Lecture 3: The BMMX ICGE Model

“Multi-regional Economic Modeling: Applications for Mexico”

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Outline

✓ What is the BMMX ICGE model?

Building blocks
A multi-sectoral, multi-regional bottom-up CGE model of Mexico’s 32 regions

- each region is modeled as an economy in its own right
- region-specific prices
- region-specific industries
- region-specific consumers

Based on the comparative-static B-MARIA and MMRF models

Database makes allowance for interregional, intra-regional and international trade

- Potential for the representation of regional and Federal government financial accounts
Stylized flows
## Embedded SAM

<table>
<thead>
<tr>
<th>REGION 1</th>
<th>REGION R</th>
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<tbody>
<tr>
<td>FACTORS</td>
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<td>AGENTS</td>
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<td>SECTORS</td>
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<td>TOTAL</td>
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</table>

**Factors**: $H, G$

**Agents**: $W'^1$

**Regions**: $W'^R$

**Local Products**: $Ch^1, Crg^1, CI^1$

**Composite Products (Demand)**: $Ch^R, Crg^R, CI^R$

**Accumulation**: $Sh^1, Seg^1$

**Row**: $M$

**Financial/Asset Adjustment**: $FAh^1, FArg^1$

**Total**: $W^1, Yh^1, Yrg^1, PCT^1, Ppm^1, O^1$

### Tables

<table>
<thead>
<tr>
<th>FACTORS</th>
<th>AGENTS</th>
<th>SECTORS</th>
<th>LOCAL PRODUCTS</th>
<th>COMPOSITE PRODUCTS (SUPPLY)</th>
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<td>COMPOSITE PRODUCTS (DEMAND)</td>
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<td>Sh^1</td>
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<td>M</td>
<td>Yrw</td>
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<td>FINANCIAL/ASSET ADJUSTMENT</td>
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<td>W^1</td>
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The Role of Transportation Services in BMMX*

![Diagram showing the role of transportation services in BMMX. The diagram includes nodes for International trade balance, Imports, Exports, Interregional trade balance, Local goods, Composite intermediate inputs, Composite capital goods, and Composite consumption goods. The diagram illustrates the flow of goods between Region A and Region B, with transportation services depicted on the edges.]}
### Core database

**ABSORPTION MATRIX**

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<td>J x Q</td>
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<td>J x Q</td>
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<td>Export</td>
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<tr>
<td>Other</td>
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I = number of commodities  
J = number of industries  
R = number of commodities used as margins  
Q = number of regions  
S = Q domestic regions + 1 foreign import
Features of database

Commodity flows are valued at “basic prices” ($\text{BAS}$):
- do not include user-specific taxes or margins

For each user of each imported good and each domestic good, there are numbers showing:
- tax levied on that usage ($\text{TAX}$)
- usage of margins – transport ($\text{MAR}$)*

Single-production:
- each commodity may be produced by one industry
- each industry may produce one commodity

For each industry the total cost of production is equal to the total value of output

For each commodity the total value of sales is equal to the total value of output
Features of database (cont.)

Domestic producers
  • J industries in Q regions

Investors
  • J industries in Q regions

Households
  • one representative household for each of the Q regions

Each of the I commodity types can be obtained from the region, from other regions, or imported from overseas
Features of database (cont.)

Aggregate foreign purchaser of exports

Other demand category corresponding to the Q regional governments

Other demand category corresponding to the central government in the Q regions

Commodity taxes and margins explicitly recognised
Notation

Main User Numbers
1 ↔ firms, current production;
2 ↔ firms, capital creation;
3 ↔ households;
4 ↔ foreign exports;
5 ↔ regional government;
6 ↔ Central government;
The number 0 is also used to denote basic prices and values.

Source dimensions
a ↔ all sources, i.e., 32 regional sources and 1 foreign;
r ↔ regional sources only;
t ↔ two sources, i.e., a domestic composite source and foreign;
c ↔ domestic composite source only;
o ↔ domestic-foreign composite source only.
What is the BMMX ICGE Model?

✓ Building blocks
Building blocks

- **Producer’s demands for inputs**
  - Investor demands
  - Household demands
  - Export demands
  - Government demands
  - Margins demands
  - Zero pure profits
  - Indirect tax equations
  - Market-clearing
  - Regional and national macroeconomic variables and price indexes
  - Capital accumulation and investment
  - Regional population and labor market
Production nest

1. Leontief
   - Intermediate Inputs
   - Primary Factors
   - Other Costs

2. CES
   - Domestic Source
   - Imported Source
   - Labor
   - Capital

3. CES
   - Region r Source
   - Region s Source

Work upwards
Demand for intermediate inputs, other costs and prices

\[ \text{E}_{x1a} \quad \text{# Demand for goods by all sources, User 1} \#
\]
\[ (\text{all},i,\text{COM})(\text{all},s,\text{ALLSOURCE})(\text{all},j,\text{IND})(\text{all},q,\text{REGDEST}) \]
\[ x1a(i,s,j,q)=\text{IS\_DOM}(s)*(x1c(i,j,q) - \text{SIGMA1C}(i)*(\text{plc}(i,s,j,q) - \text{plc}(i,j,q))) + \text{IS\_IMP}(s)*(\text{xlo}(i,j,q) - \text{SIGMA1O}(i)*(\text{pl}(i,\text{"foreign"},j,q) - \text{plo}(i,j,q))) ; \]

\[ \text{E}_{p1o} \quad \text{# Price of domestic/foreign composite, User 1} \#
\]
\[ (\text{all},i,\text{COM})(\text{all},j,\text{IND})(\text{all},q,\text{REGDEST}) \]
\[ (\text{TINY}+\text{PVAL1O}(i,j,q))*\text{plo}(i,j,q) = \text{sum}(s,\text{ALLSOURCE},\text{PVAL1A}(i,s,j,q)*\text{pl}(i,s,j,q)); \]

\[ \text{E}_{p1c} \quad \text{# Price of domestic composite, User 1} \#
\]
\[ (\text{all},i,\text{COM})(\text{all},j,\text{IND})(\text{all},q,\text{REGDEST}) \]
\[ (\text{TINY}+\text{PVAL1T}(i,\text{"domestic"},j,q))*\text{plc}(i,j,q) = \text{sum}(s,\text{REGSOURCE},\text{PVAL1A}(i,s,j,q)*\text{pl}(i,s,j,q)); \]
Demand for intermediate inputs, other costs and prices

\[ E_{x1c} \quad \# \text{Demand for domestic composite, User 1} \# \]
\[(\text{all}, i, \text{COM}) (\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST})\]
\[x_{1c}(i, j, q) = x_{1o}(i, j, q) - \Sigma 1 O(i) \times (p_{1c}(i, j, q) - p_{1o}(i, j, q));\]

\[ E_{x1o} \quad \# \text{Demand for dom./for. composite inputs, User 1} \# \]
\[(\text{all}, i, \text{COM}) (\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST})\]
\[x_{1o}(i, j, q) = z(j, q) + a_1(j, q);\]

\[ E_{x1oct} \quad \# \text{Industry demand for other cost tickets} \# \]
\[(\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST})\]
\[x_{1oct}(j, q) = z(j, q) + a_1(j, q) + a_{1oct}(j, q);\]

\[ E_{p1oct} \quad \# \text{Indexing of prices of other cost tickets} \# \]
\[(\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST})\]
\[p_{1oct}(j, q) = x_3(i, q) + f_{1oct}(j, q);\]
Demand for primary factors

\[
E_{\text{efflab}} \# \text{Industry demand for effective labor} \# \\
(\text{all}, \text{j}, \text{IND}) (\text{all}, \text{q}, \text{REGDEST}) \\
efflab(j, q) = \text{MRL}(j, q) \times x1prim(j, q) + a1lab(j, q) \\
- \text{SIGMA1FAC}(j, q) \times [p1lab(j, q) + a1lab(j, q) - \text{xi_fac}(j, q)];
\]

\[
E_{\text{curcap}} \# \text{Industry demand for capital} \# \\
(\text{all}, \text{j}, \text{IND}) (\text{all}, \text{q}, \text{REGDEST}) \\
curcap(j, q) = \text{MRK}(j, q) \times x1prim(j, q) + a1cap(j, q) \\
- \text{SIGMA1FAC}(j, q) \times [p1cap(j, q) + a1cap(j, q) - \text{xi_fac}(j, q)] + \text{IL2}(j, q) \times \text{interest};
\]

\[
E_{\text{n}} \# \text{Industry demand for land} \# \\
(\text{all}, \text{j}, \text{IND}) (\text{all}, \text{q}, \text{REGDEST}) \\
n(j, q) = \text{MRN}(j, q) \times x1prim(j, q) + a1land(j, q) \\
- \text{SIGMA1FAC}(j, q) \times [p1land(j, q) + a1land(j, q) - \text{xi_fac}(j, q)];
\]

\[
E_{\text{xi_fac}} \# \text{Effective price term for factor demand equations} \# \\
(\text{all}, \text{j}, \text{IND}) (\text{all}, \text{q}, \text{REGDEST}) \\
(TINY + \text{TOTFACIND}(j, q)) \times \text{xi_fac}(j, q) = \text{LABOR}(j, q) \times (p1lab(j, q) + a1lab(j, q)) \\
+ \text{CAPITAL}(j, q) \times (p1cap(j, q) + a1cap(j, q)) + \text{LAND}(j, q) \times (p1land(j, q) + a1land(j, q));
\]
Demand for primary factors

\( E_{x1laboi} \) # Demand for labor by industry and skill group # 
\( (all, m, OCC) (all, j, IND) (all, q, REGDEST) \) 
\[ x1laboi(j, q, m) = eflab(j, q) - \text{SIGMA1LAB}(j, q) \times [p1laboi(j, q, m) - p1lab(j, q)] \]
\[ + IL(m, j, q) \times \text{interest}; \]

\( E_{p1lab} \) # Price to each industry of labor in general # 
\( (all, j, IND) (all, q, REGDEST) \) 
\[ (TINY + LABOR(j, q)) \times p1lab(j, q) = \text{sum}(m, OCC, LAB_OCC_IND(m, j, q) \times p1laboi(j, q, m)); \]

\( E_{labind} \) # Employment by industry # 
\( (all, j, IND) (all, q, REGDEST) \) 
\[ (TINY + LABOR(j, q)) \times labind(j, q) = \text{sum}(m, OCC, LAB_OCC_IND(m, j, q) \times x1laboi(j, q, m)); \]

\( E_{x1prim} \) # Demand for the primary-factor composite # 
\( (all, j, IND) (all, q, REGDEST) \) 
\[ x1prim(j, q) = MRP(j, q) \times z(j, q) + a1(j, q) + a1prim(j, q); \]
Building blocks

- Producer’s demands for inputs
- **Investor demands**
- Household demands
- Export demands
- Government demands
- Margins demands
- Zero pure profits
- Indirect tax equations
- Market-clearing
- Regional and national macroeconomic variables and price indexes
- Capital accumulation and investment
- Regional population and labor market
Investment demand

Capital good

Leontief

Good 1

up to

Good I

CES

CES

Imported Good 1

Domestic Good 1

Imported Good I

Domestic Good I

CES

from Region 1

from Region 2

up to

from Region 33

KEY

Inputs or Outputs

Functional Form
Investment demand

\[
E_{x2a} \quad \text{# Demand for goods by source, User 2 #}
\]
\[
(all, i, COM) (all, s, ALLSOURCE) (all, j, IND) (all, q, REGDEST)
\]
\[
x2a(i, s, j, q) = IS_{DOM}(s) \times (x2c(i, j, q) - SIGMA2C(i) \times (p2a(i, s, j, q) - p2c(i, j, q)))
\]
\[
+ IS_{IMP}(s) \times (x2o(i, j, q) - SIGMA2O(i) \times (p2a(i, "foreign", j, q) - p2o(i, j, q)))
\]

\[
E_{p2o} \quad \text{# Price of domestic/foreign composite, User 2 #}
\]
\[
(all, i, COM) (all, j, IND) (all, q, REGDEST)
\]
\[
(TINY + PVAL2O(i, j, q)) \times p2o(i, j, q) = \text{sum}(s, ALLSOURCE, PVAL2A(i, s, j, q) \times p2a(i, s, j, q))
\]

\[
E_{p2c} \quad \text{# Price of domestic composite, User 2 #}
\]
\[
(all, i, COM) (all, j, IND) (all, q, REGDEST)
\]
\[
(TINY + PVAL2T(i, "domestic", j, q)) \times p2c(i, j, q)
\]
\[
= \text{sum}(s, REGSOURCE, PVAL2A(i, s, j, q) \times p2a(i, s, j, q))
\]
Investment demand

\[ E_{x2c} \] # Demand for domestic composite, User 2 #
\[
(\text{all}, i, \text{COM}) (\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST})
\]
\[ x_{2c}(i,j,q) = x_{2o}(i,j,q) - \Sigma_2O(i) \times (p_{2c}(i,j,q) - p_{2o}(i,j,q)); \]

\[ E_{x2o} \] # Demands for domestic/foreign composite, User 2 #
\[
(\text{all}, i, \text{COM}) (\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST})
\]
\[ x_{2o}(i,j,q) = y(j,q) + a_{2ind}(j,q); \]
Building blocks

✓ Producer’s demands for inputs
✓ Investor demands
✓ **Household demands**
✓ Export demands
✓ Government demands
✓ Margins demands
✓ Zero pure profits
✓ Indirect tax equations
✓ Market-clearing
✓ Regional and national macroeconomic variables and price indexes
✓ Capital accumulation and investment
✓ Regional population and labor market
Household demand

(1) $\text{Utility} \rightarrow \text{LES}$

(2) $\text{Commodity 1} \rightarrow \text{CES} \rightarrow \text{Domestic Source}$
$\text{Commodity 1} \rightarrow \text{CES} \rightarrow \text{Imported Source}$

(3) $\text{Region r Source}$
$\text{Region s Source}$
Household demand

Each regional household determines optimal consumption bundle by maximizing a Stone-Geary utility function subject to a budget constraint.

A Keynesian-type consumption function determines aggregate regional household expenditure.
Household demand

\[ E_{x3o} \quad \# \quad \text{Household demand for composite commodities} \]
\[ (\text{all,}i,\text{COM}) \quad (\text{all,}q,\text{REGDEST}) \]
\[ x3o(i,q) = [1 - \text{ALPHA}_I(i,q)] \times [\text{qhous}(q) + a3\text{sub}(i,q)] \]
\[ + \text{ALPHA}_I(i,q) \times [\text{luxexp}(q) + a3\text{lux}(i,q) - p3o(i,q)]; \]

\[ E_{a3lux} \quad \# \quad \text{Default setting for luxury taste shifter} \]
\[ (\text{all,}i,\text{COM}) \quad (\text{all,}q,\text{REGDEST}) \]
\[ a3\text{lux}(i,q) = a3\text{sub}(i,q) - \text{sum}(k,\text{COM}, \Delta(k,q) \times a3\text{sub}(k,q)); \]

\[ E_{a3sub} \quad \# \quad \text{Default setting for subsistence taste shifter} \]
\[ (\text{all,}i,\text{COM}) \quad (\text{all,}q,\text{REGDEST}) \]
\[ a3\text{sub}(i,q) = a3\text{com}(i,q) - \text{sum}(k,\text{COM}, S3\text{COM}(k,q) \times a3\text{com}(k,q)); \]

\[ E_{\text{utility}} \quad \# \quad \text{Change in utility disregarding taste change terms} \]
\[ (\text{all,}q,\text{REGDEST}) \]
\[ \text{utility}(q) = \text{luxexp}(q) - \text{qhous}(q) - \text{sum}(i,\text{COM}, \Delta(i,q) \times p3o(i,q)); \]
Household demand

\( E_{x3a} \) # Demand for goods by source, User 3 #
\((all, i, COM)(all, s, ALLSOURCE)(all, q, REGDEST)\)
\( x3a(i, s, q) = IS_{DOM}(s) \times (x3c(i, q) - SIGMA3C(i) \times (p3a(i, s, q) - p3c(i, q))) + IS_{IMP}(s) \times (x3o(i, q) - SIGMA3O(i) \times (p3a(i, "foreign", q) - p3o(i, q))) ; \)

\( E_{p3o} \) # Price of domestic/foreign composite, User 3 #
\((all, i, COM)(all, q, REGDEST)\)
\( (TINY + PVAL3O(i, q)) \times p3o(i, q) = \text{sum}(s, ALLSOURCE, PVAL3A(i, s, q) \times p3a(i, s, q)) ; \)

\( E_{p3c} \) # Price of domestic composite, User 3 #
\((all, i, COM)(all, q, REGDEST)\)
\( (TINY + PVAL3T(i, "domestic", q)) \times p3c(i, q) = \text{sum}(s, REGSOURCE, PVAL3A(i, s, q) \times p3a(i, s, q)) ; \)

\( E_{x3c} \) # Demand for domestic composite, User 3 #
\((all, i, COM)(all, q, REGDEST)\)
\( x3c(i, q) = x3o(i, q) - SIGMA3O(i) \times (p3c(i, q) - p3o(i, q)) ; \)
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- **Export demands**
- Government demands
- Margins demands
- Zero pure profits
- Indirect tax equations
- Market-clearing
- Regional and national macroeconomic variables and price indexes
- Capital accumulation and investment
- Regional population and labor market
Export commodities face individual downward-sloping foreign export demand functions

Exports of product i from source s are distinguished from exports of i from source r (r not equal s)
Foreign export demand

E_x4r  # Export demand functions  #
(all,i,TEXP)(all,s,REGSOURCE)
x4r(i,s)-feq(i)=EXP_ELAST(i)*[p4r(i,s)-fep(i)-natfep];

E_aggnt_x4r  # Export demand functions, non-trad aggregate  #
(all,s,REGSOURCE)
aggnt_x4r(s)-aggnt_feq(s)=EXP_ELAST("OP0")*[aggnt_p4r(s)-
aggnt_fep(s)-natfep];

E_nt_x4r  # Export demand functions, non-trad  #
(all,i,NTEXP)(all,s,REGSOURCE)
x4r(i,s)=aggnt_x4r(s)+faggnt_i(i)+faggnt_s(s)+faggnt_is(i,s);

E_aggnt_p4r  # Export price, non-trad aggregate  #
(all,s,REGSOURCE)
AGGEXPNT(s)*aggnt_p4r(s)=\sum(i,NTEXP,PVAL4r(i,s)*p4r(i,s))+faggnt_p4r(s);
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- Export demands
- **Government demands**
- Margins demands
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- Capital accumulation and investment
- Regional population and labor market
Government demand

Recognise regional governments and Federal government demands for goods and services for current consumption (not properly calibrated yet!)

Neither modelled explicitly

Default:

- aggregate regional government demand in region q moves with regional government revenue, with structure of demand exogenous
- aggregate Federal government demand in region q moves with national government revenue, with structure of demand exogenous
Government demand

\[ \text{E}_x5a \] # Regional government demand #
\[(\text{all}, i, \text{COM})(\text{all}, s, \text{ALLSOURCE})(\text{all}, q, \text{REGDEST})\]
\[x5a(i,s,q) = f5a(i,s,q) + f5gen(q) + \text{natf5gen} + \text{taxind}(q) - xi5(q);\]

\[ \text{E}_x6a \] # Federal government demand #
\[(\text{all}, i, \text{COM})(\text{all}, s, \text{ALLSOURCE})(\text{all}, q, \text{REGDEST})\]
\[x6a(i,s,q) = f6a(i,s,q) + f6gen(q) + \text{natf6gen} + \text{nattaxind} - \text{natxi6};\]
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- Export demands
- Government demands
- Margins demands
- Zero pure profits
- Indirect tax equations
- Market-clearing
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- Capital accumulation and investment
- Regional population and labor market
Margins commodities (identified in the set MARGCOM) provide freight services

- these commodities are consumed directly and used indirectly to facilitate the movements of products
- latter type of use is margins demand

Margins demand for margin commodity i is assumed to be proportional to the volume of the underlying flow

- e.g., margins use of transportation services in taking agricultural products to manufacturing is modelled as proportional to the volume of agricultural product used in manufacturing
Demand for margin (transportation) services

\[ E_{x1marg} \# \text{Margins on sales to producers} \#
(\text{all}, i, \text{COM}) (\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST}) (\text{all}, s, \text{ALLSOURCE}) (\text{all}, r, \text{MARGCOM})
\]
\[ x1marg(i, s, j, q, r) = \Theta(i, s, q) * x1a(i, s, j, q) + a1marg_{ij}(s, q, r) + \text{amarg}_i(s, q, r); \]

\[ E_{x2marg} \# \text{Margins on sales to capital creators} \#
(\text{all}, i, \text{COM}) (\text{all}, j, \text{IND}) (\text{all}, q, \text{REGDEST}) (\text{all}, s, \text{ALLSOURCE}) (\text{all}, r, \text{MARGCOM})
\]
\[ x2marg(i, s, j, q, r) = \Theta(i, s, q) * x2a(i, s, j, q) + a2marg_{ij}(s, q, r) + \text{amarg}_i(s, q, r); \]

\[ E_{x3marg} \# \text{Margins on sales to household consumption} \#
(\text{all}, i, \text{COM}) (\text{all}, s, \text{ALLSOURCE}) (\text{all}, q, \text{REGDEST}) (\text{all}, r, \text{MARGCOM})
\]
\[ x3marg(i, s, q, r) = \Theta(i, s, q) * x3a(i, s, q) + a3marg_i(s, q, r) + \text{amarg}_i(s, q, r); \]

\[ E_{x4marg} \# \text{Margins on exports: factory gate to port} \#
(\text{all}, i, \text{COM}) (\text{all}, r, \text{MARGCOM}) (\text{all}, s, \text{REGSOURCE})
\]
\[ x4marg(i, s, r) = x4r(i, s) + a4marg_i(s, r); \]
Progress so far through the core...

Done
- All demand equations

To do
- Zero pure profits
- Indirect tax equations
- Market-clearing
- Regional and national macroeconomic variables and price indexes
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- Export demands
- Government demands
- Margins demands
- **Zero pure profits**
- Indirect tax equations
- Market-clearing
- Regional and national macroeconomic variables and price indexes
- Capital accumulation and investment
- Regional population and labor market
Zero pure profits

Critical assumptions

- no pure profits in the production or distribution of commodities
- price received by the producer is uniform across all customers

Zero pure profits in current production imposed by setting unit prices received by producers equal to unit costs

Zero pure profits in distribution imposed by setting the prices paid by users equal to producer price plus commodity tax plus margins
Zero pure profits

\[ E_{p0a} \quad \# \text{Zero pure profits in current production} \# \]
(\text{all}, j, \text{IND})(\text{all}, q, \text{REGDEST})
(TINY+COSTS(j, q))*\{p0a(j, q)-a(j, q)\}=
\sum(i, \text{COM}, \sum(s, \text{ALLSOURCE}, PVAL1A(i, s, j, q)*p1a(i, s, j, q)))
+\sum(m, \text{OCC}, \text{LAB_OCC_IND}(m, j, q)*p1laboi(j, q, m))
+\text{CAPITAL}(j, q)*p1cap(j, q)+\text{LAND}(j, q)*p1land(j, q)
+\text{OTHCOST}(j, q)*p1oct(j, q);

\[ E_{pi} \quad \# \text{Zero pure profits in capital creation} \# \]
(\text{all}, j, \text{IND})(\text{all}, q, \text{REGDEST})
(TINY+INVEST(j, q))*(pi(j, q)-a2ind(j, q))=
\sum(i, \text{COM}, \sum(s, \text{ALLSOURCE}, PVAL2A(i, s, j, q)*p2a(i, s, j, q)))

\[ E_{p0ab} \quad \# \text{Zero pure profits in importing} \# \]
(\text{all}, i, \text{COM})
p0a(i, "foreign")=pm(i)+natphi+powtaxm(i);
Zero pure profits

\[ E_{p1a} \ # \ Purchasers \ prices \ - \ User \ 1 \ # \]
\[
\text{(all, i, COM)(all, j, IND)(all, q, REGDEST)(all, s, ALLSOURCE)}
\]
\[
\text{(TINY+PVAL1A(i, s, j, q))*p1a(i, s, j, q)}
\]
\[
= \left[ \text{BAS1(i, s, j, q)+TAX1(i, s, j, q)} \right] \cdot p0a(i, s)
\]
\[
+ \text{BAS1(i, s, j, q)*deltax1(i, s, j, q)+sum(r,MARGCOM,MAR1(i, s, j, q, r)}
\]
\[
\quad \quad \text{(p0a(r, q)+almarg_ij(s, q, r)+amarg_i(s, q, r))};
\]

\[ E_{p2a} \ # \ Purchasers \ prices \ - \ User \ 2 \ # \]
\[
\text{(all, i, COM)(all, j, IND)(all, q, REGDEST)(all, s, ALLSOURCE)}
\]
\[
\text{(TINY+PVAL2A(i, s, j, q))*p2a(i, s, j, q)=\left[ \text{BAS2(i, s, j, q)+TAX2(i, s, j, q)} \right] \cdot p0a}
\]
\[
\quad \quad \quad (i, s)
\]
\[
+ \text{BAS2(i, s, j, q)*deltax2(i, s, j, q)+sum(r,MARGCOM,MAR2(i, s, j, q, r)}
\]
\[
\quad \quad \quad \text{(p0a(r, q)+a2marg_ij(s, q, r)+amarg_i(s, q, r))};
\]

\[ E_{p3a} \ # \ Purchasers \ prices \ - \ User \ 3 \ # \]
\[
\text{(all, i, COM)(all, q, REGDEST)(all, s, ALLSOURCE)}
\]
\[
\text{(TINY+PVAL3A(i, s, q))*p3a(i, s, q)=\left[ \text{BAS3(i, s, q)+TAX3(i, s, q)} \right] \cdot p0a(i, s)}
\]
\[
+ \text{BAS3(i, s, q)*deltax3(i, s, q)+sum(r,MARGCOM,MAR3(i, s, q, r)}
\]
\[
\quad \quad \quad \text{(p0a(r, q)+a3marg_i(s, q, r)+amarg_i(s, q, r))};
\]
Zero pure profits

\[ \text{E}_{\text{p4r}} \quad \# \quad \text{Purchasers prices - User 4} \quad \#
\]
\[
(\text{all},i,\text{COM})(\text{all},s,\text{REGSOURCE})
\]
\[
(\text{TINY}+\text{PVAL4R}(i,s))(\text{natphi}+\text{p4r}(i,s)) = [\text{BAS4}(i,s)+\text{TAX4}(i,s)]*\text{p0a}(i,s)
\]
\[
+\text{BAS4}(i,s)*\text{deltax4}(i,s)+\text{sum}(r,\text{MARGCOM},\text{MAR4}(i,s,r)) *
\]
\[
(\text{p0a}(r,s)+\text{a4marg_i}(s,r)));
\]

\[ \text{E}_{\text{p5a}} \quad \# \quad \text{Purchasers prices - User 5} \quad \#
\]
\[
(\text{all},i,\text{COM})(\text{all},q,\text{REGDEST})(\text{all},s,\text{ALLSOURCE})
\]
\[
(\text{TINY}+\text{PVAL5A}(i,s,q)) * \text{p5a}(i,s,q) = [\text{BAS5}(i,s,q)+\text{TAX5}(i,s,q)]*\text{p0a}(i,s)
\]
\[
+\text{BAS5}(i,s,q)*\text{deltax5}(i,s,q)+\text{sum}(r,\text{MARGCOM},\text{MAR5}(i,s,q,r)) *
\]
\[
(\text{p0a}(r,q)+\text{a5marg_i}(s,q,r)+\text{amarg_i}(s,q,r)));
\]

\[ \text{E}_{\text{p6a}} \quad \# \quad \text{Purchasers prices - User 6} \quad \#
\]
\[
(\text{all},i,\text{COM})(\text{all},s,\text{ALLSOURCE})(\text{all},q,\text{REGDEST})
\]
\[
(\text{TINY}+\text{PVAL6A}(i,s,q)) * \text{p6a}(i,s,q) = [\text{BAS6}(i,s,q)+\text{TAX6}(i,s,q)]*\text{p0a}(i,s)
\]
\[
+\text{BAS6}(i,s,q)*\text{deltax6}(i,s,q)+\text{sum}(r,\text{MARGCOM},\text{MAR6}(i,s,q,r)) *
\]
\[
(\text{p0a}(r,s)+\text{a6marg_i}(s,q,r)+\text{amarg_i}(s,q,r)));
\]
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- Export demands
- Government demands
- Margins demands
- Zero pure profits
- **Indirect tax equations**
- Market-clearing
- Regional and national macroeconomic variables and price indexes
- Capital accumulation and investment
- Regional population and labor market
Indirect taxes

Equations have been added to enable flexible handling of indirect taxes on all flows of goods and services.

Equations allow for variations in tax rates across commodities, their sources and destinations.
Indirect taxes

E_deltax1 # Tax rate on sales to User 1 #
(all,i,COM) (all,s,ALLSOURCE) (all,j,IND) (all,q,REGDEST)
deltax1(i,s,j,q)=deltax(i)+deltax1all+deltaxsource(s)+deltaxdest(q);

E_deltax2 # Tax rate on sales to User 2 #
(all,i,COM) (all,s,ALLSOURCE) (all,j,IND) (all,q,REGDEST)
deltax2(i,s,j,q)=deltax(i)+deltax2all+deltaxsource(s)+deltaxdest(q);

E_deltax3 # Tax rate on sales to User 3 #
(all,i,COM) (all,s,ALLSOURCE) (all,j,IND) (all,q,REGDEST)
deltax3(i,s,q)=deltax(i)+deltax3all+deltaxsource(s)+deltaxdest(q);

E_deltax4 # Tax rate on sales to User 4 #
(all,i,COM) (all,s,REGSOURCE)
deltax4(i,s)=deltax(i)+deltax4all+deltaxsource(s) +deltaxdest("foreign");
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- Export demands
- Government demands
- Margins demands
- Zero pure profits
- Indirect tax equations

**Market-clearing**

- Regional and national macroeconomic variables and price indexes
- Capital accumulation and investment
- Regional population and labor market
Market-clearing

Equations that impose market clearing (demand equals supply) for:
- domestically produced margin and non-margin commodities
- imported commodities
Market-clearing

\[ E_{\text{mkt\_clear\_margins}} \] # Demand equals supply for margin commodities #

\[
(\text{all}, r, \text{MARGCOM}) (\text{all}, s, \text{REGSOURCE}) \\
(TINY+\text{SALES}(r,s)) \ast z(r,s) = \sum(j, \text{IND}, \sum(q, \text{REGDEST}, \text{BAS1}(r,s,j,q) \ast x_{1a}(r,s,j,q) \\
+ \text{BAS2}(r,s,j,q) \ast x_{2a}(r,s,j,q)) \ast \sum(q, \text{REGDEST}, \text{BAS3}(r,s,q) \ast x_{3a}(r,s,q)) \\
+ \text{BAS4}(r,s) \ast x_{4a}(r,s) + \sum(q, \text{REGDEST}, \text{BAS5}(r,s,q) \ast x_{5a}(r,s,q)) \\
+ \sum(q, \text{REGDEST}, \text{BAS6}(r,s,q) \ast x_{6a}(r,s,q)) \\
+ \sum(j, \text{IND}, \sum(i, \text{COM}, \sum(ss, \text{ALLSOURCE}, \text{MAR1}(i,ss,j,s,r) \ast x_{1\text{marg}}(i,ss,j,s,r) \\
+ \text{MAR2}(i,ss,j,s,r) \ast x_{2\text{marg}}(i,ss,j,s,r))) \\
+ \sum(i, \text{COM}, \sum(ss, \text{ALLSOURCE}, \text{MAR3}(i,ss,s,r) \ast x_{3\text{marg}}(i,ss,s,r)) \\
+ \sum(i, \text{COM}, \sum(ss, \text{ALLSOURCE}, \text{MAR4}(i,s,r) \ast x_{4\text{marg}}(i,s,r)) \\
+ \sum(i, \text{COM}, \sum(ss, \text{ALLSOURCE}, \text{MAR5}(i,ss,s,r) \ast x_{5\text{marg}}(i,ss,s,r)) \\
+ \sum(i, \text{COM}, \sum(ss, \text{ALLSOURCE}, \text{MAR6}(i,ss,s,r) \ast x_{6\text{marg}}(i,ss,s,r)));
\]
Market-clearing

E_mkt_clear_nomarg  # Demand equals supply for non-margin commodities #
(all,r,NONMARGCOM)(all,s,REGSOURCE)
(TINY+SALES(r,s))*z(r,s)=sum(j,IND,sum(q,REGDEST,BAS1(r,s,j,q)*x1a(r,s,j,q))
+sum(j,IND,sum(q,REGDEST,BAS2(r,s,j,q)*x2a(r,s,j,q)))
+sum(q,REGDEST,BAS3(r,s,q)*x3a(r,s,q))+BAS4(r,s)*x4r(r,s)
+sum(q,REGDEST,BAS5(r,s,q)*x5a(r,s,q))+sum(q,REGDEST,BAS6(r,s,q)*x6a(r,s,q));

E_x0impa  # Import volume of commodities by region #
(all,i,COM)(all,q,REGDEST)
(TINY+IMPORTS(i,q))*x0imp(i,q)=
sum(j,IND,BAS1(i,"foreign",j,q)*x1a(i,"foreign",j,q)
+BAS2(i,"foreign",j,q)*x2a(i,"foreign",j,q))
+BAS3(i,"foreign",q)*x3a(i,"foreign",q)
+BAS5(i,"foreign",q)*x5a(i,"foreign",q)+BAS6(i,"foreign",q)*x6a(i,"foreign",q);
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
- Export demands
- Government demands
- Margins demands
- Zero pure profits
- Indirect tax equations
- Market-clearing
- **Regional and national macro variables and price indexes**
- Capital accumulation and investment
- Regional population and labor market
Macro aggregates

Wide range of national and regional macro variables defined...

Two concepts of the real wage rate

- consumer real wage rate (PLAB/CPI)
- producer real wage rate (PLAB/PGDP)
Progress so far...

Done
- Core CGE equations relating to demand, supply, prices, indirect taxes, market-clearing and summary macro variables

To do
- Capital accumulation and investment
- Regional population and labor market
Building blocks

- Producer’s demands for inputs
- Investor demands
- Household demands
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- Government demands
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- **Capital accumulation and investment**
- Regional population and labor market
Investment “dynamics”

Capital, investment and expected rates of return

\[ K_{j,q}(t+1) = (1 - DEP_{j,q}) \times K_{j,q}(t) + Y_{j,q}(t) \]

Given starting point for capital (t=0) and an explanation of investment, we can trace out time path for capital
Investment “dynamics”

Investment explained by assuming that:

\[
\frac{K_{j,q}(t+1)}{K_{j,q}(t)} - 1 = F_{j,q}^{t}[EROR_{j,q}(t)]
\]

Growth in capital related to expected rate of return

- In BMMX ICGE only assume static expectations, though rational is possible
Rates of return and investment

For static expectations case, the actual rate of return is:

\[ RO_i(j,q) = \frac{P_i(j,q)}{\Pi_i(j,q)} - D(j,q) \]

\[ ro(j,q) = p_i(j,q) - \pi_i(j,q) \]

\[ ro(j,q) = QCOEF(j,q)[p_i(j,q) - \pi_i(j,q)] \]

*QCOEF*: relação entre taxa bruta e taxa líquida de retorno (> 1)
In long-run comparative-static simulations:

- aggregate capital adjusts to maintain $R_{INT} (natr_tot)$
- capital allocated in line with equation $E_f\_rate\_xx$
  - industries with relatively large increases in capital require relatively high rates of return
  - industries with relatively small increases in capital require relatively low rates of return
- industry investment determined by fixed ratios of investment to capital (equation $E_y$)
Rates of return and investment

Equalization in the rates of return

\[
\left( \frac{K(j,q)}{K(q)} \right)^{-\beta(j,q)} \quad RO(j,q) = R_{int}
\]

\[
ro(j,q) - r_{int} = \beta_t(j,q)[k(j,q) - k(q)] + f\_rate(j,q)
\]

*beta*: risk/return ratio

Short-run: *f_rate* endogenous, *k* exogenous

Long-run: *f_rate* exogenous, *k* endogenous
Investment “dynamics”

Growth rate of capital stocks and investment in the short-run:

\[ k_{t+1}(j,q) - k_t(j,q) = 0 \]  \quad \text{% change in capital stocks}

\[ y_t(j,q) = 0 \]  \quad \text{% change in investment}
Investment “dynamics”

Growth rate of capital stocks and investment in the long-run:

\[
\frac{K_{j,q}(t+1)}{K_{j,q}(t)} = \left( \frac{K_{j,q}(t)}{K_{j,q}(0)} \right)^{1/T}
\]

\[
k_{t+1}(j,q) = \left( 1 + \frac{1}{T} \right) k_t(j,q)
\]
Investment in the short run

Fixed capital stocks in the base year values:

- $curcap(j,q)$ exogenous ($=0$)
- relationship between sectoral rates of return, $r0(j,q)$, and reference interest rate, $natr_tot$, is endogenous ($f_rate_xx(j,q)$ endogenous)

Percentage change in sectoral investment, $y(j,q)$ is zero; this can be guaranteed by setting the shift term, $delf_rate(j,q)$, exogenous and zero

By hypothesis, not only the capital stocks are fixed but also firms’ investment plans
Investment in the short run

\[ E_{r0} \quad \# \quad \text{Definition of rates of return to capital} \# \]
\[ r_0(j,q) = QCOEF(j,q) \times (p1cap(j,q) - pi(j,q)); \]

\[ E_{f\_rate\_xx} \quad \# \quad \text{Capital growth rates related to rates of return} \# \]
\[ (r_0(j,q) - natr\_tot) = BETA_R(j,q) \times [\text{curcap}(j,q) - kt(q)] + \text{f\_rate\_xx}(j,q); \]

\[ E_{\text{curcapT1}} \quad \# \quad \text{Capital stock in period T+1} \# \]
\[ \text{curcap\_t1}(j,q) - \text{curcap}(j,q) = 0; \]

\[ E_{yT} \quad \# \quad \text{Investment in period T} \# \]
\[ \text{curcap}(j,q) - y(j,q) - 100 \times \text{delf\_rate}(j,q) = 0; \]
Investment in the long run

Capital stocks endogenously determined:
- \( \text{curcap}(j,q) \) endogenous
- relationship between sectoral rates of return, \( r_0(j,q) \), and reference interest rate, \( \text{natr_tot} \), is given (\( f_{\text{rate_xx}}(j,q) \) exogenous)

Percentage change in sectoral investment, \( y(j,q) \) is endogenous

Firms’ investment plans are carried out, reestablishing returns differentials in the base year

Rate of capital accumulation, but **not the level** of capital stock, remains constant
Investment in the long run

E_r0  # Definition of rates of return to capital #
\[ r_0(j, q) = QCOEF(j, q) \times (plcap(j, q) - pi(j, q)) \];

E_f_rate_xx  # Capital growth rates related to rates of return #
\[ (r_0(j, q) - natr_tot) = BETA_R(j, q) \times [curcap(j, q) - kt(q)] + f_rate_xx(j, q); \]

E_curcapT1  # Capital stock in period T+1 #
\[ curcap_t1(j, q) - K_TERM \times curcap(j, q) = 0; \]

E_yT  # Investment in period T #
\[ VALK_T1(j, q) \times curcap_t1(j, q) = VALKT(j, q) \times DEP(j) \times curcap(j, q) + (INVEST(j, q)) \times y(j, q) - 100 \times (VALK_0(j, q) \times (1 - DEP(j))) \]

endog.  exog.
Building blocks

✓ Producer’s demands for inputs
✓ Investor demands
✓ Household demands
✓ Export demands
✓ Government demands
✓ Margins demands
✓ Zero pure profits
✓ Indirect tax equations
✓ Market-clearing
✓ Regional and national macro variables and price indexes
✓ Capital accumulation and investment
✓ **Regional population and labor market**
Regional population and labor market

Critical variables:
- regional population
- regional migration
- regional unemployment
- regional participation rates
- regional wage relativities

Various closures
Regional population and labor market

(1) Fixed
- wage relativities (determining employment by region),
  participation and unemployment rates (determining population by
  region)

(1) Endogenous
- regional migration

(2) Fixed
- regional migration, participation rates, wage relativities

(2) Endogenous
- unemployment rates

(3) Fixed
- regional migration, participation and unemployment rates

(3) Endogenous
- wage relativities
Labor market in the short-run

\[ E_{\text{wage_diff}} \quad \# \quad \text{Region real-wage diff } \quad (\text{all}, q, \text{REGDEST}) \]

\[ \text{wage_diff}(q) = \text{pwage}(q) - \text{natxi3} - \text{natrealwage}; \]

\[ E_{\text{del_labsup}} \quad \# \quad \text{P-point changes in regional unemployment rates } \quad (\text{all}, q, \text{REGDEST}) \]

\[ C_{\text{labsup}}(q) \ast \text{del_unr}(q) = C_{\text{EMPLOY}}(q) \ast (\text{labsup}(q) - \text{employ}(q)); \]

\[ \text{del_unr}(q) \quad \# \quad \text{Percentage-point changes in regional unemployment rate } \#; \]

endog.  exog.
Labor market in the long-run

\[ E_{\text{wage\_diff}} \]  # Region real-wage diff #(all, q, REGDEST)

\[ \text{wage\_diff}(q) = \text{pwage}(q) - \text{natxi3} - \text{natrealwage}; \]

\[ E_{\text{del\_labsup}} \]  # P-point changes in regional unemployment rates #(all, q, REGDEST)

\[ \text{C\_labsup}(q) * \text{del\_unr}(q) = \text{C\_EMPLOY}(q) * (\text{labsup}(q) - \text{employ}(q)); \]

\[ \text{del\_unr}(q) \]  # Percentage-point changes in regional unemployment rate #;
Closures

Each equation explains a variable

More variables than equations

Endogenous variables: explained by model

Exogenous variables: set by user

Closure: choice of exogenous variables

Many possible closures

\[ \text{Number of endogenous variables} = \text{Number of equations} \]
Length of run, $T$

$T$ is related to our choice of closure

With short-run closure we assume that:
- $T$ is long enough for price changes to be transmitted throughout the economy, and for price-induced substitution to take place
- $T$ is not long enough for investment decisions to greatly affect the useful size of sectoral capital stocks [new buildings and equipment take time to produce and install]

$T$ might be 2 years. So results mean:
- A 10% consumption increase might lead to employment in 2 years time being 1.2% higher than it would be (in 2 years time) if the consumption increase did not occur
Different closures

Many closures might be used for different purposes

No unique natural or correct closure

Must be at least one exogenous variable measured in local currency units

Normally just one — called the *numéraire*

Often the exchange rate, *natphi*, or *natxi3*, the CPI.

Some quantity variables must be exogenous, such as:
- primary factor endowments
- final demand aggregates
Short-run environment

Gross Regional Product = HH consump. + Invest. + Governm. consump. + Trade balance

National employment

Employment

Real wage (national)

Rate of return on capital

Capital stock

Endogenous

Exogenous

Public and private

Central and regional

Domestic and foreign
Long-run environment


- National employment
- Endogenous
- Exogenous
- Rate of return on capital → Capital stock
- Public and private
- Central and regional
- Domestic and foreign