NEREUS

Núcleo de Economia Regional e Urbana
da Universidade de São Paulo

The University of São Paulo
Regional and Urban Economics Lab

Lecture 3: The BMMX ICGE Model

"Multi-regional Economic Modeling: Applications for Mexico"

January 22-23, 2020 – Centro de Investigación y Docencia Económicas, Ciudad de México, México

Eduardo A. Haddad and Inácio Araújo

Outline

- ✓ What is the BMMX ICGE model?
- Building blocks

BMMX ICGE, a bottom-up spatial CGE model of Mexico

A multi-sectoral, multi-regional bottom-up CGE model of Mexico's 32 regions

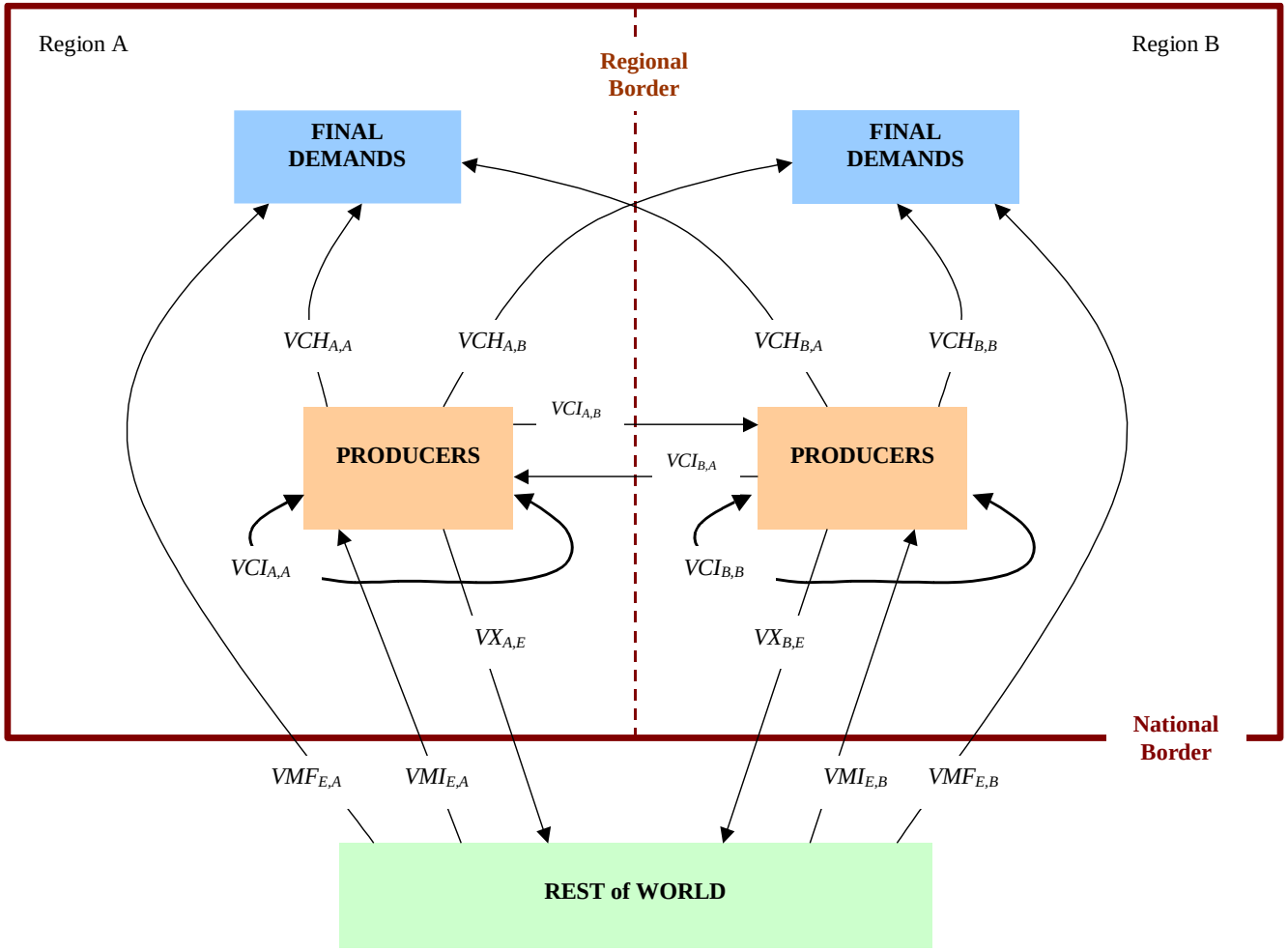
- each region is modeled as an economy in its own right
- region-specific prices
- region-specific industries
- region-specific consumers

Based on the comparative-static B-MARIA and MMRF models

Database makes allowance for interregional, intra-regional and international trade

- Potential for the representation of regional and Federal government financial accounts

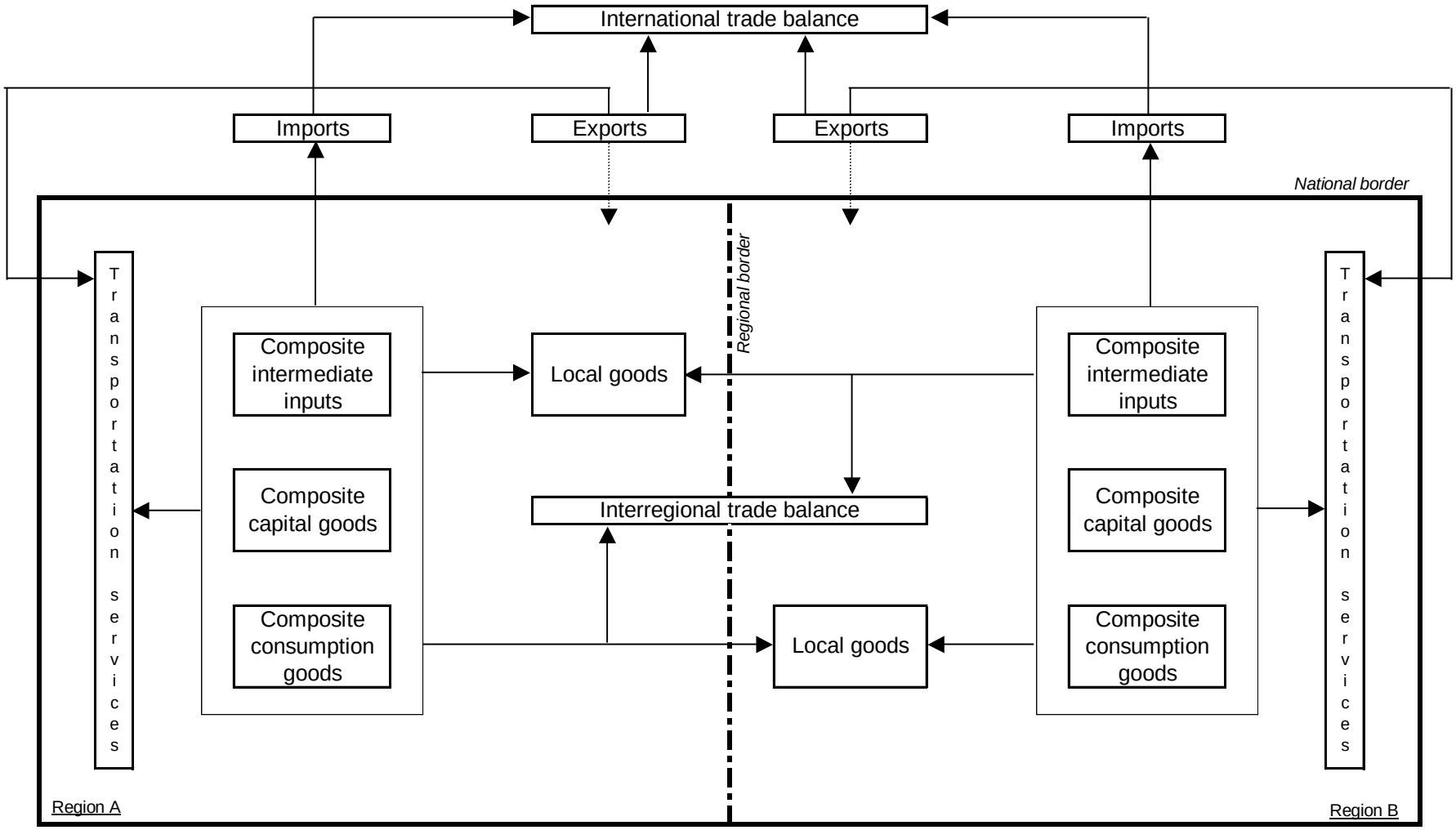
Stylized flows



Embedded SAM

		REGION 1					REGION R										
		FACTORS	AGENTS H	AGENTS G	SECTORS	LOCAL PRODUCTS	COMPOSITE PRODUCTS (SUPPLY)	FACTORS	AGENTS H	AGENTS G	SECTORS	LOCAL PRODUCTS	COMPOSITE PRODUCTS (SUPPLY)	FEDERAL GOV.	IMPORTS	EXPORTS	ACCUM.	ROW	TOTAL
REGION 1	FACTORS				W^1														W^1
REGION 1	AGENTS H	Wh^{11}		$Thrg^1$				Wh^{1R}						$Thfg^1$					Yh^1
REGION 1	AGENTS G		$IDrgh^1$			$TIrg^1$								Tfg^1					Yrg^1
REGION 1	SECTORS					PMI^1										E^1			PCF^1
REGION 1	LOCAL PRODUCTS						Ppm^{11}						Ppm^{1R}						Ppm^1
REGION 1	COMPOSITE PRODUCTS (DEMAND)		Ch^1	Crg^1	CI^1									Cfg^1			I^1		D^1
REGION R	FACTORS										W^R								W^R
REGION R	AGENTS H	Wh^{R1}						Wh^{RR}		$Thrg^R$				$Thfg^R$					Yh^R
REGION R	AGENTS G								$IDrgh^R$			$TIrg^R$		Tfg^R					Yrg^R
REGION R	SECTORS											PMI^R				E^R			PCF^1
REGION R	LOCAL PRODUCTS						Ppm^{R1}						Ppm^{RR}						Ppm^R
REGION R	COMPOSITE PRODUCTS (DEMAND)								Ch^R	Crg^R	CI^R			Cfg^R			I^R		D^R
	FEDERAL GOVERNMENT		$IDfgh^1$			$TIfg^1$			$IDfgh^R$			$TIfg^R$			Tim	Tex			Yfg
	IMPORTS						Ipm^1						Ipm^R						Ipm
	EXPORTS																	Epm	Epm
	ACCUMULATION		Sh^1	Srg^1					Sh^R	Srg^R				Sfg				SBC	I
	ROW														M				Yrw
	FINANCIAL/ASSET ADJUSTMENT		FAh^1	$FArg^1$					FAh^R	$FArg^R$				FAfg					FA
	TOTAL	W^1	Yh^1	Yrg^1	PCF^1	Ppm^1	O^1	W^R	Yh^R	Yrg^R	PCF^R	Ppm^R	O^R	Yfg	Ipm	Epm	I	Yrw	

The Role of Transportation Services in BMMX*



Core database

		ABSORPTION MATRIX					
		1	2	3	4	5	6
		Producers	Investors	Household	Export	Regional Govt.	Federal Govt.
	Size	J x Q	J x Q	Q	1	Q	Q
Basic Flows	I x S	BAS1	BAS2	BAS3	BAS4	BAS5	BAS6
Margins	I x S x R	MAR1	MAR2	MAR3	MAR4	MAR5	MAR6
Taxes	I x S	TAX1	TAX2	TAX3	TAX4	TAX5	TAX6
Labor	1	LABR					
Capital	1	CPTL					
Other	1	OCTS					

I = number of commodities
J = number of industries
R = number of commodities used as margins
Q = number of regions
S = Q domestic regions + 1 foreign import

Features of database

Commodity flows are valued at "basic prices" (**BAS**):

- do not include user-specific taxes or margins

For each user of each imported good and each domestic good, there are numbers showing:

- tax levied on that usage (**TAX**)
- usage of margins – transport (**MAR**)*

Single-production:

- each commodity may be produced by one industry
- each industry may produce one commodity

For each industry the total cost of production is equal to the total value of output

For each commodity the total value of sales is equal to the total value of output

Features of database (cont.)

Domestic producers

- J industries in Q regions

Investors

- J industries in Q regions

Households

- one representative household for each of the Q regions

Each of the I commodity types can be obtained from the region, from other regions, or imported from overseas

Features of database (cont.)

Aggregate foreign purchaser of exports

Other demand category corresponding to the Q regional governments

Other demand category corresponding to the central government in the Q regions

Commodity taxes and margins explicitly recognised

Notation

Main User Numbers

1 ⇔ firms, current production;
2 ⇔ firms, capital creation;
3 ⇔ households;
4 ⇔ foreign exports;
5 ⇔ regional government;
6 ⇔ Central government;

The number 0 is also used to denote basic prices and values.

Source dimensions

a ⇔ all sources, i.e., 32 regional sources and 1 foreign;
r ⇔ regional sources only;
t ⇔ two sources, i.e., a domestic composite source and foreign;
c ⇔ domestic composite source only;
o ⇔ domestic-foreign composite source only.

Outline

What is the BMMX ICGE Model?

✓ Building blocks

Building blocks

- ✓ **Producer's demands for inputs**
- ✓ Investor demands
- ✓ Household demands
- ✓ Export demands
- ✓ Government demands
- ✓ Margins demands
- ✓ Zero pure profits
- ✓ Indirect tax equations
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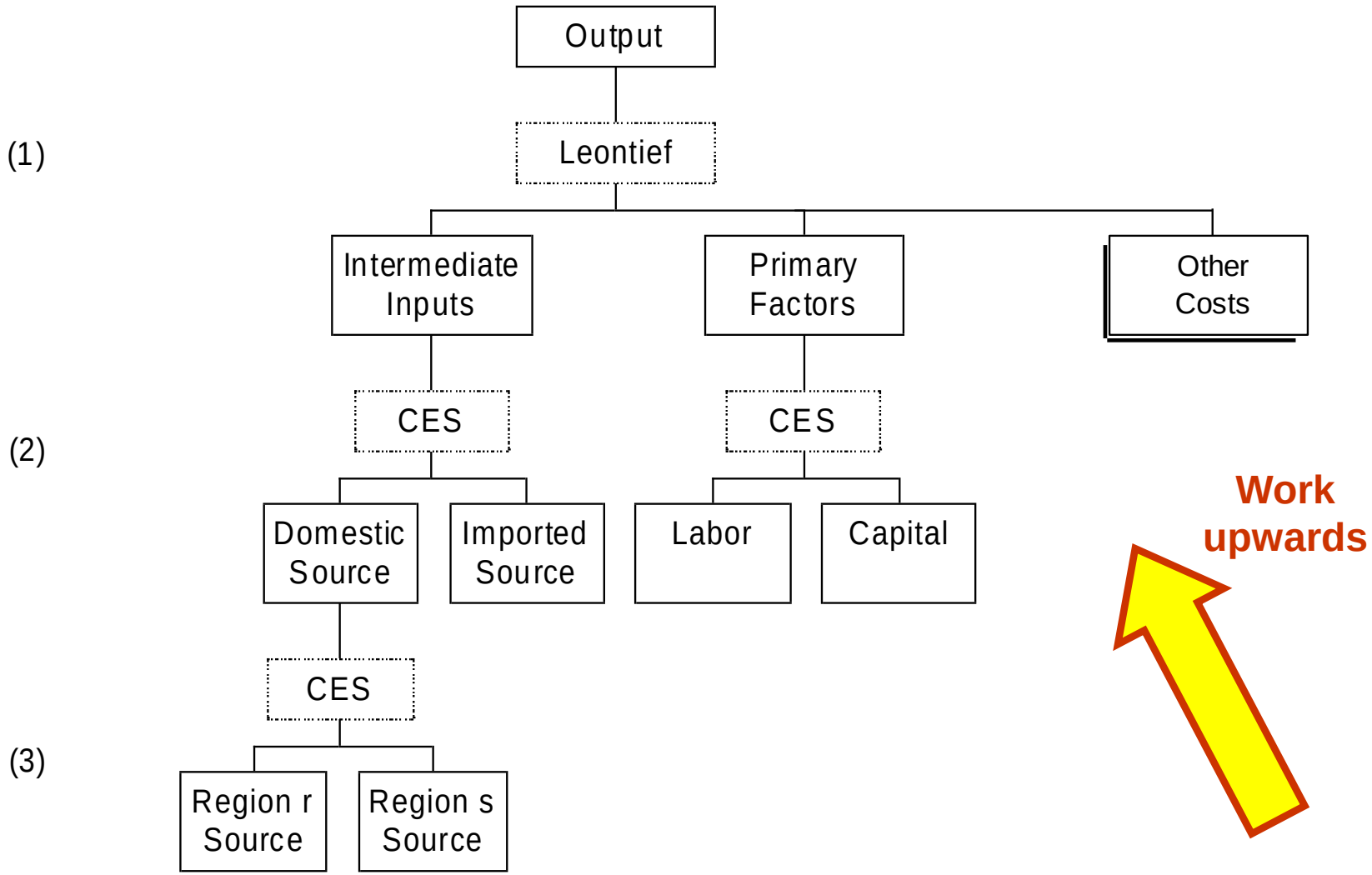
Production nest



Demand for intermediate inputs, other costs and prices

```
E_x1a # Demand for goods by all sources, User 1 #
(all,i,COM)(all,s,ALLSOURCE)(all,j,IND)(all,q,REGDEST)
x1a(i,s,j,q)=IS_DOM(s)*(x1c(i,j,q)-
SIGMA1C(i)*(p1a(i,s,j,q)-p1c(i,j,q)))
+IS_IMP(s)*(x1o(i,j,q)-SIGMA1O(i)*(p1a(i,"foreign",j,q)-
p1o(i,j,q)));

E_p1o # Price of domestic/foreign composite, User 1 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
(TINY+PVAL1O(i,j,q))*p1o(i,j,q) =
sum(s,ALLSOURCE,PVAL1A(i,s,j,q)*p1a(i,s,j,q));

E_p1c # Price of domestic composite, User 1 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
(TINY+PVAL1T(i,"domestic",j,q))*p1c(i,j,q)
=sum(s,REGSOURCE,PVAL1A(i,s,j,q)*p1a(i,s,j,q));
```

Demand for intermediate inputs, other costs and prices

```
E_x1c # Demand for domestic composite, User 1 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
x1c(i,j,q)=x1o(i,j,q)-SIGMA1O(i)*(p1c(i,j,q)-p1o(i,j,q));

E_x1o # Demand for dom./for. composite inputs, User 1 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
x1o(i,j,q)=z(j,q)+a1(j,q);

E_x1oct # Industry demand for other cost tickets #
(all,j,IND)(all,q,REGDEST)
x1oct(j,q)=z(j,q)+a1(j,q)+a1oct(j,q);

E_p1oct # Indexing of prices of other cost tickets #
(all,j,IND)(all,q,REGDEST)
p1oct(j,q)=xi3(q)+f1oct(j,q);
```

Demand for primary factors

```
E_efflab # Industry demand for effective labor #
(all,j,IND)(all,q,REGDEST)
efflab(j,q)=MRL(j,q)*x1prim(j,q)+a1lab(j,q)
-SIGMA1FAC(j,q)*[p1lab(j,q)+a1lab(j,q)-xi_fac(j,q)];

E_curcap # Industry demand for capital #
(all,j,IND)(all,q,REGDEST)
curcap(j,q)=MRK(j,q)*x1prim(j,q)+a1cap(j,q)
-SIGMA1FAC(j,q)*[p1cap(j,q)+a1cap(j,q)-
xi_fac(j,q)]+IL2(j,q)*interest;

E_n # Industry demand for land #
(all,j,IND)(all,q,REGDEST)
n(j,q)=MRN(j,q)*x1prim(j,q)+a1land(j,q)
-SIGMA1FAC(j,q)*[p1land(j,q)+a1land(j,q)-xi_fac(j,q)];

E_xi_fac # Effective price term for factor demand equations #
(all,j,IND)(all,q,REGDEST)
(TINY+TOTFACIND(j,q))*xi_fac(j,q)=LABOR(j,q)*(p1lab(j,q)+a1lab(j,q))
+CAPITAL(j,q)*(p1cap(j,q)+a1cap(j,q))+LAND(j,q)*(p1land(j,q)+a1land(
j,q));
```

Demand for primary factors

```
E_x1laboi # Demand for labor by industry and skill group #
(all,m,OCC)(all,j,IND)(all,q,REGDEST)
x1laboi(j,q,m)=efflab(j,q)-SIGMA1LAB(j,q)*[p1laboi(j,q,m)-
p1lab(j,q)]
+IL(m,j,q)*interest;

E_p1lab # Price to each industry of labor in general #
(all,j,IND)(all,q,REGDEST)
(TINY+LABOR(j,q))*p1lab(j,q)=sum(m,OCC,LAB_OCC_IND(m,j,q)*p1laboi(j,
q,m));

E_labind # Employment by industry #
(all,j,IND)(all,q,REGDEST)
(TINY+LABOR(j,q))*labind(j,q)=sum(m,OCC,LAB_OCC_IND(m,j,q)*x1laboi(j
,q,m));

E_x1prim # Demand for the primary-factor composite #
(all,j,IND)(all,q,REGDEST)
x1prim(j,q)=MRP(j,q)*z(j,q)+a1(j,q)+a1prim(j,q);
```

Building blocks

- ✓ Producer's demands for inputs
- ✓ **Investor demands**
- ✓ Household demands
- ✓ Export demands
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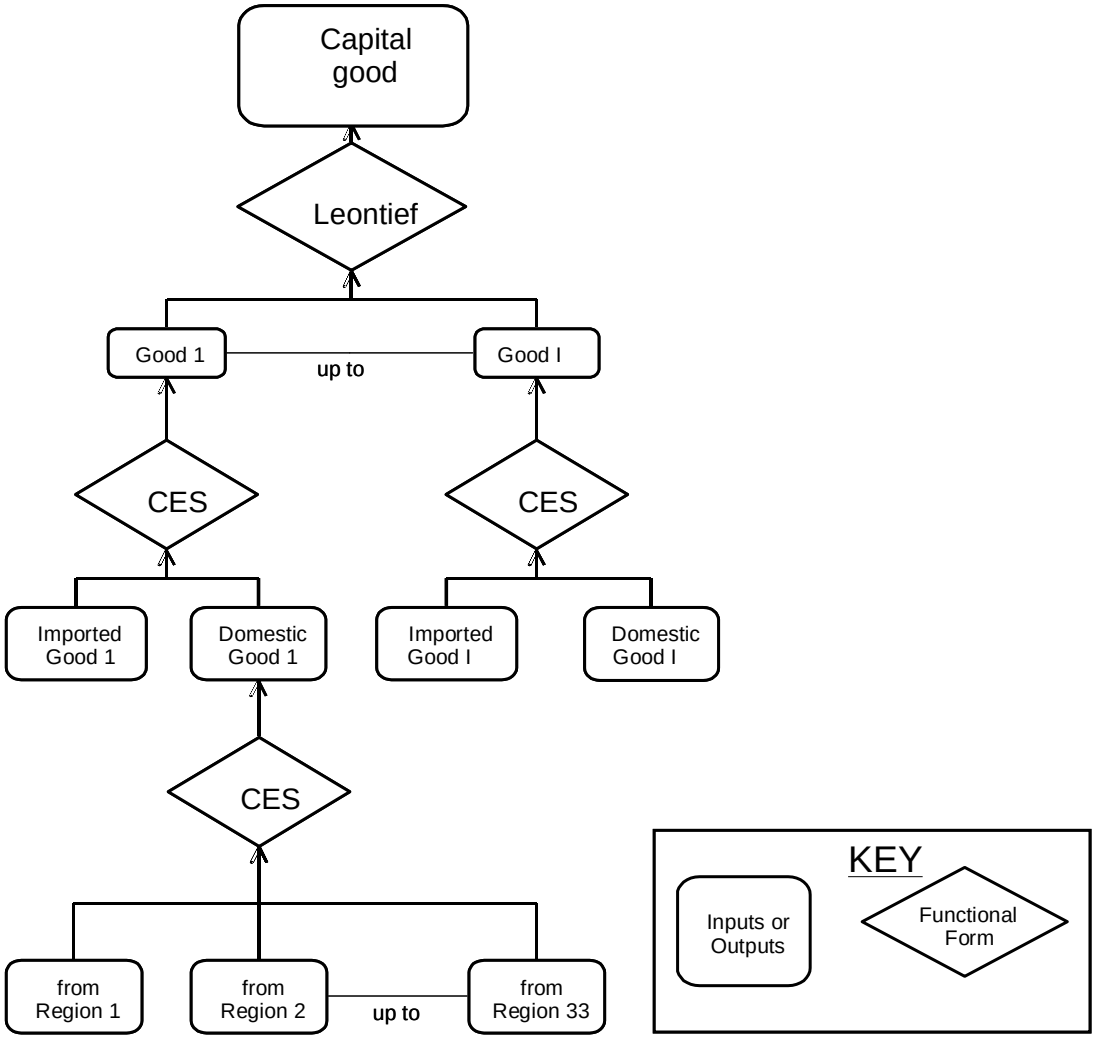
Investment demand



Investment demand

```
E_x2a # Demand for goods by source, User 2 #
(all,i,COM)(all,s,ALLSOURCE)(all,j,IND)(all,q,REGDEST)
x2a(i,s,j,q)=IS_DOM(s)*(x2c(i,j,q)-SIGMA2C(i)*(p2a(i,s,j,q)-
p2c(i,j,q)))
+IS_IMP(s)*(x2o(i,j,q)-SIGMA2O(i)*(p2a(i,"foreign",j,q)-
p2o(i,j,q)));

E_p2o # Price of domestic/foreign composite, User 2 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
(TINY+PVAL2O(i,j,q))*p2o(i,j,q)=sum(s,ALLSOURCE,PVAL2A(i,s,j,q)
*p2a(i,s,j,q));

E_p2c # Price of domestic composite, User 2 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
(TINY+PVAL2T(i,"domestic",j,q))*p2c(i,j,q)
=sum(s,REGSOURCE,PVAL2A(i,s,j,q)*p2a(i,s,j,q));
```

Investment demand

```
E_x2c # Demand for domestic composite, User 2 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
x2c(i,j,q)=x2o(i,j,q)-SIGMA2O(i)*(p2c(i,j,q)-p2o(i,j,q));

E_x2o # Demands for domestic/foreign composite, User 2 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)
x2o(i,j,q)=y(j,q)+a2ind(j,q);
```

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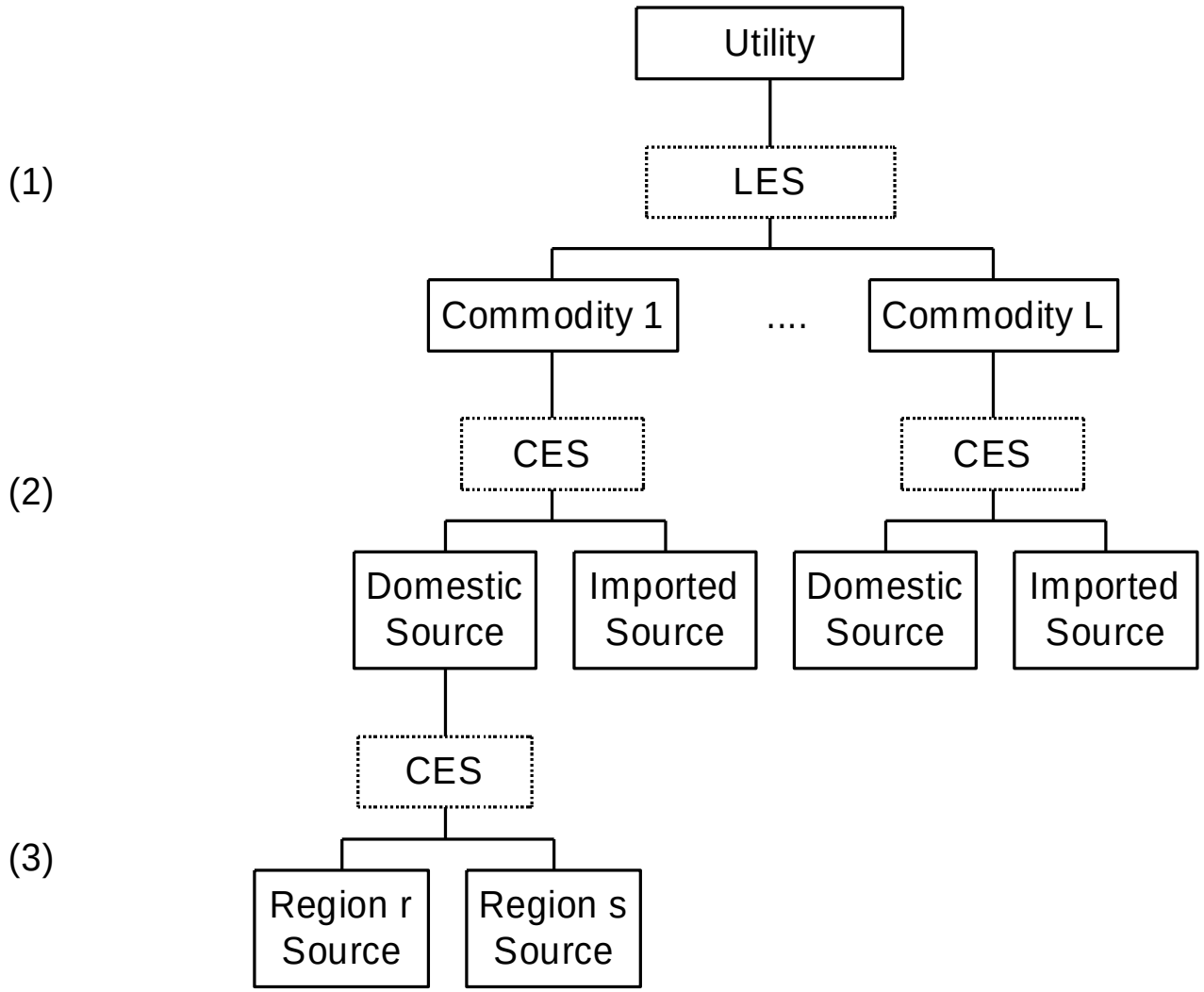
Household demand



Household demand

Each regional household determines optimal consumption bundle by maximizing a Stone-Geary utility function subject to a budget constraint

A Keynesian-type consumption function determines aggregate regional household expenditure

Household demand

```
E_x3o # Household demand for composite commodities #
(all,i,COM)(all,q,REGDEST)
x3o(i,q)=[1-ALPHA_I(i,q)]*[qhous(q)+a3sub(i,q)]
+ALPHA_I(i,q)*[luxexp(q)+a3lux(i,q)-p3o(i,q)];

E_a3lux # Default setting for luxury taste shifter #
(all,i,COM)(all,q,REGDEST)
a3lux(i,q)=a3sub(i,q)-sum(k,COM,DELTA(k,q)*a3sub(k,q));

E_a3sub # Default setting for subsistence taste shifter #
(all,i,COM)(all,q,REGDEST)
a3sub(i,q)=a3com(i,q)-sum(k,COM,S3COM(k,q)*a3com(k,q));

E_utility # Change in utility disregarding taste change terms #
(all,q,REGDEST)
utility(q)=luxexp(q)-qhous(q)-sum(i,COM,DELTA(i,q)*p3o(i,q));
```

Household demand

```
E_x3a # Demand for goods by source, User 3 #
(all,i,COM)(all,s,ALLSOURCE)(all,q,REGDEST)
x3a(i,s,q)=IS_DOM(s)*(x3c(i,q)-SIGMA3C(i)*(p3a(i,s,q)-p3c(i,q)))
+IS_IMP(s)*(x3o(i,q)-SIGMA3O(i)*(p3a(i,"foreign",q)-p3o(i,q)));

E_p3o # Price of domestic/foreign composite, User 3 #
(all,i,COM)(all,q,REGDEST)
(TINY+PVAL3O(i,q))*p3o(i,q)=sum(s,ALLSOURCE,PVAL3A(i,s,q)*p3a(i,s,q));

E_p3c # Price of domestic composite, User 3 #
(all,i,COM)(all,q,REGDEST)
(TINY+PVAL3T(i,"domestic",q))*p3c(i,q)
=sum(s,REGSOURCE,PVAL3A(i,s,q)*p3a(i,s,q));

E_x3c # Demand for domestic composite, User 3 #
(all,i,COM)(all,q,REGDEST)
x3c(i,q)=x3o(i,q)-SIGMA3O(i)*(p3c(i,q)-p3o(i,q));
```

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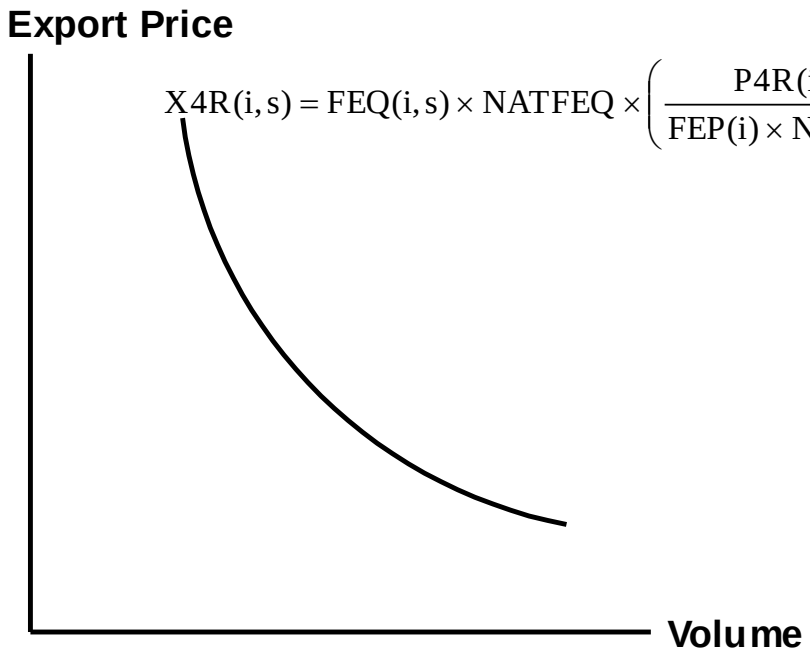
Foreign export demand



$$X4R(i,s) = FEQ(i,s) \times NATFEQ \times \left(\frac{P4R(i,s)}{FEP(i) \times NATFEP} \right)^{EXP\_ELAST(i)}$$

Export commodities face individual downward-sloping foreign export demand functions

Exports of product i from source s are distinguished from exports of i from source r (r not equal s)

Foreign export demand

```
E_x4r # Export demand functions #
(all,i,TEXP)(all,s,REGSOURCE)
x4r(i,s)-feq(i)=EXP_ELAST(i)*[p4r(i,s)-fep(i)-natfep];

E_aggnt_x4r # Export demand functions, non-trad aggregate #
(all,s,REGSOURCE)
aggnt_x4r(s)-aggnt_feq(s)=EXP_ELAST("OP0")*[aggnt_p4r(s)-
aggnt_fep(s)-natfep];

E_nt_x4r # Export demand functions, non-trad #
(all,i,NTEXP)(all,s,REGSOURCE)
x4r(i,s)=aggnt_x4r(s)+faggnt_i(i)+faggnt_s(s)+faggnt_is(i,s);

E_aggnt_p4r # Export price, non-trad aggregate #
(all,s,REGSOURCE)
AGGEXPNT(s)*aggnt_p4r(s)=sum(i,NTEXP,PVAL4r(i,s)*p4r(i,s))+fagg
nt_p4r(s);
```

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Government demand

Recognise regional governments and Federal government demands for goods and services for current consumption (not properly calibrated yet!)

Neither modelled explicitly

Default:

- aggregate regional government demand in region q moves with regional government revenue, with structure of demand exogenous
- aggregate Federal government demand in region q moves with national government revenue, with structure of demand exogenous

Government demand

```
E_x5a # Regional government demand #
(all,i,COM)(all,s,ALLSOURCE)(all,q,REGDEST)
x5a(i,s,q)=f5a(i,s,q)+f5gen(q)+natf5gen+taxind(q)-xi5(q);

E_x6a # Federal government demand #
(all,i,COM)(all,s,ALLSOURCE)(all,q,REGDEST)
x6a(i,s,q)=f6a(i,s,q)+f6gen(q)+natf6gen+nattaxind-natxi6;
```

Building blocks

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Demand for margin (transportation) services*

Margins commodities (identified in the set MARGCOM) provide freight services

- these commodities are consumed directly and used indirectly to facilitate the movements of products
- latter type of use is margins demand

Margins demand for margin commodity i is assumed to be proportional to the volume of the underlying flow

- e.g., margins use of transportation services in taking agricultural products to manufacturing is modelled as proportional to the volume of agricultural product used in manufacturing

Demand for margin (transportation) services

```
E_x1marg # Margins on sales to producers #
(all,i,COM)(all,j,IND)(all,q,REGDEST)(all,s,ALLSOURCE)(all,r,MARGCOM)
x1marg(i,s,j,q,r)=THETA(i,s,q)*x1a(i,s,j,q)+a1marg_ij(s,q,r)+amarg_i(s,
q,r);

E_x2marg # Margins on sales to capital creators #
(all,i,COM)(all,j,IND)(all,q,REGDEST)(all,s,ALLSOURCE)(all,r,MARGCOM)
x2marg(i,s,j,q,r)=THETA(i,s,q)*x2a(i,s,j,q)+a2marg_ij(s,q,r)+amarg_i(s,
q,r);

E_x3marg # Margins on sales to household consumption #
(all,i,COM)(all,s,ALLSOURCE)(all,q,REGDEST)(all,r,MARGCOM)
x3marg(i,s,q,r)=THETA(i,s,q)*x3a(i,s,q)+a3marg_i(s,q,r)+amarg_i(s,q,r);

E_x4marg # Margins on exports: factory gate to port #
(all,i,COM)(all,r,MARGCOM)(all,s,REGSOURCE)
x4marg(i,s,r)=x4r(i,s)+a4marg_i(s,r);
```

Progress so far through the core...

Done

- All demand equations

To do

- Zero pure profits
- Indirect tax equations
- Market-clearing
- Regional and national macroeconomic variables and price indexes

Building blocks

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Zero pure profits

Critical assumptions

- no pure profits in the production or distribution of commodities
- price received by the producer is uniform across all customers

Zero pure profits in current production imposed by setting unit prices received by producers equal to unit costs

Zero pure profits in distribution imposed by setting the prices paid by users equal to producer price plus commodity tax plus margins

Zero pure profits

```
E_p0a # Zero pure profits in current production #
(all,j,IND)(all,q,REGDEST)
(TINY+COSTS(j,q))*{p0a(j,q)-a(j,q)}=
sum(i,COM,sum(s,ALLSOURCE,PVAL1A(i,s,j,q)*p1a(i,s,j,q)))
+sum(m,OCC,LAB_OCC_IND(m,j,q)*p1laboi(j,q,m))
+CAPITAL(j,q)*p1cap(j,q)+LAND(j,q)*p1land(j,q)
+OTHCOST(j,q)*p1oct(j,q);

E_pi # Zero pure profits in capital creation #
(all,j,IND)(all,q,REGDEST)
(TINY+INVEST(j,q))*(pi(j,q)-a2ind(j,q))=
sum(i,COM,sum(s,ALLSOURCE,PVAL2A(i,s,j,q)*p2a(i,s,j,q)));

E_p0ab # Zero pure profits in importing #
(all,i,COM)
p0a(i,"foreign")=pm(i)+natphi+powtaxm(i);
```

Zero pure profits

```
E_p1a # Purchasers prices - User 1 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)(all,s,ALLSOURCE)
(TINY+PVAL1A(i,s,j,q))*p1a(i,s,j,q)
=[BAS1(i,s,j,q)+TAX1(i,s,j,q)]*p0a(i,s)
+BAS1(i,s,j,q)*deltax1(i,s,j,q)+sum(r,MARGCOM,MAR1(i,s,j,q,r)*
(p0a(r,q)+a1marg_ij(s,q,r)+amarg_i(s,q,r)));

E_p2a # Purchasers prices - User 2 #
(all,i,COM)(all,j,IND)(all,q,REGDEST)(all,s,ALLSOURCE)
(TINY+PVAL2A(i,s,j,q))*p2a(i,s,j,q)=[BAS2(i,s,j,q)+TAX2(i,s,j,q)]*p0a
(i,s)
+BAS2(i,s,j,q)*deltax2(i,s,j,q)+sum(r,MARGCOM,MAR2(i,s,j,q,r)*
(p0a(r,q)+a2marg_ij(s,q,r)+amarg_i(s,q,r)));

E_p3a # Purchasers prices - User 3 #
(all,i,COM)(all,q,REGDEST)(all,s,ALLSOURCE)
(TINY+PVAL3A(i,s,q))*p3a(i,s,q)=[BAS3(i,s,q)+TAX3(i,s,q)]*p0a(i,s)
+BAS3(i,s,q)*deltax3(i,s,q)+sum(r,MARGCOM,MAR3(i,s,q,r)*
(p0a(r,q)+a3marg_i(s,q,r)+amarg_i(s,q,r)));
```

Zero pure profits

```
E_p4r # Purchasers prices - User 4 #
(all,i,COM)(all,s,REGSOURCE)
(TINY+PVAL4R(i,s))*(natphi+p4r(i,s)) =[BAS4(i,s)+TAX4(i,s)]*p0a(i,s)
+BAS4(i,s)*deltax4(i,s)+sum(r,MARGCOM,MAR4(i,s,r)*
(p0a(r,s)+a4marg_i(s,r)));

E_p5a # Purchasers prices - User 5 #
(all,i,COM)(all,q,REGDEST)(all,s,ALLSOURCE)
(TINY+PVAL5A(i,s,q))*p5a(i,s,q)=[BAS5(i,s,q)+TAX5(i,s,q)]*p0a(i,s)
+BAS5(i,s,q)*deltax5(i,s,q)+sum(r,MARGCOM,MAR5(i,s,q,r)*
(p0a(r,q)+a5marg_i(s,q,r)+amarg_i(s,q,r)));

E_p6a # Purchasers prices - User 6 #
(all,i,COM)(all,s,ALLSOURCE) (all,q,REGDEST)
(TINY+PVAL6A(i,s,q))*p6a(i,s,q)=[BAS6(i,s,q)+TAX6(i,s,q)]*p0a(i,s)
+BAS6(i,s,q)*deltax6(i,s,q)+sum(r,MARGCOM,MAR6(i,s,q,r)*
(p0a(r,s)+a6marg_i(s,q,r)+amarg_i(s,q,r)));
```

Building blocks

- ✓ Producer's demands for inputs
- ✓ Investor demands
- ✓ Household demands
- ✓ Export demands
- ✓ Government demands
- ✓ Margins demands
- ✓ Zero pure profits
- ✓ **Indirect tax equations**
- ✓ Market-clearing
- ✓ Regional and national macroeconomic variables and price indexes
- ✓ Capital accumulation and investment
- ✓ Regional population and labor market

Indirect taxes

Equations have been added to enable flexible handling of indirect taxes on all flows of goods and services

Equations allow for variations in tax rates across commodities, their sources and destinations

Indirect taxes

```
E_deltax1 # Tax rate on sales to User 1 #
(all,i,COM)(all,s,ALLSOURCE)(all,j,IND)(all,q,REGDEST)
deltax1(i,s,j,q)=deltax(i)+deltax1all+deltaxsource(s)+deltaxdest(q);

E_deltax2 # Tax rate on sales to User 2 #
(all,i,COM)(all,s,ALLSOURCE)(all,j,IND)(all,q,REGDEST)
deltax2(i,s,j,q)=deltax(i)+deltax2all+deltaxsource(s)+deltaxdest(q);

E_deltax3 # Tax rate on sales to User 3 #
(all,i,COM)(all,s,ALLSOURCE)(all,q,REGDEST)
deltax3(i,s,q)=deltax(i)+deltax3all+deltaxsource(s)+deltaxdest(q);

E_deltax4 # Tax rate on sales to User 4 #
(all,i,COM)(all,s,REGSOURCE)
deltax4(i,s)=deltax(i)+deltax4all+deltaxsource(s)
+deltaxdest("foreign");
```

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Market-clearing

Equations that impose market clearing (demand equals supply) for:

- domestically produced margin and non-margin commodities
- imported commodities

Market-clearing

```
E_mkt_clear_margins # Demand equals supply for margin commodities #
(all,r,MARGCOM)(all,s,REGSOURCE)
(TINY+SALES(r,s))*z(r,s)=sum(j,IND,sum(q,REGDEST,BAS1(r,s,j,q)*x1a(r,s,j,q)
+BAS2(r,s,j,q)*x2a(r,s,j,q)))+sum(q,REGDEST,BAS3(r,s,q)*x3a(r,s,q))
+BAS4(r,s)*x4r(r,s)+sum(q,REGDEST,BAS5(r,s,q)*x5a(r,s,q))
+sum(q,REGDEST,BAS6(r,s,q)*x6a(r,s,q))
+sum(j,IND,sum(i,COM,sum(ss,ALLSOURCE,MAR1(i,ss,j,s,r)*x1marg(i,ss,j,s,r)
+MAR2(i,ss,j,s,r)*x2marg(i,ss,j,s,r))))
+sum(i,COM, sum(ss,ALLSOURCE,MAR3(i,ss,s,r)*x3marg(i,ss,s,r)))
+sum(i,COM,MAR4(i,s,r)*x4marg(i,s,r))
+sum(i,COM,sum(ss,ALLSOURCE,MAR5(i,ss,s,r)*x5marg(i,ss,s,r)))
+sum(i,COM,sum(ss,ALLSOURCE,MAR6(i,ss,s,r)*x6marg(i,ss,s,r)));
```

Market-clearing

```
E_mkt_clear_nomarg # Demand equals supply for non-margin commodities #
(all,r,NONMARGCOM)(all,s,REGSOURCE)
(TINY+SALES(r,s))*z(r,s)=sum(j,IND,sum(q,REGDEST,BAS1(r,s,j,q)*x1a(r,s,j,q)))
+sum(j,IND,sum(q,REGDEST,BAS2(r,s,j,q)*x2a(r,s,j,q)))
+sum(q,REGDEST,BAS3(r,s,q)*x3a(r,s,q))+BAS4(r,s)*x4r(r,s)
+sum(q,REGDEST,BAS5(r,s,q)*x5a(r,s,q))+sum(q,REGDEST,BAS6(r,s,q)*x6a(r,s,q));

E_x0impa # Import volume of commodities by region #
(all,i,COM)(all,q,REGDEST)
(TINY+IMPORTS(i,q))*x0imp(i,q)=
sum(j,IND,BAS1(i,"foreign",j,q)*x1a(i,"foreign",j,q)
+BAS2(i,"foreign",j,q)*x2a(i,"foreign",j,q))
+BAS3(i,"foreign",q)*x3a(i,"foreign",q)
+BAS5(i,"foreign",q)*x5a(i,"foreign",q)+BAS6(i,"foreign",q)*x6a(i,"foreign",q);
```

Building blocks

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Macro aggregates

Wide range of national and regional macro variables defined…

Two concepts of the real wage rate

- consumer real wage rate (PLAB/CPI)
- producer real wage rate (PLAB/PGDP)

Progress so far...

Done

- Core CGE equations relating to demand, supply, prices, indirect taxes, market-clearing and summary macro variables

To do

- Capital accumulation and investment
- Regional population and labor market

Building blocks

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Investment "dynamics"

Capital, investment and expected rates of return

$$K_{j,q}(t+1) = (1 - DEP_{j,q}) \times K_{j,q}(t) + Y_{j,q}(t)$$

Given starting point for capital (t=0) and an explanation of investment, we can trace out time path for capital

Investment "dynamics"

Investment explained by assuming that:

$$\frac{K_{j,q}(t+1)}{K_{j,q}(t)} - 1 = F_{j,q}^{t}[EROR_{j,q}(t)]$$

Growth in capital related to expected rate of return

- In BMMX ICGE only assume static expectations, though rational is possible

Rates of return and investment

For static expectations case, the actual rate of return is:

$$RO_t(j,q) = \frac{P_t(j,q)}{\Pi_t(j,q)} - D(j,q)$$

$$ro(j,q) = p_t(j,q) - \pi_t(j,q)$$

$$ro(j,q) = QCOEF\ (j,q)\left[p_t(j,q) - \pi_t(j,q)\right]$$

QCOEF: relação entre taxa bruta e taxa líquida de retorno (> 1)

Rates of return and investment

In long-run comparative-static simulations:

- aggregate capital adjusts to maintain R_{INT} (*natr_tot*)
- capital allocated in line with equation **E_f_rate_xx**
  - industries with relatively large increases in capital require relatively high rates of return
  - industries with relatively small increases in capital require relatively low rates of return
- industry investment determined by fixed ratios of investment to capital (equation **E_y**)

Rates of return and investment

Equalization in the rates of return

$$\left(\frac{K(j,q)}{K(q)}\right)^{-\beta(j,q)} RO(j,q) = R_{\text{int}}$$

$$ro(j,q) - r_{\text{int}} = \beta_t(j,q)\left[k(j,q) - k(q)\right] + f\_rate(j,q)$$

beta: risk/return ratio

Short-run: *f_rate* endogenous, *k* exogenous

Long-run: *f_rate* exogenous, *k* endogenous

Investment "dynamics"

Growth rate of capital stocks and investment in the short-run:

$$k_{t+1}(j,q) - k_t(j,q) = 0$$ % change in capital stocks

$$y_t(j,q) = 0$$ % change in investment

Investment "dynamics"

Growth rate of capital stocks and investment in the long-run:

$$\frac{K_{j,q}(t+1)}{K_{j,q}(t)} = \left(\frac{K_{j,q}(t)}{K_{j,q}(0)} \right)^{1/T}$$

$$k_{t+1}(j,q) = \left(1 + \frac{1}{T} \right) k_t(j,q)$$

Investment in the short run

Fixed capital stocks in the base year values:

- *curcap(j,q)* exogenous (=0)
- relationship between sectoral rates of return, *r0(j,q)*, and reference interest rate, *natr_tot*, is endogenous (*f_rate_xx(j,q)* endogenous)

Percentage change in sectoral investment, *y(j,q)* is zero; this can be guaranteed by setting the shift term, *delf_rate(j,q)*, exogenous and zero

By hypothesis, not only the capital stocks are fixed but also firms' investment plans

Investment in the short run

```
E_r0 # Definition of rates of return to capital #
r0(j,q) = QCOEF(j,q)*(p1cap(j,q) - pi(j,q));

E_f_rate_xx # Capital growth rates related to rates of
return #
(r0(j,q) - natr_tot) = BETA_R(j,q)*[curcap(j,q) -
kt(q)] + f_rate_xx(j,q);

E_curcapT1 # Capital stock in period T+1 #
  curcap_t1(j,q) - curcap(j,q) =0;

E_yT # Investment in period T #
curcap(j,q) - y(j,q)- 100*delf_rate(j,q)=0;
```

endog. exog.

Investment in the long run

Capital stocks endogenously determined:

- *curcap(j,q)* endogenous
- relationship between sectoral rates of return,, *r0(j,q)*, and reference interest rate, *natr_tot*, is given (*f_rate_xx(j,q)* exogenous)

Percentage change in sectoral investment, *y(j,q)* is endogenous

Firms' investment plans are carried out, reestablishing returns differentials in the base year

Rate of capital accumulation, but **not the level** of capital stock, remains constant

Investment in the long run

```
E_r0 # Definition of rates of return to capital #
r0(j,q) = QCOEF(j,q)*(p1cap(j,q) - pi(j,q));

E_f_rate_xx # Capital growth rates related to rates of
return #
(r0(j,q) - natr_tot) = BETA_R(j,q)*[curcap(j,q) - kt(q)] +
f_rate_xx(j,q);

E_curcapT1 # Capital stock in period T+1 #
curcap_t1(j,q) - K_TERM*curcap(j,q)=0;

E_yT # Investment in period T #
VALK_T1(j,q)*curcap_t1(j,q)= VALKT(j,q)*DEP(j)*curcap(j,q)
+(INVEST(j,q))*y(j,q)-100*(VALK_0(j,q)*(1-DEP(j))
```

endog. exog.

(DEP(j) = 0.96)

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Regional population and labor market

Critical variables:

- regional population
- regional migration
- regional unemployment
- regional participation rates
- regional wage relativities

Various closures

Regional population and labor market

(1) Fixed
- wage relativities (determining employment by region), participation and unemployment rates (determining population by region)

(1) Endogenous
- regional migration

(2) Fixed
- **regional migration, participation rates, wage relativities**

(2) Endogenous
- **unemployment rates**

(3) Fixed
- regional migration, participation and unemployment rates

(3) Endogenous
- wage relativities

Labor market in the short-run

```
E_wage_diff # Region real-wage diff #(all,q,REGDEST)

wage_diff(q) = pwage(q) - natxi3 - natrealwage;

E_del_labsup # P-point changes in regional
unemployment rates #(all,q,REGDEST)

C_labsup(q)*del_unr(q)=C_EMPLOY(q)*(labsup(q)-
                                    employ(q));
```

del_unr(q) # *Percentage-point changes in regional unemployment rate* #;

endog. exog.

Labor market in the long-run

```
E_wage_diff # Region real-wage diff #(all,q,REGDEST)

wage_diff(q) = pwage(q) - natxi3 - natrealwage;

E_del_labsup # P-point changes in regional
unemployment rates #(all,q,REGDEST)

C_labsup(q)*del_unr(q)=C_EMPLOY(q)*(labsup(q)-
                                    employ(q));
```

del_unr(q) # *Percentage-point changes in regional unemployment rate* #;

endog. exog.

Closures

Each equation explains a variable

More variables than equations

Endogenous variables: explained by model

Exogenous variables: set by user

Closure: choice of exogenous variables

Many possible closures

Number of endogenous variables = Number of equations

Length of run, T

T is related to our choice of closure

With short-run closure we assume that:

- T is long enough for price changes to be transmitted throughout the economy, and for price-induced substitution to take place
- T is not long enough for investment decisions to greatly affect the useful size of sectoral capital stocks [new buildings and equipment take time to produce and install]

T might be 2 years. So results mean:

- A 10% consumption increase might lead to employment in 2 years time being 1.2% higher than it would be (in 2 years time) if the consumption increase did not occur

Different closures

Many closures might be used for different purposes

No unique natural or correct closure

Must be at least one exogenous variable measured in local currency units

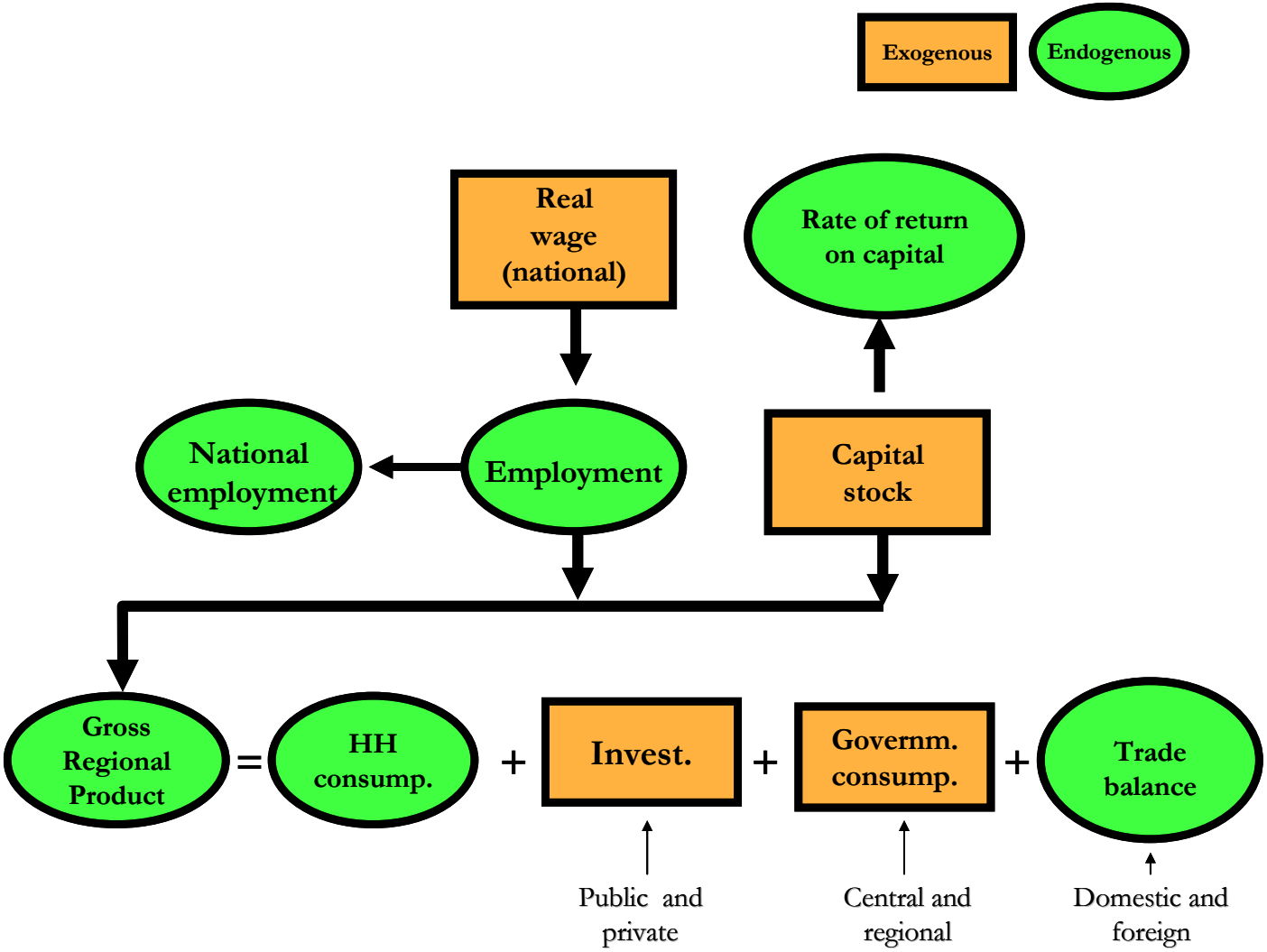
Normally just one — called the *numéraire*

Often the exchange rate, *natphi*, or *natxi3*, the CPI.

Some quantity variables must be exogenous, such as:

- primary factor endowments
- final demand aggregates

Short-run environment



Long-run environment