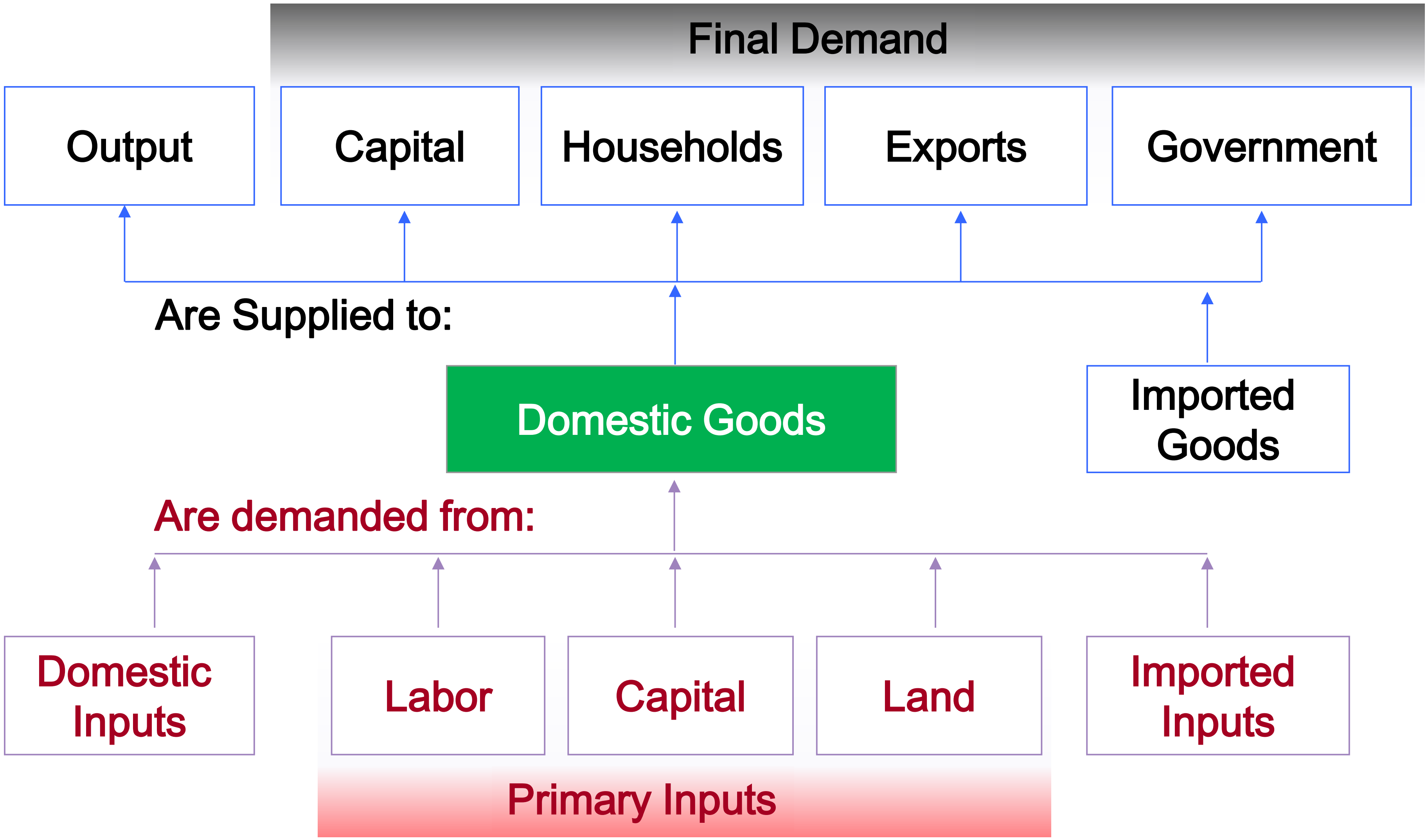
Imagine a region with *m* firms, producing a whole array of goods and services from agriculture, food processing, manufacturing, personal and business services and government.

Firms are assigned to *n* broad sectors based on their principal product.

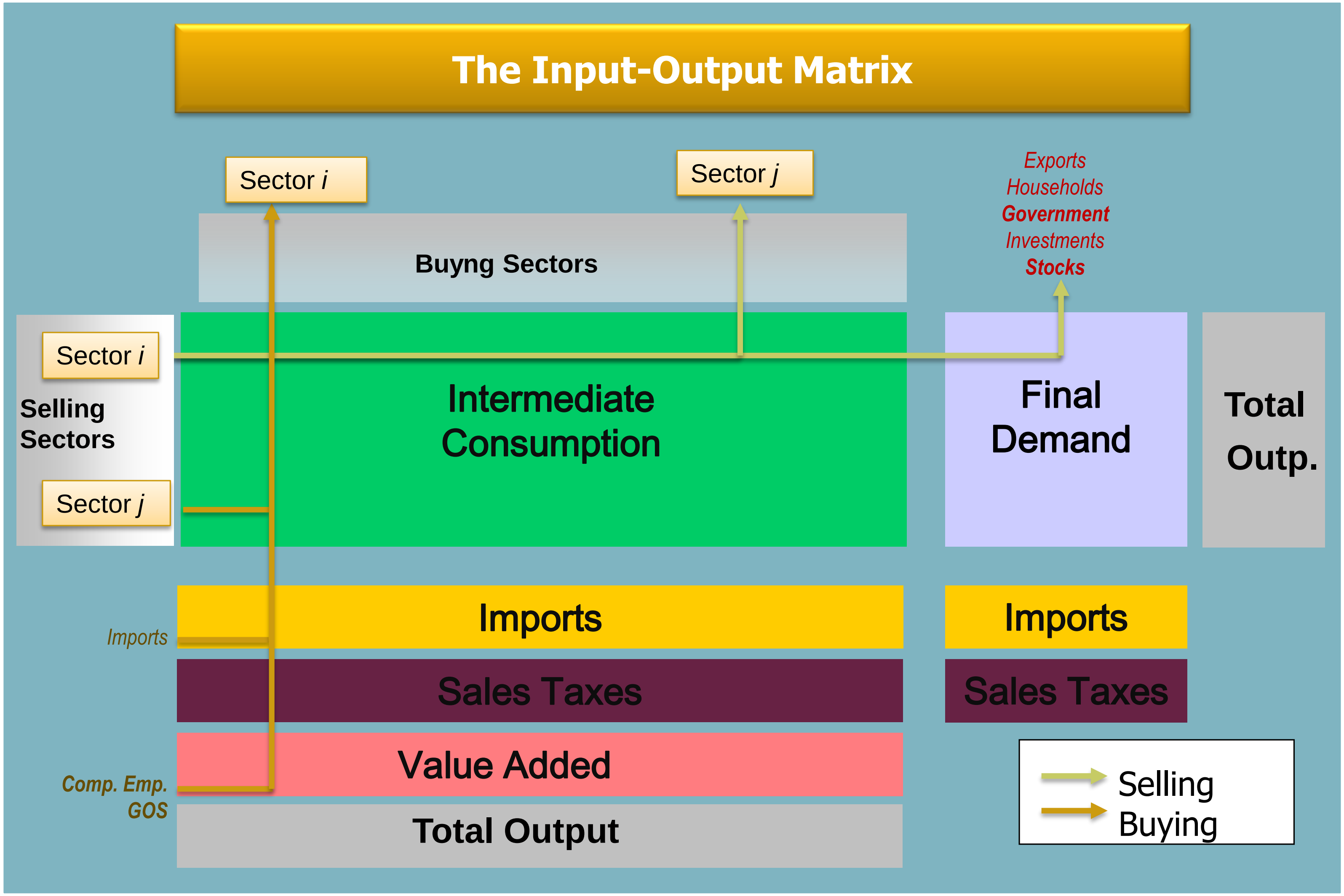
The number of sectors, *n*, ranges from 20 to several hundred and the allocation conforms to the Standard Industrial Classification (SIC).

For this presentation, only two sectors will be shown to facilitate the analysis and to avoid getting bogged down in details.

Input-output flows



Input-output table



Numerical example

IO Matrix	S1	S2	Y	X
S1	150	500	350	1000
S2	200	100	1700	2000
W	650	1400		
X	1000	2000		
Employment	300	800		

Input-output flows

The transactions between these sectors are arrayed in a matrix (*n* rows and *n* columns), as shown in the table.

Looking across the *rows*, the sales made by the firm at the left can be traced to firms listed at the top of the column.

Thus sector 2 sells $200 to sector 1, and $100 to sector 2.

The *columns* provide complementary information of the source of purchases made by the sector at the top of the column from all other sectors.

Again, following sector 2, note that it buys $500 from sector 1, and $100 from sector 2.

This part of the input-output table is referred to as the *interindustry transactions*; it provides an *economic photograph* of the ways in which one sector is linked to another sector.

Input-output flows

However, sectors also make sales to other sets of activities – consumers, government and to customers located outside the region (exports).

In addition, firms also make purchases of labor (wages and salaries), returns to capital (profits and dividends) and imports.

These are shown in the rest of the table.

The column Y is referred to as *final demand*; row W is referred to as primary inputs.

The sum of wages and salaries and profits and dividends (returns to labor and capital) are referred to as *value added*.

Input-output model

The input-output table is basically an accounting system – a double entry one similar to that prepared for a business in which sales and purchases or assets and liabilities will be shown but, in this case, for an economy.

The next step is to **prepare an economic model** so that we can trace the impact of changes in one sector on the rest of the economy.

We do this because the nature of interdependence among sectors varies.

Input-output model: key assumptions

We assume that each sector produces goods and services according to a fixed recipe (formally known as a **production function**)

$$a_{ij} = \frac{z_{ij}}{x_j} \quad , \quad i, j = 1, \ldots, n$$

- Fixed technical coefficient
- Constant returns to scale
- Sectors use inputs in fixed proportions

Input-output model: key assumptions

Inputs are expressed in monetary terms since it would be difficult to combine tons of iron ore with megawatts of electricity, or hours of labor in some consistent fashion.

This fixed recipe enables us to express the transactions in proportional form, also known as *direct coefficients*; these are shown in **the first example in the Excel file**.

The final assumption is that the economy is driven by signals emanating from final demand (consumers, government, exports); this is the *exogenous* part of the economy, while the interindustry transactions respond to these signals and are therefore *endogenous*.

Basic relations

	Buying Sectors		
Selling Sectors	Intermediate Consumption	Final Demand	Total Output
	Imports (M)	M	
	Sales Taxes on Inputs (T)	T	
	Value Added		
	Total Output		

Basic relations

$$\sum_{j=1}^{n} z_{ij} + y_i \equiv x_i \quad , \quad i = 1,\ldots,n$$

$$a_{ij} = \frac{z_{ij}}{x_j} \quad , \quad i, j = 1,\ldots,n$$

Production function

$$x_j = f\left(z_{1j},\ldots,z_{nj},W_j,M_j\right) \qquad x_j = \min\left(\frac{z_{1j}}{a_{1j}},\ldots,\frac{z_{nj}}{a_{nj}}\right)$$

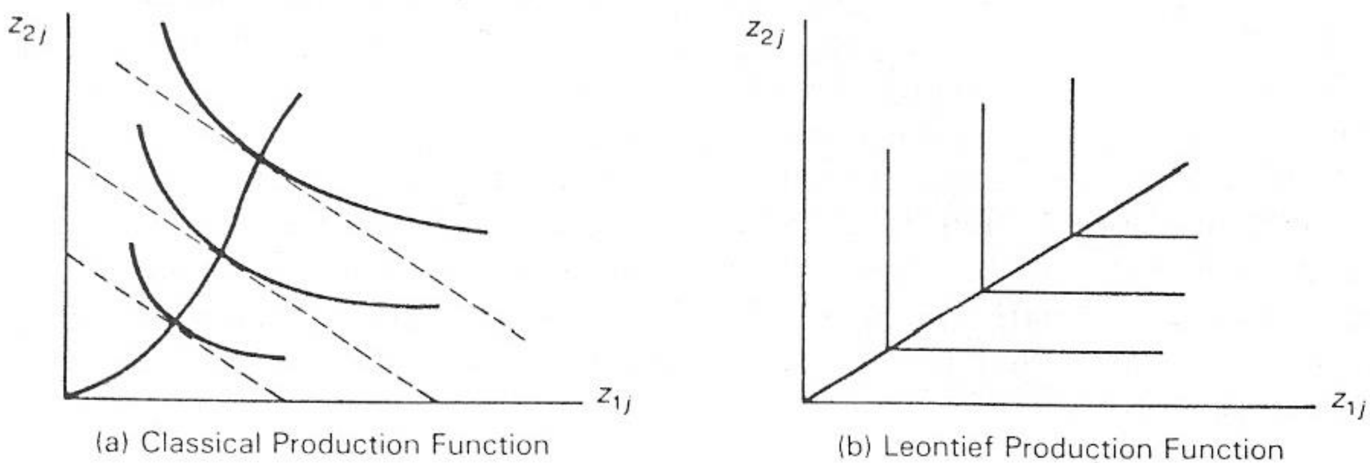


FIGURE 2-1 Production functions in input space.

The Leontief model

$$\sum_{j=1}^{n} a_{ij} x_j + y_i = x_i \qquad i = 1,\ldots,n$$

$$Ax + y = x$$

$$x = (I - A)^{-1} y$$

$$B = (I - A)^{-1}$$

The power series approximation of $(I - A)^{-1}$

$$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots)$$

Leontief matrix

$(I-A)^{-1}$ is known as the Leontief inverse matrix and is shown in the table below:

The entries reveal the direct and indirect impacts on a sector when final demand in the sector at the top of the column changes by $1 (or $1 million or $100 million).

Note that the entry on the principal diagonal is always >1; the unit value represents the increase in final demand in that sector. The remaining portion is the direct and indirect impact of expansion.

$(I-A)^{-1}$		1	2
	1	1,254	0,330
	2	0,264	1,122
	Total	1,518	1,452

Leontief matrix

At the bottom of the multiplier table there is a row labeled **total**

Note that these values vary from 1.45 (sector 2) to 1.52 (sector 1)

How should these entries be interpreted?

They provide information on the impact on the rest of the economy (including the sector in question) of a unit change in final demand in any sector.

The value 1.45 for sector 2 tells us that for every increase of $1 in that sector an additional 0.45 worth of activity is generated for a total value of production of 1.45.

Leontief matrix

Why do these values vary?

- They reflect the degree to which a sector is dependent on other sectors in the economy for its inputs and as a source of consumption for its products.
- They depend on the structure of production (the recipe).

It would be incorrect to assume that a sector's importance in the economy is directly related to the size of the multiplier

- While true in part, a sector with a large volume of production but a modest multiplier may generate a greater volume of activity in the region than the sector with the largest multiplier but a smaller volume of production.

Multipliers and generators

There are several additional multipliers that can be calculated

When a sector expands production, it will increase payments to labor generating additional wages and salaries that will be spent in the region. Further, other industries whose production has to expand to meet these new demands will also spend more on wages and salaries. Thus, we may generate an **income multiplier** that reveals the relationship between direct income generation and total income (in similar fashion to output).

We could also transform the analysis into **employment** terms.

Multipliers and generators

Coefficient

$$C_j^v = \frac{V_j}{X_j}$$

Generator

$$\begin{cases} G_j^v = \sum_{i=1}^{n} c_i^v b_{ij} \\ G^v = C^v B \rightarrow 1xn \\ \text{or} \\ G^v = \hat{C}^v B \rightarrow nxn \end{cases}$$

Multiplier

$$M_j^v = \frac{G_j^v}{C_j^V}$$

Impact analysis

$$X = (I - A)^{-1} Y$$

$$\Delta X = (I - A)^{-1} \Delta Y$$

$$\Delta V = \hat{C}^{v} \Delta X$$

$$\Delta V = \hat{C}^{v} B \Delta Y$$

$$G^{v} = \hat{C}^{v} B$$

Closing the IO model to households

Households earn incomes (at least in part) in payment for their labor inputs to production processes, and, as consumers, spend their income in rather well patterned ways.

One could move the household sector from the final-demand column and labor-input row and place it inside the technically interrelated table, making it one of the ***endogenous*** sectors.

$$\Delta X \rightarrow \Delta W \rightarrow \Delta Y$$

(Second example in the Excel file)

Closing the IO model to households

$$\overline{A} = \begin{bmatrix} A & H_c \\ H_r & 0 \end{bmatrix} \qquad \overline{Y} = \begin{bmatrix} Y^* \\ Y^*_{n+1} \end{bmatrix} \qquad \overline{X} = \begin{bmatrix} X \\ X_{n+1} \end{bmatrix}$$

$$Y^* = Y - Y^h$$

$$\overline{X} = \overline{B}\,\overline{Y}$$

$$\overline{B} = (I - \overline{A})^{-1}$$

Regional IO models

	Buying Sectors		
Selling Sectors	Intermediate Consumption	Final Demand	Total Output
	Imports from the rest of the Country (MC)	MC	
	Imports from the rest of the World (MW)	MW	
	Sales Taxes on Inputs (T)	T	
	Value Added		
	Total Output		

Interregional IO models

	Buying Sectors Region L	Buying Sectors Region M			
Selling sectors **Region L**	Interindustry Inputs LL	Interindustry Inputs LM	FD LL	FD LM	TO L
Selling sectors **Region M**	Interindustry Inputs ML	Interindustry Inputs MM	FD ML	FD MM	TO M
	Imports from the World	Imports from the World	M	M	M
	Sales Taxes	Sales Taxes	T	T	T
	Value Added	Value Added			
	Total Output L	Total Output M			

Interregional IO models

Large amount of data

Problem?!

Interregional IO models

Estimation of regional models

Early studies:

$$p_j^R = \frac{\left(X_j^R - E_j^R\right)}{\left(X_j^R - E_j^R + M_j^R\right)}$$

where:

X_j^R is the total output of good j in region R;

E_j^R is the total exports of good j from region R;

M_j^R is the total imports of good j by region R.

$$A^R = \hat{P}A \longrightarrow X^R = \left(I - \hat{P}A\right)^{-1} Y^R$$

Estimation of regional models

Example:

$$\hat{P} = \begin{bmatrix} .8 & 0 \\ 0 & .6 \end{bmatrix}, \qquad A^R = \hat{P}A = \begin{bmatrix} .8 & 0 \\ 0 & .6 \end{bmatrix} \begin{bmatrix} .15 & .25 \\ .20 & .05 \end{bmatrix} = \begin{bmatrix} .12 & .20 \\ .12 & .03 \end{bmatrix},$$

$$(I - A^R)^{-1} = \begin{bmatrix} 1.169 & .241 \\ .145 & 1.061 \end{bmatrix}$$

(Third example in the Excel file)

Regional coefficients

Regional coefficients:

$$a_{ij}^{LL} = \frac{z_{ij}^{LL}}{X_j^L}$$

Regional production:

$$X^L = \left(I - A^{LL}\right)^{-1} Y^L$$

Interregional IO models

Interregional flows – intermediate consumption:

$$Z = \begin{bmatrix} Z^{LL} & Z^{LM} \\ Z^{ML} & Z^{MM} \end{bmatrix}$$

Total output:

$$X_i = z_{i1} + z_{i2} + \ldots + z_{ii} + \ldots + z_{in} + Y_i$$

$$X_1^L = z_{11}^{LL} + z_{12}^{LL} + z_{11}^{LM} + z_{12}^{LM} + Y_1^L$$

Interregional IO models

Interregional coefficients:

$$a_{ij}^{LL} = \frac{z_{ij}^{LL}}{X_j^{L}} \qquad a_{ij}^{LM} = \frac{z_{ij}^{LM}}{X_j^{M}}$$

$$a_{ij}^{ML} = \frac{z_{ij}^{ML}}{X_j^{L}} \qquad a_{ij}^{MM} = \frac{z_{ij}^{MM}}{X_j^{M}}$$

Deriving the interregional IO model

$$A = \begin{bmatrix} A^{LL} & \vdots & A^{LM} \\ \cdots & \cdots & \cdots \\ A^{ML} & \vdots & A^{MM} \end{bmatrix} \qquad Y = \begin{bmatrix} Y^{L} \\ \cdots \\ Y^{M} \end{bmatrix} \qquad X = \begin{bmatrix} X^{L} \\ \cdots \\ X^{M} \end{bmatrix}$$

$$\left\{ \begin{bmatrix} I & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & I \end{bmatrix} - \begin{bmatrix} A^{LL} & \vdots & A^{LM} \\ \cdots & \cdots & \cdots \\ A^{ML} & \vdots & A^{MM} \end{bmatrix} \right\} \begin{bmatrix} X^{L} \\ \cdots \\ X^{M} \end{bmatrix} = \begin{bmatrix} Y^{L} \\ \cdots \\ Y^{M} \end{bmatrix}$$

$$(I - A)X = Y,$$

$$X = \left(I - A\right)^{-1} Y$$

(Fourth example in the Excel file)

Multipliers in (inter)regional IO models

Multipliers vary not only across sectors but also across regions.

A small regional economy, with a modest representation of industry, may not be able to provide all the necessary inputs required by local industry. Thus, there will be considerable importation of inputs (sometimes referred to as leakages).

In general, the larger the value of the imports, the lower the value of the multiplier.

We would expect multipliers to decrease as we move from the country as a whole to a macro-region, an individual province, a metropolitan region and finally to a municipality.