3 Input–Output Models at the Regional Level

3.1 Introduction

Originally, applications of the input–output model were carried out at national levels – for example, to assess the impact on the individual sectors of the US economy of a change from war to peacetime production as the end of World War II approached. Over time, interest in economic analysis at the regional level – whether for a group of states (as in a federal reserve district), an individual state, a county or a metropolitan area – has led to modifications of the input–output model which attempt to reflect the peculiarities of a regional (subnational) problem. There are at least two basic features of a regional economy that influence the characteristics of a regional input–output study.

First, although the data in a national input–output coefficients table are obviously some kind of averages of data from individual producers located in specific regions, the structure of production in a particular region may be identical to or it may differ markedly from that recorded in the national input–output table. Soft drinks of a particular brand that are bottled in Boston probably incorporate basically the same ingredients in the same proportions as are present in that brand of soft drink produced in Kansas City or Atlanta or in any other bottling plant in the United States. On the other hand, electricity produced in eastern Washington by water power (Coulee Dam) represents quite a different mix of inputs from electricity that is produced from coal in Pennsylvania or by means of nuclear power or "wind farms" elsewhere. For these reasons, the early methodology for regional input–output applications – which used national input coefficients with some minor modifications – has given way to coefficients tables that are tailored to a particular region on the basis of data specific to that region.

Secondly, it is generally true that the smaller the economic area, the more dependent that area's economy is on trade with "outside" areas – transactions that cross the region's borders – both for sales of regional outputs and purchases of inputs needed for production. That is, one of the elements that contributed to the exogenous final-demand sector in the model described in Chapter 2 – exports – now will generally be relatively much more important and a higher proportion of inputs will be imported from producers located outside of the region. To exaggerate, a one-world economy would have no "foreign trade," since all sales and purchases would be internal to the worldwide

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"region," whereas an urban area depends very much on imports and exports (imports of components to aircraft production and exports of Boeing airliners from the Seattle area).

In this chapter we will explore some of the attempts that have been made to incorporate these features of a regional economy into an input–output framework. Such regional input–output models may deal with a single region or with two or more regions and their interconnections. The several-region case is termed *interregional input–output analysis* (in one version) or *multiregional input–output analysis* (in another version). We will examine each of these kinds of regionalized input–output models, as well as what is known as the *balanced regional* model.

There has been an enormous amount of input–output work at the regional level. Examples of some of the earliest single-region applications are found in Moore and Petersen (1955), Isard and Kuenne (1953), Miller (1957), and Hirsch (1959). A very thorough discussion and documentation of the details involved in producing a regional input–output table during the early period in the development of this area of application is provided by Isard and Langford (1971) – in this case the region was the Philadelphia Standard Metropolitan Statistical Area – and in Miernyk *et al.* (1967) for Boulder, Colorado, and Miernyk *et al.* (1970) for West Virginia. Overviews of early regional input–output models can be found in Polenske (1980, Chapter 3) and in Miernyk (1982). For an idea of the large amount of continuing work in this area, the reader is referred to annual indexes in such journals as *Economic Systems Research, Journal of Regional Science, International Regional Science Review* and *Papers in Regional Science*.[1] In addition, many regional input–output tables and studies using these tables have been published by the appropriate sub-national agencies (state and local governments or their counterparts outside the USA) for whom the analysis was done, or by universities where the work was done.

In section 3.6 we indicate some examples of how the geographic scale of connected-region models has evolved in both the micro- and macroscopic directions from these earliest applications – down to models of as small an area as an inner-city neighborhood and up to what are often referred to as "world" models, encompassing several blocs of mega-nations. Examples of regional applications will also be discussed in Chapter 6 on multipliers and in Chapter 8 on estimating regional data. Much of the material on regional and interregional input–output models in this chapter and several chapters later in this book is covered (in less detail) in Miller (1998).

3.2 Single-Region Models

3.2.1 National Coefficients

Generally, regional input–output studies attempt to quantify the impacts on the producing sectors located in a particular region that are caused by new final demands for

[1] Other relevant journals include *Environment and Planning A, Annals of Regional Science, Regional Studies, Growth and Change, Urban Studies, Land Economics, Regional Science and Urban Economics, Regional Science Perspectives*, and *Economic Geography*.

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products made in the region. Early regional studies (Isard and Kuenne, 1953; Miller, 1957) used a national table of technical coefficients in conjunction with an adjustment procedure that was designed to capture some of the characteristics of the regional economies, since specific coefficients tables for the particular regions did not exist.[2]

We use a superscript r to designate "region r" in the same way that subscript i denoted "sector i" in the discussion in Chapter 2. Thus, just as x_i was used to denote the gross output of sector i, let $\mathbf{x}^r = [x_i^r]$ denote the vector of gross outputs of sectors in region r. Similarly, $\mathbf{f}^r = [f_i^r]$ represents the vector of exogenous demands for goods made in region r. For example, if r denotes Washington State, one element of $\mathbf{f}^r$ could be an order from a foreign airline for commercial aircraft from Boeing in Washington.

The problem in these early regional studies was that only a national technical coefficients matrix, $\mathbf{A}$, was available, but what was needed, essentially, was a matrix showing inputs from firms *in the region* to production *in that region*. Denote this unknown matrix by $\mathbf{A}^{rr} = [a_{ij}^{rr}]$, where a_{ij}^{rr} is the amount of input from sector i in r per dollar's worth of output of sector j in r. (This anticipates notation later for many-region models, where we will need two superscripts to identify origin and destination regions, just as i and j are origin and destination sectors.) Assume, in the absence of evidence to the contrary, that local producers use the same production recipes as are shown in the national coefficients table, meaning that the *technology* of production in each sector in region r is the same as in the nation as a whole. Nonetheless, in order to translate regional final demands into outputs of *regional* firms ($\mathbf{x}^r$), the national coefficients matrix must be modified to produce $\mathbf{A}^{rr}$ (*locally produced* goods in local production).

Early studies carried out this modification through the use of estimated *regional supply percentages*, one for each sector in the regional economy, designed to show the percentage of the total required outputs from each sector that could be expected to originate within the region. One straightforward way to estimate these percentages, using data that may often be obtainable at the regional level, requires knowledge of (1) total regional output of each sector i, x_i^r, (2) exports of the product of each sector i from region r, e_i^r, and (3) imports of good i into region r, m_i^r. Then, one can form an expression for the *proportion* of the total amount of good i available in region r that was produced in r (the *regional supply proportion* of good i). We denote this by p_i^r, where

$$p_i^r = \frac{(x_i^r - e_i^r)}{(x_i^r - e_i^r + m_i^r)}$$

The numerator is the *locally produced* amount of i that is available to purchasers in r; the denominator is the *total* amount of i available in r, either produced locally or imported. (Thus $p_i^r \times 100$ is an estimate of the regional supply percentage for sector i in region r – the percentage of good i available in r that was produced there.)

Assuming that we can estimate such proportions for each sector in the economy, each element in the ith row of the national coefficients matrix could be multiplied by

[2] The "regions" were the Greater New York–Philadelphia urban-industrial region (consisting of 2 counties in Connecticut, 11 in New York, 19 in New Jersey, and 5 in Pennsylvania) in the first case and the states of Washington, Oregon, and Idaho in the second.

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p_i^r to generate a row of locally produced direct input coefficients of good i to each local producer. If we arrange these proportions in an n-element column vector, $\mathbf{p}^r$, then our working estimate of the regional matrix will be $\mathbf{A}^{rr} = \hat{\mathbf{p}}^r\mathbf{A}$. For a two-sector model, this is

$$\mathbf{A}^{rr} = \hat{\mathbf{p}}^r\mathbf{A} = \begin{bmatrix} p_1^r & 0 \\ 0 & p_2^r \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} p_1^r a_{11} & p_1^r a_{12} \\ p_2^r a_{21} & p_2^r a_{22} \end{bmatrix}$$

For any $\mathbf{f}^r$ we could then find $\mathbf{x}^r = (\mathbf{I} - \hat{\mathbf{p}}^r\mathbf{A})^{-1}\mathbf{f}^r$. This uniform modification of the elements in a row of $\mathbf{A}$ is a strong assumption. It means, for example, that if the aircraft, kitchen equipment, and pleasure boat sectors in Washington all use aluminum (sector i) as an input, all three sectors buy the same percentage, p_i^r, of their total aluminum needs from firms located within the state.

In the two-sector example in Chapter 2 we had $\mathbf{A} = \begin{bmatrix} .15 & .25 \\ .20 & .05 \end{bmatrix}$. Assume that this is a national table, and that we want to create $\mathbf{A}^{rr}$ from it, and that there is no evidence that the basic structure of production in the region differs from the national average structure reflected in $\mathbf{A}$. The unique features of the region, however, are captured in the regional supply percentages. Using regional output, export and import data, suppose we estimate that 80 percent of sector 1 goods will come from firms in that sector within the region, but only 60 percent of sector 2 goods can be expected to be supplied by regional firms in sector 2, so $\mathbf{p}^r = \begin{bmatrix} .8 \\ .6 \end{bmatrix}$. Suppose that the projected (new) final demand in the region is $\mathbf{f}^r = \begin{bmatrix} 600 \\ 1500 \end{bmatrix}$ (this is the final demand vector that was used for some of the numerical examples in Chapter 2). Then

$$\hat{\mathbf{p}}^r = \begin{bmatrix} .8 & 0 \\ 0 & .6 \end{bmatrix},\ \mathbf{A}^{rr} = \hat{\mathbf{p}}^r\mathbf{A} = \begin{bmatrix} .8 & 0 \\ 0 & .6 \end{bmatrix} \begin{bmatrix} .15 & .25 \\ .20 & .05 \end{bmatrix} = \begin{bmatrix} .12 & .20 \\ .12 & .03 \end{bmatrix},$$

$$(\mathbf{I} - \mathbf{A}^{rr})^{-1} = \begin{bmatrix} 1.169 & 0.241 \\ 0.145 & 1.061 \end{bmatrix}$$

and using this regional inverse directly,

$$\mathbf{x}^r = (\mathbf{I} - \mathbf{A}^{rr})^{-1}\mathbf{f}^r = \begin{bmatrix} 1.169 & 0.241 \\ 0.145 & 1.061 \end{bmatrix} \begin{bmatrix} 600 \\ 1500 \end{bmatrix} = \begin{bmatrix} 1062.90 \\ 1678.50 \end{bmatrix} \tag{3.1}$$

This tells us that the total output that will need to be produced in the region by sectors 1 and 2 is \$1062.90 and \$1678.50, respectively.

In more recent regional input–output analyses, attempts have been made to model the characteristics of a regional economy more precisely. We examine these briefly in the following section, and we return to the "regionalization" problem in Chapter 8.

3.2.2 Regional Coefficients

We noted above that electricity produced in Washington will most likely have a different production recipe (column of technical coefficients) from electricity made in

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Pennsylvania. These regionally produced electricities are really two different products – "hydroelectric power" and "coal-fired electrical power." As another example, consider the aircraft sector. In a national table, this would include the manufacture of a mix of commercial, business, and personal aircraft. One input to this sector would be the huge jet engines used on Boeing commercial airliners. On the other hand, the aircraft sector in a regional table for the state of Florida might reflect the manufacture of small personal aircraft, for which the jumbo jet engines are not an input at all; in a Washington table, however, jet engines are an extremely important input.

Sectors in even very disaggregated national input–output tables will be made up of a variety of products – as in the aircraft sector example. And firms within that sector, located in various regions of the country, will generally produce only a small number of those products – Boeing in Washington does not produce small propeller-driven airplanes; Piper in Florida does not produce jet airliners that can carry upwards of 300 passengers. This illustrates the so-called *product-mix* problem in input–output; firms classified in the same sector actually produce different sets of products. The most straightforward way to avoid this problem is to survey firms in the region and construct what is called a survey-based regional input–output table. In conducting such a survey, one can pose essentially two variants of the basic question. In asking firms in sector j in a particular region about their use of various inputs, the question can be:

1. How much sector i product did you buy last year in making your output? (For example, how much aluminum did aircraft manufacturers in Washington State buy last year?), or
2. How much sector i product did you buy last year from firms located in the region? (For example, how much aluminum used by aircraft producers in Washington was purchased from producers in Washington?)[3]

In the former case a truly *regional technical coefficients* table would be produced; this would better reflect production practices in the region than does the national table – it would eliminate the input of large jet engines into the manufacture of private aircraft in Florida, for example. But it would not address the question of how much of each required input came from within the region and how much was imported. On the other hand, a set of coefficients based on inputs supplied from firms within the region for outputs of firms in the region would reflect regional production technology. These might be termed *regional input coefficients*. They are to be distinguished from regional technical coefficients since they do not always accurately describe the technology of regional firms, but rather only the way in which local firms use local inputs. (*Intraregional input coefficients* would be an even more precise, although cumbersome, description.[4])

Rather than adapt a national coefficients table through application of regional supply proportions, some regional analysts have tried to derive true regional input coefficient

[3] If it is also possible to determine how much came from firms located outside the state then one has the beginnings of an interregional or multiregional model. These are discussed below in sections 3.3 and 3.4.

[4] Tiebout (1969, p. 335) used "direct intraregional interindustry coefficient," which is completely precise but also rather cumbersome.

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tables through surveys of regional establishments using variants of question 2. A series of tables for Washington State illustrates this kind of survey-based modeling effort, specifically for the state for 1963, 1967, 1972, 1982, 1987, and 2002. (There is also a Washington table for 1997 produced mainly by a nonsurvey estimating technique; nonsurvey approaches are explored in Chapters 7 and 8. The 1997 and 2002 tables are available at www.ofm.wa.gov/economy/io.) The Washington tables can be found in Bourque and Weeks (1969), Beyers *et al.* (1970), Bourque and Conway (1977), Bourque (1987), and Chase, Bourque and Conway (1993). These data have been the basis of many comparative studies.

To examine this kind of extension, we need more complicated notation. We continue to use a superscript r for the region in question. Then let z_{ij}^{rr} denote the dollar flow of goods from sector i in region r to sector j in region r.[5] Just as the order of subscripts is "from–to" with respect to sectors, the order of superscripts indicates "from–to" with respect to geographic locations. If we had a complete set of data on z_{ij}^{rr} for all n sectors in the regional economy, and also data on gross outputs (x_j^r) of each sector in the region, a set of regional input coefficients could be derived as

$$a_{ij}^{rr} = \frac{z_{ij}^{rr}}{x_j^r} \tag{3.2}$$

Let $\underset{(n\times n)}{\mathbf{Z}^{rr}} = [z_{ij}^{rr}]$ and $\underset{(n\times 1)}{\mathbf{x}^r} = [x_j^r]$; then the regional input coefficients matrix is

$$\mathbf{A}^{rr} = \mathbf{Z}^{rr}(\hat{\mathbf{x}}^r)^{-1} \tag{3.3}$$

(This is what was approximated in the early regional studies described above by $\hat{\mathbf{p}}^r\mathbf{A}$.) Then the impacts on *regional* production of a final-demand change in region r would be found as

$$\mathbf{x}^r = (\mathbf{I} - \mathbf{A}^{rr})^{-1}\mathbf{f}^r \tag{3.4}$$

3.2.3 Closing a Regional Model with respect to Households

The Washington State models noted above were closed with respect to households in the manner described in Chapter 2 – by adding a household consumption column and a labor input row. One extension to the process of endogenizing households in an input–output model is to add more than one row and column to the direct input coefficients matrix. This approach is frequently implemented at a regional level, although it can apply equally well to national models. As usual, the impacts of projected increases in final demand will be increased sectoral outputs and therefore increased payments for labor services. The basic idea is that a distinction should be made between consumption habits of various kinds of consumers – for example, at a sub-national level, those of established residents of the region, who may experience an increase in their incomes (for example, due to productivity increases) and the consumption patterns of new residents, who may

[5] We need double superscripts because later we will also measure interindustry flows between regions – as in z_{ij}^{sr}.

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move into the region in anticipation of employment (new income). This distinction apparently originated with Tiebout (1969), where they are designated *intensive* and *extensive* income growth, respectively.

The reason for the distinction is that current residents may spend each dollar of new income according to a set of *marginal* consumption coefficients, while new residents may distribute their purchases according to a set of *average* consumption coefficients.

> The presumption should be clear: as new residents move in to fill jobs at the same wage rate as established residents, average consumption propensities are relevant. Insofar as regional income rises because of increased per capita incomes, marginal consumption propensities apply (Tiebout, 1969, p. 336).

If sales, by sector, could be broken down into those to new residents and those to existing residents, and if labor payments, by sector, could be similarly disaggregated, then marginal and average household consumption coefficients could be derived. Similarly, knowing each sector's outputs, "old" and "new" labor inputs per dollar's worth of output could be found. These would form two additional rows and columns with which to close the model.

In practice, such data are not so conveniently available. Tiebout (1969) describes the derivation of extensive and intensive coefficients in a regional model for the state of Washington. Miernyk *et al.* (1967) investigate essentially the same issue for their pioneering Boulder, Colorado, input–output study.[6] In addition, an attempt was made in the Boulder study to disaggregate the income increases to existing residents by income class, with lower marginal consumption propensities in higher income classes. (See Miernyk *et al.* 1967, esp. Chapter V.)

Instead of disaggregating households into "old" and "new" residents, Blackwell (1978) proposes a tripartite division into intensive and extensive (current residents and new residents, respectively, as above) and also redistributive, which is that portion of any new income that goes to previously unemployed local residents. The distinction between currently employed and currently unemployed workers is also explored in some detail by Madden and Batey (1983, and elsewhere).[7] The considerable work of Madden and Batey and their colleagues on "extended" input–output models is representative of a large body of research linking population and economic models. It is summarized in Batey and Madden (1999), which also contains references to a great deal of earlier work by them and by others. Miyazawa (1976) also investigates extensions to multiple categories of consumption spending and income recipients. We further explore various model closures (including the Miyawaza formulation) in Chapter 6 when we investigate input–output multipliers.

[6] Tiebout's contribution in formulating this distinction between extensive and intensive consumption propensities in a region is noted by Miernyk *et al.* (1967, p. 104, n. 9). A draft of Tiebout's paper was completed by 1967 and was published posthumously in 1969, following his death in January, 1968.

[7] Other early examples of "extended" models with households included (by no means an exhaustive list) include Schinnar (1976), Beyers (1980), Gordon and Ledent (1981), Ledent and Gordon (1981), and Joun and Conway (1983). These combined models are sometimes referred to as demo-economic – or also as eco-demographic. The demo-economic components reflect inputs from various labor (household) groups, and the eco-demographic components capture activity such as consumption by various household types.

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3.3 Many-Region Models: The Interregional Approach

Single-region models of the sort described in the previous section represent one approach to modeling a regional economy in input–output terms. What they fail to do, however, is to recognize in an operational way the interconnections between regions. The one region of interest (in the above, this was region r) was essentially "disconnected" from the rest of the country within which it is located, in the sense that its production recipes are reflected in an intraregional matrix, $\mathbf{A}^{rr}$. For a country made up of several regions, a number of important questions have several-region implications. Next year's national defense budget might include a large order for a certain type of aircraft built in California, the overhaul of one or more ships in Virginia, and modernization and upgrading of an army base in New Jersey. Each of these activities can be expected to have ramifications not only within the region (state, in this example) where the activity takes place, but also in other states. The total economic effect is therefore likely to be larger than the sum of the regional effects in California, Virginia, and New Jersey. Firms outside California will produce goods that will be imported to California for aircraft production; those firms, in turn, may import goods from other states for *their* production. Materials for ship overhaul will come to Virginia from suppliers outside that state. Electronic parts for the base upgrading in New Jersey may be imported from elsewhere and the electronics firms, in turn, will need both local (wherever they are located) and imported inputs, and so on.

A fundamental problem in many-region input–output modeling is therefore the estimation of the transactions between regions. One approach, the *interregional* model, requires a complete (ideal) set of both intra- and interregional data. For the two-region case, this means knowing $\mathbf{x}^r = [x_i^r]$, $\mathbf{x}^s = [x_i^s]$, $\mathbf{Z}^{rr} = [z_{ij}^{rr}]$ and $\mathbf{Z}^{ss} = [z_{ij}^{ss}]$ along with $\mathbf{Z}^{rs} = [z_{ij}^{rs}]$ – recording transactions from sector i in region r to sector j in region s – and $\mathbf{Z}^{sr} = [z_{ij}^{sr}]$ – in which flows from s to r are captured. It is the last two matrices that cause the most trouble. In practice, it is never the case that one has such detailed information, and the requirements grow quickly with the number of regions – a three-region model has six interregional matrices, a four-region model has 12, and so on.

Alternative forms of many-region input–output models were created and elaborated by members of the Harvard Economic Research Project (HERP) under Leontief's direction, from its inception through the 1960s.[8] Taken chronologically, the interregional input–output model (IRIO) structure was first described by Isard (1951) and elaborated in Isard *et al.* (1960). (This is often labeled the "Isard model".) Leontief *et al.* (1953) sketched the framework of an intranational input–output model (often referred to as a "balanced regional model;" section 3.5, below). This was later applied to assess the sectoral and regional impact of a cut in US arms spending in Leontief *et al.* (1965). The multiregional input–output model (MRIO) was (almost simultaneously) described in Chenery (1953) (a two-region model for Italy) and in Moses (1955) (a nine-region US model) – thus the label "Chenery–Moses model." Finally, Leontief and Strout (1963)

[8] HERP was started at Harvard by Leontief in 1948 and continued until 1972. Thorough accounts of this formative work can be found in Polenske (1995, 2004).

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Table 3.1 Interindustry, Interregional Flows of Goods

		Purchasing Sector				
		Region r			Region s	
Selling Sector		1	2	3	1	2
Region r	1	z_{11}^{rr}	z_{12}^{rr}	z_{13}^{rr}	z_{11}^{rs}	z_{12}^{rs}
	2	z_{21}^{rr}	z_{22}^{rr}	z_{23}^{rr}	z_{21}^{rs}	z_{22}^{rs}
	3	z_{31}^{rr}	z_{32}^{rr}	z_{33}^{rr}	z_{31}^{rs}	z_{32}^{rs}
Region s	1	z_{11}^{sr}	z_{12}^{sr}	z_{13}^{sr}	z_{11}^{ss}	z_{12}^{ss}
	2	z_{21}^{sr}	z_{22}^{sr}	z_{23}^{sr}	z_{21}^{ss}	z_{22}^{ss}

proposed a gravity-model approach to estimation of interregional flows in a connected-region input–output model.[9] In this section we explore the interregional input–output (IRIO) model.

3.3.1 *Basic Structure of Two-Region Interregional Input–Output Models*

For purposes of illustration, we consider a two-region economy (for example, in Italy, northern Italy and southern Italy; or, in the United States, New England and the rest of the United States). Using r and s, as before, for the two regions, let there be three producing sectors (1, 2, 3) in region r and two (1, 2) in region s. Suppose that one has information for region r on both *intra*regional flows, z_{ij}^{rr}, and *inter*regional flows, z_{ij}^{sr}. There will be nine of the former and six of the latter. Suppose, further, that the same kind of information is available (perhaps through a survey) on the use of inputs by firms located in region s, z_{ij}^{rs} and z_{ij}^{ss}. This complete table of intraregional and interregional data can be represented as

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{rr} & \mathbf{Z}^{rs} \\ \mathbf{Z}^{sr} & \mathbf{Z}^{ss} \end{bmatrix}$$

Table 3.1 indicates the full set of data.[10]

In the regional models of section 3.2, we utilized intraregional information only – as in (3.2), (3.3), and (3.4). We now want to incorporate much more explicitly the interregional linkages, as represented by information in $\mathbf{Z}^{rs}$ and $\mathbf{Z}^{sr}$.

[9] Isard *et al.* (1960, esp. Chapter 11) described gravity models and explored their potential for estimating interregional interactions (including commodity flows) in detail. We explore the gravity approach and others in section 8.6, below, on estimating interregional flows.

[10] To be more consistent with already-familiar subscript notation, one could denote the regions by 1 and 2, respectively. Then an element such as z_{13}^{sr} would be denoted z_{13}^{21}. However, for purposes of exposition it seems clearer to use lowercase letters to designate regions; for example, so as to avoid having z's with four different numbers attached to them.

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These off-diagonal matrices need not be square. Here $\mathbf{Z}^{rs}$ has dimensions 3×2 and $\mathbf{Z}^{sr}$ is a 2×3 matrix. The on-diagonal matrices are always square; for this example, $\mathbf{Z}^{rr}$ and $\mathbf{Z}^{ss}$ are 3×3 and 2×2, respectively. While the elements in $\mathbf{Z}^{rs}$ represent "exports" from region r and simultaneously "imports" to region s, it is usual in regional input–output work to refer to these as *interregional trade* (or simply *trade*) flows and to use the terms *export* and *import* when dealing with foreign trade that crosses national, not just regional, boundaries.

By surveying firms in both regions on their purchases of locally produced inputs and inputs from the other region, one would accumulate the data shown in the various *columns* of Table 3.1. On the other hand, the data in Table 3.1 could also be gathered by asking firms in each region how much they sold to each sector in their region and how much they sold to sectors in the other region. This would generate the figures shown in the various *rows* of Table 3.1.[11]

Consider again the basic equation for the distribution of sector i's product, as given in equation (2.1) of Chapter 2:

$$x_i = z_{i1} + z_{i2} + \cdots + z_{ij} + \cdots + z_{in} + f_i$$

One of the components recorded in the final-demand term was exports of sector i goods. In the two-region interregional input–output model, that part of f_i that represents sales of sector i's product to the productive sectors in the other region (but not to consumers in the other region) is removed from the final-demand category and specified explicitly. For our two-region example, the output of sector 1 in region r would be expressed as

$$x_1^r = \underbrace{z_{11}^{rr} + z_{12}^{rr} + z_{13}^{rr}}_{\substack{\text{Sector 1 intraregional,} \\ \text{interindustry sales}}} + \underbrace{z_{11}^{rs} + z_{12}^{rs}}_{\substack{\text{Sector 1 interregional,} \\ \text{interindustry sales}}} + \underbrace{f_1^r}_{\substack{\text{Sector 1 intraregional} \\ \text{sales to final demand}}} \tag{3.5}$$

There will be similar equations for x_2^r and x_3^r, and also for x_1^s and x_2^s. The regional input coefficients for region r were given in (3.2). There will also be a set for region s,

$$a_{ij}^{ss} = \frac{z_{ij}^{ss}}{x_j^s} \tag{3.6}$$

Interregional trade coefficients are found in the same manner, where the denominators are gross outputs of sectors in the receiving region. Here these are

$$a_{ij}^{rs} = \frac{z_{ij}^{rs}}{x_j^s} \text{ and } a_{ij}^{sr} = \frac{z_{ij}^{sr}}{x_j^r} \tag{3.7}$$

Using these regional input and trade coefficients, (3.5) can be re-expressed as

$$x_1^r = a_{11}^{rr}x_1^r + a_{12}^{rr}x_2^r + a_{13}^{rr}x_3^r + a_{11}^{rs}x_1^s + a_{12}^{rs}x_2^s + f_1^r \tag{3.8}$$

[11] Usually, one has some (not complete) information on purchases and also some (not complete) information on sales. The problem then is to produce a table from possibly inconsistent data. This reconciliation problem is discussed in section 8.9.

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Again, there will be similar expressions for x_2^r, x_3^r, x_1^s, and x_2^s. [Compare the equations (2.4) in Chapter 2, where there was no regional dimension – no superscripts r and s – and where there were n sectors.] Following the same development as in Chapter 2, by moving all terms involving $\mathbf{x}^r$ or $\mathbf{x}^s$ to the left (3.8) becomes

$$(1-a_{11}^{rr})x_1^r - a_{12}^{rr}x_2^r - a_{13}^{rr}x_3^r - a_{11}^{rs}x_1^s - a_{12}^{rs}x_2^s = f_1^r \tag{3.9}$$

There are similar equations with f_2^r, f_3^r, f_1^s, and f_2^s on the right-hand sides.

For the present example, $\mathbf{A}^{rr}$ [(3.3)] is

$$\mathbf{A}^{rr} = \begin{bmatrix} a_{11}^{rr} & a_{12}^{rr} & a_{13}^{rr} \\ a_{21}^{rr} & a_{22}^{rr} & a_{23}^{rr} \\ a_{31}^{rr} & a_{32}^{rr} & a_{33}^{rr} \end{bmatrix}$$

Also, for this example, $\mathbf{A}^{ss} = \mathbf{Z}^{ss}(\hat{\mathbf{x}}^s)^{-1}$, and the two trade coefficients matrices are $\mathbf{A}^{rs} = \mathbf{Z}^{rs}(\hat{\mathbf{x}}^s)^{-1}$ and $\mathbf{A}^{sr} = \mathbf{Z}^{sr}(\hat{\mathbf{x}}^r)^{-1}$. Using these four matrices, the five equations of which (3.9) is the first can be represented compactly as

$$\begin{aligned} (\mathbf{I}-\mathbf{A}^{rr})\mathbf{x}^r \ - \ \mathbf{A}^{rs}\mathbf{x}^s &= \mathbf{f}^r \\ -\mathbf{A}^{sr}\mathbf{x}^r \ + (\mathbf{I}-\mathbf{A}^{ss})\mathbf{x}^s &= \mathbf{f}^s \end{aligned} \tag{3.10}$$

where $\mathbf{f}^r$ is the three-element vector of final demands for region r goods, and $\mathbf{f}^s$ is the two-element vector of final demands for region s goods.

We define the complete coefficients matrix for a two-region interregional model as consisting of the four submatrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{rr} & \mathbf{A}^{rs} \\ \mathbf{A}^{sr} & \mathbf{A}^{ss} \end{bmatrix}$$

For the current example, this will be a 5×5 matrix. Similarly, let

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^r \\ \mathbf{x}^s \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} \mathbf{f}^r \\ \mathbf{f}^s \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} \underset{(3\times3)}{\mathbf{I}} & \underset{(3\times2)}{\mathbf{0}} \\ \underset{(2\times3)}{\mathbf{0}} & \underset{(2\times2)}{\mathbf{I}} \end{bmatrix}$$

Then (3.10) can be expressed as

$$(\mathbf{I}-\mathbf{A})\mathbf{x} = \mathbf{f} \tag{3.11}$$

as in (2.10) in Chapter 2. To highlight the structure of (3.11), it can be expressed less compactly as

$$\left\{ \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{A}^{rr} & \mathbf{A}^{rs} \\ \mathbf{A}^{sr} & \mathbf{A}^{ss} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{x}^r \\ \mathbf{x}^s \end{bmatrix} = \begin{bmatrix} \mathbf{f}^r \\ \mathbf{f}^s \end{bmatrix} \tag{3.12}$$

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Note that in using an interregional model of this kind for analysis, not only is stability of the (intra)regional input coefficients necessary (the elements of $\mathbf{A}^{rr}$ and $\mathbf{A}^{ss}$), but also interregional input coefficients in $\mathbf{A}^{rs}$ and $\mathbf{A}^{sr}$ are assumed unvarying over time. Thus both the structure of production in each region and interregional trade patterns are "frozen" in the model. For a given level of final demands in either or both regions, the necessary gross outputs in both regions can be found in the usual input–output fashion as $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$. As is clear from (3.12), this complete $(\mathbf{I} - \mathbf{A})$ matrix will be larger than that for the single-region model – if both regions are divided into n sectors, the single-region matrix would be of size $n \times n$ and the full two-region interregional model would be $2n \times 2n$, which means four times as many (possible) elements of information are needed (many of which may be zero, of course). However, aside from these dimensionality effects, the analysis proceeds along similar lines.

The advantage is that the model captures the magnitude of effects on each sector in each region; interregional linkages are made specific by sector in the supplying region and by sector in the receiving region. The accompanying disadvantages are primarily the greatly increased data needs and the necessary assumptions of constancy of interregional trading relationships. If it is not always easy to accept the idea of constant input coefficients in general, in the national input–output model, it may be even more difficult to believe that imports of good i per dollar's worth of sector j output in a specific region remain constant, no matter how much sector j's output changes.

3.3.2 *Interregional Feedbacks in the Two-Region Model*

Consider an increase in the demand by a foreign airline for commercial aircraft produced in Washington State (region r). Certain subassemblies and parts will be purchased from sectors outside the region (for example, jet engines from Connecticut, region s). This stimulus of new output in Connecticut because of new output in Washington is often called an interregional spillover. The increased demand for aircraft will increase the demand for engines and consequently for all of the direct and indirect inputs to the manufacture of jet engines, one of which might be extruded aluminum components made in Washington. This idea is illustrated in Figure 3.1.

The downward arrow connecting Washington output to Connecticut output represents an interregional spillover effect; the upward arrow from Connecticut to Washington

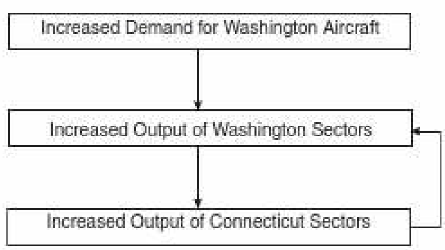

Figure 3.1 Increases in Washington Final Demands Affecting Washington Outputs via Connecticut

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is also an interregional spillover – the first originates in Washington ($r \rightarrow s$), the second originates in Connecticut ($s \rightarrow r$). The loop (two arrows) connecting Washington output to itself, via Connecticut output, represents an interregional feedback effect ($r \rightarrow r$); in other words, Washington needs more inputs from Connecticut and therefore Connecticut needs more inputs from everywhere, including Washington. The interregional model in its two-matrix-equation form [in (3.10)] allows one to isolate exactly the magnitude of such interregional feedbacks.

Suppose, in (3.10), that we read $\mathbf{x}^r$, $\mathbf{x}^s$, $\mathbf{f}^r$ and $\mathbf{f}^s$ as "changes in" – that is, $\Delta\mathbf{x}^r$, $\Delta\mathbf{x}^s$, $\Delta\mathbf{f}^r$, and $\Delta\mathbf{f}^s$. Given a vector of changes in final demands in the two regions, we can find the consequent changes in gross outputs in both regions. Assume, for simplicity, that $\Delta\mathbf{f}^s = \mathbf{0}$; we are assessing the impacts in both regions of a change in final demands in region r only. Under these conditions, solving the second equation in (3.10) for $\mathbf{x}^s$ gives

$$\mathbf{x}^s = (\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{sr}\mathbf{x}^r$$

and putting this into the first equation, we have

$$(\mathbf{I} - \mathbf{A}^{rr})\mathbf{x}^r - \mathbf{A}^{rs}(\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{sr}\mathbf{x}^r = \mathbf{f}^r \tag{3.13}$$

Note that a single-region model (for region r), as in (3.4), would be $(\mathbf{I} - \mathbf{A}^{rr})\mathbf{x}^r = \mathbf{f}^r$. The "extra" (second) term, subtracted on the left in (3.13),

$$\mathbf{A}^{rs}(\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{sr}\mathbf{x}^r \tag{3.14}$$

represents exactly the added demands made on the output of region r because of interregional trade linkages; it is an interregional feedback term. Consider the various parts, starting at the right: (a) $\mathbf{A}^{sr}\mathbf{x}^r$ captures the magnitude of flows from s to r because of increased output in r [the value of engines that are shipped from Connecticut to Washington for installation in the new airplanes], (b) $(\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{sr}\mathbf{x}^r$ then translates these flows into total direct and indirect needs in s to produce the required shipments from s (Connecticut production in all sectors needed to supply the engines for shipment to Washington), (c) $\mathbf{A}^{rs}(\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{sr}\mathbf{x}^r$ indicates the magnitude of the additional sales from r to s that will be necessary to sustain the total s-based production found in (b) [new outputs from Washington sectors to satisfy Connecticut demand for inputs to Connecticut production quantified in (b)].[12]

Thus the strength and importance of interregional linkages depend not only on the elements of the interregional input coefficients matrices – $\mathbf{A}^{rs}$ and $\mathbf{A}^{sr}$, in this example – but also on the full set of regional input coefficients in the other region, as represented by $(\mathbf{I} - \mathbf{A}^{ss})^{-1}$. It is precisely these kinds of spatial linkages that distinguish complete interregional models from single-region models. Since the feedback term is subtracted from $(\mathbf{I} - \mathbf{A}^{rr})\mathbf{x}^r$ in (3.13), a given value of $\mathbf{f}^r$ will generate a larger $\mathbf{x}^r$ than in a single-region analysis in order that the required shipments to region s can be met, as well

[12] The arrows in Figure 3.1 indicate the directions of transmission of demands to producers. The output responses to those demands travel in the opposite direction along the arrows.

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Table 3.2 Flow Data for a Hypothetical Two-Region Interregional Case

		Purchasing Sector				
		Region r			Region s	
Selling Sector		1	2	3	1	2
Region r	1	150	500	50	25	75
	2	200	100	400	200	100
	3	300	500	50	60	40
Region s	1	75	100	60	200	250
	2	50	25	25	150	100

as the usual intraregional shipments, $\mathbf{A}^{rr}\mathbf{x}^r$. In terms of outputs, the single- and two-region models will generate $x^r = (\mathbf{I} - \mathbf{A}^{rr})^{-1}\mathbf{f}^r$ and $\mathbf{x}^r = (\mathbf{I} - \mathbf{A}^{rr} - \mathbf{A}^{rs}\mathbf{L}^{ss}\mathbf{A}^{sr})^{-1}\mathbf{f}^r$, respectively.

3.3.3 *Numerical Example: Hypothetical Two-Region Interregional Case*

To illustrate for the two-region case, suppose that the figures in Table 3.2 represent the data in Table 3.1. Also, let

$$\mathbf{f}^r = \begin{bmatrix} 200 \\ 1000 \\ 50 \end{bmatrix} \text{ and } \mathbf{f}^s = \begin{bmatrix} 515 \\ 450 \end{bmatrix}, \text{ so that } \mathbf{f} = \begin{bmatrix} \mathbf{f}^r \\ \mathbf{f}^s \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \\ 50 \\ \\ 515 \\ 450 \end{bmatrix}$$

Thus

$$\mathbf{x}^r = \begin{bmatrix} 1000 \\ 2000 \\ 1000 \end{bmatrix}, \ \mathbf{x}^s = \begin{bmatrix} 1200 \\ 800 \end{bmatrix}, \text{ and } \mathbf{x} = \begin{bmatrix} \mathbf{x}^r \\ \mathbf{x}^s \end{bmatrix} = \begin{bmatrix} 1000 \\ 2000 \\ 1000 \\ \\ 1200 \\ 800 \end{bmatrix}$$

and $\mathbf{A}^{rr}$ is found to be

$$\mathbf{A}^{rr} = \begin{bmatrix} .150 & .250 & .050 \\ .200 & .050 & .400 \\ .300 & .250 & .050 \end{bmatrix}$$

Similarly,

$$\mathbf{A}^{ss} = \begin{bmatrix} .1667 & .3125 \\ .1250 & .1250 \end{bmatrix}, \ \mathbf{A}^{rs} = \begin{bmatrix} .0208 & .0938 \\ .1667 & .1250 \\ .0500 & .0500 \end{bmatrix}, \ \mathbf{A}^{sr} = \begin{bmatrix} .0750 & .0500 & .0600 \\ .0500 & .0125 & .0250 \end{bmatrix}$$

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so

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{rr} & \mathbf{A}^{rs} \\ \mathbf{A}^{sr} & \mathbf{A}^{ss} \end{bmatrix} = \begin{bmatrix} 0.1500 & 0.2500 & 0.0500 & 0.0208 & 0.0938 \\ 0.2000 & 0.0500 & 0.4000 & 0.1667 & 0.1250 \\ 0.3000 & 0.2500 & 0.0500 & 0.0500 & 0.0500 \\ 0.0750 & 0.0500 & 0.0600 & 0.1667 & 0.3125 \\ 0.0500 & 0.0125 & 0.0250 & 0.1250 & 0.1250 \end{bmatrix}$$

and define

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix} = \begin{bmatrix} 1.4234 & 0.4652 & 0.2909 & 0.1917 & 0.3041 \\ 0.6346 & 1.4237 & 0.6707 & 0.4092 & 0.4558 \\ 0.6383 & 0.5369 & 1.3363 & 0.2501 & 0.3108 \\ 0.2672 & 0.2000 & 0.1973 & 1.3406 & 0.5473 \\ 0.1468 & 0.0908 & 0.0926 & 0.2155 & 1.2538 \end{bmatrix}$$

We use $\mathbf{L}_{11}$, $\mathbf{L}_{12}$ and so on because later it will be necessary to refer to these individual submatrices in $\mathbf{L}$, and they are to be distinguished from $\mathbf{L}^{rr} = (\mathbf{I} - \mathbf{A}^{rr})^{-1}$ and $\mathbf{L}^{ss} = (\mathbf{I} - \mathbf{A}^{ss})^{-1}$, which are often used to denote Leontief inverses associated with regional direct input coefficients matrices.

Impacts on the sectors in both regions of various new final-demand vectors in either or both regions can now be found. For example, with new demand of 100 for the output of sector 1 in region r, $(\mathbf{f}^{new})' = [\,100 \;\; 0 \;\; 0 \quad 0 \;\; 0\,]$, and, using $\mathbf{L}$, above,

$$\mathbf{x}^{new} = \begin{bmatrix} (\mathbf{x}^r)^{new} \\ (\mathbf{x}^s)^{new} \end{bmatrix} = \mathbf{L}\mathbf{f}^{new} = \begin{bmatrix} 142.34 \\ 63.46 \\ 63.83 \\ 26.72 \\ 14.68 \end{bmatrix}$$

The new outputs in region s of sectors 1 (26.72) and 2 (14.68) that result from the new demand in region r reflect *interregional spillovers* – economic stimulus in a region other than the one in which the exogenous change occurs (in this case spillovers from region r to region s).

It is to be emphasized that the final demands in the interregional input–output model are for outputs produced in a particular region. That is, $f_1^r = 100$ means that there is a final demand of 100 for sector 1 goods that are produced in region r. If sector 1 were aircraft production and region r were Washington, new orders from a foreign airline for Boeing commercial airliners would be represented in the value for f_1^r.

Using these hypothetical data, we can illustrate the differences between the results from a single-region model for region r alone and the results from this two-region interregional model. From the information on $\mathbf{A}^{rr}$ alone we find

$$\mathbf{L}^{rr} = (\mathbf{I} - \mathbf{A}^{rr})^{-1} = \begin{bmatrix} 1.3651 & .4253 & .2509 \\ .5273 & 1.3481 & .5954 \\ .5698 & .4890 & 1.2885 \end{bmatrix}$$

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Using this single-region model with $(\mathbf{f}^r)^{new} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$ and ignoring interregional linkages, as in (3.4), we have

$$\mathbf{x}_S^r = \mathbf{L}^{rr}\mathbf{f}^r = \begin{bmatrix} 1.3651 & .4253 & .2509 \\ .5273 & 1.3481 & .5954 \\ .5698 & .4890 & 1.2885 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 136.51 \\ 52.73 \\ 56.99 \end{bmatrix}$$

We use a subscript S to make clear that these are outputs in the *single*-region model, and we drop the superscript "*new*." With the complete two-region model we had, for region r,

$$\mathbf{x}_T^r = \begin{bmatrix} 142.34 \\ 63.46 \\ 63.83 \end{bmatrix}$$

Here, $\mathbf{x}_T^r$ reminds us that these are outputs in the *two*-region interregional model. The difference in results for region r is seen to be

$$\mathbf{x}_T^r - \mathbf{x}_S^r = \begin{bmatrix} 142.34 \\ 63.46 \\ 63.83 \end{bmatrix} - \begin{bmatrix} 136.51 \\ 52.73 \\ 56.99 \end{bmatrix} = \begin{bmatrix} 5.83 \\ 10.73 \\ 6.84 \end{bmatrix}$$

Each region r output is larger in the interregional model because the interregional feedbacks are captured in that model. One measure of the "error" that would be involved in ignoring these feedbacks – in using a single-region model as opposed to an interregional model – would be given by the percentage of total output in region r that one fails to capture when using a single-region model only. Total output over all sectors in region r in the two-region model is $\mathbf{i}'\mathbf{x}_T^r = 269.63$. Total output estimated in the single-region model is $\mathbf{i}'\mathbf{x}_S^r = 246.23$. By this measure, the underestimate that occurs in using the single-region model is $\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{x}_S^r = 23.40$, or $(23.40/269.63) \times 100 = 8.7$ percent of the total true (two-region model) output. Formally, this *overall percentage error* measure is found as

$$OPE = [(\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{x}_S^r)/\mathbf{i}'\mathbf{x}_T^r] \times 100 = [\mathbf{i}'(\mathbf{x}_T^r - \mathbf{x}_S^r)/\mathbf{i}'\mathbf{x}_T^r] \times 100$$

It thus becomes an interesting empirical question to try to assess the importance of interregional feedbacks in real-world regional input–output models. If it turned out that the error caused by ignoring interregional linkages when assessing the impact of new region r final demands on region r outputs was quite small, then one might argue that (at least for such questions) the apparatus of an interregional model would be unnecessary. The answer will depend, in part, upon the relative strengths of the interregional linkages; in the two-region model this means on the magnitudes of the elements in $\mathbf{A}^{rs}$ and $\mathbf{A}^{sr}$. Precisely this question has been investigated; however, the results are inconclusive. The conclusion from an early set of experiments was that interregional feedback effects were likely to be very small (less than one half of one percent, using the overall

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percentage error measure presented above for illustration). (See Miller, 1966, 1969.) Other studies have tended to confirm the relative smallness of interregional feedback effects by comparing output multipliers from single- and many-region input–output models. (Chapter 6.) There has been work on derivation of upper limits on the percentage error that could be expected in certain interregional input–output models when the interregional feedbacks are ignored (in particular, Gillen and Guccione, 1980; Miller, 1986; Guccione *et al.*, 1988).

The error caused by ignoring interregional feedbacks is strongly influenced by the level of self-sufficiency in region r – whether or not region r is relatively dependent on inputs from region s. This is because higher dependence is reflected in larger coefficients in $\mathbf{A}^{sr}$ which, again as in (3.14), generate a larger feedback term. Self-sufficiency is also a function of the geographic size of the region. In a two-region model with Nebraska (region r) and the rest of the United States (region s), the average element in $\mathbf{A}^{sr}$ will be larger than in a two-region model in which region r is the United States west of the Mississippi and region s is the United States east of the Mississippi. However, in the Nebraska (r)/rest-of-the-United States (s) example, the elements in $\mathbf{A}^{rs}$ (reflecting rest-of-the-United States dependence on inputs from Nebraska) will be generally much smaller than in the United States West (r)/United States East (s) example. Thus it is not easy to generalize on how the geographical size of the respective regions ultimately influences the size of the interregional feedbacks.

In any case, a single-region model, by definition, cannot capture effects outside of that region (spillovers) in regional/sectoral detail, and there are many kinds of economic impact questions that have important ramifications in more than one region of a national economy. In these cases, some kind of connected-region model is essential. The interregional input–output framework provides one such approach. Feedbacks and spillovers in input–output models will be examined again in Chapter 6 when we discuss multiplier decompositions.

Some analysts (for example, Oosterhaven, 1981) suggest that measurement of feedback effects should be based not on total impacts (direct and indirect) but rather should be found as percentages of indirect impacts only – without the first term in the power series or with $\mathbf{f}$ netted out from gross outputs in $OPE = [(\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{x}_S^r)/\mathbf{i}'\mathbf{x}_T^r] \times 100$. This means

$$\begin{aligned} OPE^n &= \{[(\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{f}) - (\mathbf{i}'\mathbf{x}_S^r - \mathbf{i}'\mathbf{f})]/(\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{f})\} \times 100 \\ &= [(\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{x}_S^r)/(\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{f})] \times 100 \end{aligned}$$

This "net" measure is larger than OPE (except in the trivial case when $\mathbf{f} = \mathbf{0}$); namely $OPE^n = (OPE)\left(\frac{\mathbf{i}'\mathbf{x}_T^r}{\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{f}}\right)$. In our numerical example, $\left(\frac{\mathbf{i}'\mathbf{x}_T^r}{\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{f}}\right) = 1.59$ and $OPE^n = 13.8$. Alternatively, $100 \times (OPE/OPE^n) = 10 \times \left(\frac{\mathbf{i}'\mathbf{x}_T^r - \mathbf{i}'\mathbf{f}}{\mathbf{i}'\mathbf{x}_T^r}\right)$ indicates the percentage of the net measure that is captured by the original measure. In the example, this is 63 percent.

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3.3.4 Interregional Models with more than Two Regions

The fundamental structure of models with more than two regions is identical to the two-region case in section 3.3.1, although the numbers of matrices and their sizes increase. The objective is to capture explicitly the various economic connections between and among the several regions in a multiregional economy. For example, in a three-region model (regions 1, 2, and 3), the complete coefficients matrix would be

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{11} & \mathbf{A}^{12} & \mathbf{A}^{13} \\ \mathbf{A}^{21} & \mathbf{A}^{22} & \mathbf{A}^{23} \\ \mathbf{A}^{31} & \mathbf{A}^{32} & \mathbf{A}^{33} \end{bmatrix} \tag{3.15}$$

and the parallel to (3.10) is

$$\begin{aligned} (\mathbf{I}-\mathbf{A}^{11})\mathbf{x}^1 - \mathbf{A}^{12}\mathbf{x}^2 - \mathbf{A}^{13}\mathbf{x}^3 &= \mathbf{f}^1 \\ -\mathbf{A}^{21}\mathbf{x}^1 + (\mathbf{I}-\mathbf{A}^{22})\mathbf{x}^2 - \mathbf{A}^{23}\mathbf{x}^3 &= \mathbf{f}^2 \\ -\mathbf{A}^{31}\mathbf{x}^1 - \mathbf{A}^{32}\mathbf{x}^2 + (\mathbf{I}-\mathbf{A}^{33})\mathbf{x}^3 &= \mathbf{f}^3 \end{aligned} \tag{3.16}$$

With $\mathbf{x} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \mathbf{x}^3 \end{bmatrix}$, $\mathbf{f} = \begin{bmatrix} \mathbf{f}^1 \\ \mathbf{f}^2 \\ \mathbf{f}^3 \end{bmatrix}$ and $\mathbf{I} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$, the complete three-region interregional input–output model is still represented as $(\mathbf{I}-\mathbf{A})\mathbf{x} = \mathbf{f}$. The underlying logic is the same as that for the two-region model, and the equations in (3.16) can be built up in the same way as were those in (3.10). Also, the magnitudes of the interregional feedback effects can be made specific.

The extension to a p-region model is straightforward. (For example, there are nine-region models for Japan, noted in section 3.3.5, below.) The parallel to (3.16) is

$$\begin{aligned} &(\mathbf{I}-\mathbf{A}^{11})\mathbf{x}^1 - \mathbf{A}^{12}\mathbf{x}^2 - \cdots - \mathbf{A}^{1p}\mathbf{x}^p = \mathbf{f}^1 \\ &\vdots \\ &-\mathbf{A}^{p1}\mathbf{x}^1 - \mathbf{A}^{p2}\mathbf{x}^2 - \cdots + (\mathbf{I}-\mathbf{A}^{pp})\mathbf{x}^p = \mathbf{f}^p \end{aligned} \tag{3.17}$$

(The interested reader can construct the parallel expressions for **A**, **I**, **f** and **x**.)

The data requirements increase quickly with the number of regions. Assuming that all regions are divided into n sectors (not a necessary requirement at all – each region could have a different number of sectors), a complete two-region interregional model requires data for four coefficients matrices of size $n \times n$, a three-region model contains nine $n \times n$ matrices, a four-region model has sixteen such matrices, and a p-region model has p^2 such $n \times n$ matrices. However, interregional models with a relatively small number of regions may be useful, since one region can always be defined as the "rest of the country" or the "rest of the world." A three-region model might concentrate on a particular county, region 2 could be the "rest of the state" and region 3 the "rest of the nation" (outside the state).

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3.3.5 *Implementation of the IRIO Model*

Clearly, the interregional input–output model requires a large amount of detailed data. For this reason, there have been few real-world applications. Perhaps the most ambitious attempts at implementation are contained in the impressive series of Japanese survey-based interregional tables, with nine regions and (ultimately) 25 sectors, beginning with 1960 and updated every five years. [See Ministry of International Trade and Industry (MITI), various years; this was reorganized as the Ministry of Economy, Trade and Industry (METI) in 2001.] This very rich data source has generated a number of Japanese comparative regional studies (see, for example, Akita, 1994, 1999; Akita and Kataoka, 2002).

3.4 Many-Region Models: The Multiregional Approach

While a complete interregional model of the sort described in section 3.3 is generally impossible to implement for very many regions and/or sectors because of the enormous amounts of data that it requires, the approach has inspired modifications and simplifications in the direction of a more operational framework. One attempt in this direction uses the "Chenery–Moses" approach (noted in section 3.3, above) for consistent estimation of the intra- and interregional transactions required in the IRIO model. It has come to be known as a multiregional input–output model. It contains counterparts to the regional input coefficients matrices – as in $\mathbf{A}^{rr}$ – and the interregional input (trade) coefficients matrices – as in $\mathbf{A}^{rs}$. In both cases the attempt has been to specify a model in which the data are more easily obtained.

Polenske examined and implemented three versions of the MRIO model – the Chenery–Moses version (also known as a "column-coefficient" model for reasons that will become clear below), an alternative row-coefficient version, and one using the gravity model approach of Leontief and Strout (1963).[13] Problems with the latter two approaches ultimately precluded their use, and the column-coefficient model was chosen as the structure on which to develop the US MRIO model. [Polenske, 1970a, 1970b, 1980, 1995 (section 2), 2004 (section 8); Bon, 1984.]

3.4.1 *The Regional Tables*

The multiregional input–output model uses a regional *technical* coefficients matrix, $\mathbf{A}^r$, in place of the regional *input* coefficients matrix, $\mathbf{A}^{rr}$. These regional technical coefficients, a_{ij}^r, can be produced from responses to the question "How much sector i product did you buy last year in making your output?" [Question (1) in section 3.2], where they were contrasted with the regional input coefficients, a_{ij}^{rr}. Information regarding the region of origin of a given input is ignored; one only needs information on the dollars' worth of input from sector i used by sector j in region r. These transactions are usually

[13] Leontief and Strout (1963) "devised the multiregional input–output (MRIO) accounts" (Polenske and Hewings, 2004, p. 274).

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denoted by $z_{ij}^{\cdot r}$, where the dot indicates that all possible geographical locations for sector i are lumped together.[14] These coefficients are defined as $a_{ij}^{r} = \frac{z_{ij}^{\cdot r}}{x_j^r}$ and $\mathbf{A}^r = [a_{ij}^r]$.

In practice, when actual regional data on technology are not available, estimates of regional technical coefficients matrices are sometimes made using what is known as the product-mix approach. The basic assumption is that input requirements per unit of output are constant from region to region at a very fine level of industrial classification, but that an important distinguishing characteristic of production at the regional level is the composition of sector outputs, when one is dealing with more aggregate sectors. To return to our earlier illustration of the product-mix problem, when two-engine commercial jets are made in Washington (or anywhere else), they use, among other things, two jet engines as inputs; when single-engine propeller-driven private aircraft are made in Florida or in any other state, they use one propeller engine as one of the inputs to production. But the important fact to capture is that the output of the sector designated "aircraft" in a Washington table is composed of a vastly different mix of products (commercial jets) than the "aircraft" sector in Florida (private/corporate airplanes).

To illustrate, assume that sector 2 is food and kindred products, and that it contains only three subsectors, which can be designated by their outputs: tomato soup (sector 2.1), chocolate bars (sector 2.2), and guava jelly (sector 2.3). Assume that the *national* technical coefficients from sector 8, paper and allied products, to each of these subsectors are: 0.005, 0.009, and 0.003. (These represent various aspects of packaging – labels, wrappers, etc.) Suppose that we want to derive coefficients for inputs from sector 8 to sector 2, a_{82}, for New Jersey (region J) and for Florida (region F). The data that we would need are shown in Table 3.3, where N designates *national* data. The food and kindred products sector was composed of only tomato soup ($700,000) and chocolate bars ($300,000) output (no guava jelly) in New Jersey; in Florida it was made up of tomato soup ($80,000) and guava jelly ($420,000) – no chocolate bars.

Purchases of paper and allied products as inputs to New Jersey food and kindred products production over the period covered by the output figures in Table 3.3 are then assumed to be the sum of

$$a_{8,2.1}^{N}\, x_{2.1}^{J} = (.005)(700,000) = 3500$$
$$a_{8,2.2}^{N}\, x_{2.2}^{J} = (.009)(300,000) = 2700$$
$$a_{8,2.3}^{N}\, x_{2.3}^{J} = (.003)(0) = 0$$

for a total of $6200 in necessary inputs from sector 8 to production in sector 2 in New Jersey. Since $x_2^J = x_{2.1}^J + x_{2.2}^J + x_{2.3}^J = 1,000,000$,

$$a_{82}^{J} = 6200/1,000,000 = .0062$$

[14] Sometimes a small $\circ$ or a larger dot is used, primarily because it is easier to read.

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Table 3.3 Data Needed for Conversion of National to Regional Coefficients via the Product-Mix Approach

National Data			
To sector 2: Food and Kindred Products			
Subsectors	2.1 (tomato soup)	2.2 (chocolate bars)	2.3 (guava jelly)
From sector 8: Paper and Allied Products			
	$a^N_{8,2.1} = .005$	$a^N_{8,2.2} = .009$	$a^N_{8,2.3} = .003$
Regional Data			
Outputs (in 1000 dollars) by subsector of sector 2			
	(New Jersey)	(Florida)	
	$x^J_{2.1} = 700$	$x^F_{2.1} = 80$	
	$x^J_{2.2} = 300$	$x^F_{2.2} = 0$	
	$x^J_{2.3} = 0$	$x^F_{2.3} = 420$	
Total Outputs (Sector 2)			
	$x^J_2 = 1000$	$x^F_2 = 500$	

Similarly, for Florida,

$$a^N_{8,2.1}x^F_{2.1} = (.005)(80,000) = 400$$
$$a^N_{8,2.2}x^F_{2.2} = (.009)(0) = 0$$
$$a^N_{8,2.1}x^F_{2.3} = (.003)(420,000) = 1260$$

The total Florida inputs from sector 8 would be estimated as \$1660. Since $x^F_2 = 500{,}000$, we have

$$a^F_2 = 1660/500,000 = .0033$$

Formally,

$$a^J_{82} = \frac{(a^N_{8,2.1}x^J_{2.1} + a^N_{8,2.2}x^J_{2.2} + a^N_{8,2.3}x^J_{2.3})}{x^J_2} = a^N_{8,2.1}\left(\frac{x^J_{2.1}}{x^J_2}\right) + a^N_{8,2.2}\left(\frac{x^J_{2.2}}{x^J_2}\right) + a^N_{8,2.3}\left(\frac{x^J_{2.3}}{x^J_2}\right)$$

$$a^F_{82} = \frac{(a^N_{8,2.1}x^F_{2.1} + a^N_{8,2.2}x^F_{2.2} + a^N_{8,2.3}x^F_{2.3})}{x^F_2} = a^N_{8,2.1}\left(\frac{x^F_{2.1}}{x^F_2}\right) + a^N_{8,2.2}\left(\frac{x^F_{2.2}}{x^F_2}\right) + a^N_{8,2.3}\left(\frac{x^F_{2.3}}{x^F_2}\right)$$

The *regional* coefficients derived in this way are *weighted* averages of the national detailed coefficients, where the weights are the proportions of subsector outputs to total output of the sector (e.g., $x^J_{2.1}/x^J_2$) in each state.

3.4.2 The Interregional Tables

The interconnections among regions in the multiregional input–output model are captured in an entirely different way from the interregional input–output framework. Trade

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Table 3.4 Interregional Shipments of Commodity i

	Receiving Region					
Shipping Region	1	2	$\cdots$	s	$\cdots$	p
1	z_i^{11}	z_i^{12}	$\cdots$	z_i^{1s}	$\cdots$	z_i^{1p}
2	z_i^{21}	z_i^{22}	$\cdots$	z_i^{2s}	$\cdots$	z_i^{2p}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$
r	z_i^{r1}	z_i^{r2}	$\cdots$	z_i^{rs}	$\cdots$	z_i^{rp}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
p	z_i^{p1}	z_i^{p2}	$\cdots$	z_i^{ps}	$\cdots$	z_i^{pp}
Total	T_i^1	T_i^2	$\cdots$	T_i^s	$\cdots$	T_i^p

flows in the multiregional model are estimated by sector, again to take advantage of the kinds of data likely to be available. For sector i, let z_i^{rs} denote the dollar flow of good i from region r to region s, irrespective of the sector of destination in the receiving region.[15] These flows will include shipments to the producing sectors in region s as well as to final demand in s. Thus there is, for each sector, a shipments matrix of the sort shown in Table 3.4.

Note that each of the column sums in this table represents the total shipments of good i into that region from all of the regions in the model; this total, for column s, is denoted in the table for good i by T_i^s:

$$T_i^s = z_i^{1s} + z_i^{2s} + \cdots + z_i^{rs} + \cdots + z_i^{ps} \tag{3.18}$$

If each element in column s is divided by this total, we have coefficients denoting the *proportion* of all of good i used in s that comes from each region r ($r = 1, \ldots, p$). These proportions are denoted c_i^{rs}:

$$c_i^{rs} = \frac{z_i^{rs}}{T_i^s}$$

For later use, these coefficients are rearranged as follows. For each possible origin-destination pair of regions, denote by $\mathbf{c}^{rs}$ the n-element column vector

$$\mathbf{c}^{rs} = \begin{bmatrix} c_1^{rs} \\ \vdots \\ c_n^{rs} \end{bmatrix}$$

[15] To be consistent with the notation $z_{ij}^{\cdot r}$ or z_{ij}^{or}, above, this should properly be $z_{i\cdot}^{rs}$ or $z_{i\circ}^{rs}$. However, when the blank space is in the second subscript position, it is easier to distinguish than when it is in the first superscript position, and so we avoid the double subscript option.

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These elements show, for region s, the proportion of the total amount of each good used in s that comes from region r. Finally, construct $\hat{\mathbf{c}}^{rs}$,

$$\hat{\mathbf{c}}^{rs} = \begin{bmatrix} c_1^{rs} & 0 & \cdots & 0 \\ 0 & c_2^{rs} & & \\ \vdots & & & \\ 0 & 0 & \cdots & c_n^{rs} \end{bmatrix} \tag{3.19}$$

for $r, s = 1, \ldots, p$. Note that there will be *intra*regional matrices in this set. For example, there will be a matrix $\hat{\mathbf{c}}^{ss}$, namely

$$\hat{\mathbf{c}}^{ss} = \begin{bmatrix} c_1^{ss} & 0 & \cdots & 0 \\ 0 & c_2^{ss} & & \\ \vdots & & & \\ 0 & 0 & \cdots & c_n^{ss} \end{bmatrix} \tag{3.20}$$

whose elements, $c_i^{ss} = z_i^{ss}/T_i^s$, indicate the proportion of good i used in region s that came from within region s.

3.4.3 The Multiregional Model[16]

Consider a small two-sector, two-region example, where

$$\mathbf{A}^r = \begin{bmatrix} a_{11}^r & a_{12}^r \\ a_{21}^r & a_{22}^r \end{bmatrix}, \quad \mathbf{A}^s = \begin{bmatrix} a_{11}^s & a_{12}^s \\ a_{21}^s & a_{22}^s \end{bmatrix}$$

$$\hat{\mathbf{c}}^{rs} = \begin{bmatrix} c_1^{rs} & 0 \\ 0 & c_2^{rs} \end{bmatrix}, \quad \hat{\mathbf{c}}^{ss} = \begin{bmatrix} c_1^{ss} & 0 \\ 0 & c_2^{ss} \end{bmatrix}$$

Then the multiregional input–output model uses the matrix

$$\hat{\mathbf{c}}^{rs}\mathbf{A}^s = \begin{bmatrix} c_1^{rs}a_{11}^s & c_1^{rs}a_{12}^s \\ c_2^{rs}a_{21}^s & c_2^{rs}a_{22}^s \end{bmatrix}$$

as an estimate of $\mathbf{A}^{rs}$ in the interregional input–output model. Similarly,

$$\hat{\mathbf{c}}^{ss}\mathbf{A}^s = \begin{bmatrix} c_1^{ss}a_{11}^s & c_1^{ss}a_{12}^s \\ c_2^{ss}a_{21}^s & c_2^{ss}a_{22}^s \end{bmatrix}$$

in the multiregional model replaces $\mathbf{A}^{ss}$ in the interregional model. Therefore the multiregional input–output model embodies the same assumption as was used in the earlier regional models with estimated supply percentages. Looking at the top rows of the

[16] In this section we emphasize the structural parallels between the multiregional model and the interregional model. In Appendix 3.1 to this chapter the basic relationships in the multiregional model are derived from standard economic and input–output theory.

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$\hat{\mathbf{c}}^{rs}\mathbf{A}^s$ and $\hat{\mathbf{c}}^{ss}\mathbf{A}^s$ matrices, note that both sectors 1 and 2 in region s are assumed to have the same proportion of their total use of commodity 1 supplied from region r, namely c_1^{rs}, and the same proportion supplied from within region $s - c_1^{ss}$.

Suppose that sector 1 in both regions r and s is electricity production and sector 2 in region s is automobile production, then if $c_1^{rs} = 0.6$, this means that 60 percent of all electricity used in making electricity in region s comes from region r and 60 percent of all electricity used in automobile manufacture in region s also comes from region r. And similarly, since in this two-region model it would be true that $c_1^{ss} = 0.4$, 40 percent of the electricity used in both electricity production and automobile production in s comes from within that region.

Since the interregional shipments recorded in Table 3.4 include sales to both producing sectors and final-demand users in the receiving region, the final demands in region s are met in part by firms within the region ($\hat{\mathbf{c}}^{ss}\mathbf{f}^s$) and in part by purchases from firms in region r ($\hat{\mathbf{c}}^{rs}\mathbf{f}^s$). To continue the illustration with $c_1^{rs}= 0.6$, where sector 1 is electricity production, 60 percent of the final demand for electricity in region s will *also* be satisfied by producers in region r.

The multiregional input–output counterpart to (3.10) for the interregional model is therefore

$$\begin{aligned} (\mathbf{I} - \hat{\mathbf{c}}^{rr}\mathbf{A}^r)\mathbf{x}^r - \hat{\mathbf{c}}^{rs}\mathbf{A}^s\mathbf{x}^s &= \hat{\mathbf{c}}^{rr}\mathbf{f}^r + \hat{\mathbf{c}}^{rs}\mathbf{f}^s \\ -\hat{\mathbf{c}}^{sr}\mathbf{A}^r\mathbf{x}^r + (\mathbf{I} - \hat{\mathbf{c}}^{ss}\mathbf{A}^s)\mathbf{x}^s &= \hat{\mathbf{c}}^{sr}\mathbf{f}^r + \hat{\mathbf{c}}^{ss}\mathbf{f}^s \end{aligned} \tag{3.21}$$

Let

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^r & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^s \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{rr} & \hat{\mathbf{c}}^{rs} \\ \hat{\mathbf{c}}^{sr} & \hat{\mathbf{c}}^{ss} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \mathbf{x}^r \\ \mathbf{x}^s \end{bmatrix}, \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^r \\ \mathbf{f}^s \end{bmatrix}$$

so that (3.21) can be represented as

$$(\mathbf{I} - \mathbf{CA})\mathbf{x} = \mathbf{Cf} \tag{3.22}$$

and the solution will be given by

$$\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf} \tag{3.23}$$

The extension to more than two regions is straightforward. Equations for the three-region model would be

$$\begin{aligned} (\mathbf{I} - \hat{\mathbf{c}}^{11}\mathbf{A}^1)\mathbf{x}^1 - \hat{\mathbf{c}}^{12}\mathbf{A}^2\mathbf{x}^2 - \hat{\mathbf{c}}^{13}\mathbf{A}^3\mathbf{x}^3 &= \hat{\mathbf{c}}^{11}\mathbf{f}^1 + \hat{\mathbf{c}}^{12}\mathbf{f}^2 + \hat{\mathbf{c}}^{13}\mathbf{f}^3 \\ -\hat{\mathbf{c}}^{21}\mathbf{A}^1\mathbf{x}^1 + (\mathbf{I} - \hat{\mathbf{c}}^{22}\mathbf{A}^2)\mathbf{x}^2 - \hat{\mathbf{c}}^{23}\mathbf{A}^3\mathbf{x}^3 &= \hat{\mathbf{c}}^{21}\mathbf{f}^1 + \hat{\mathbf{c}}^{22}\mathbf{f}^2 + \hat{\mathbf{c}}^{23}\mathbf{f}^3 \\ -\hat{\mathbf{c}}^{31}\mathbf{A}^1\mathbf{x}^1 - \hat{\mathbf{c}}^{32}\mathbf{A}^2\mathbf{x}^2 + (\mathbf{I} - \hat{\mathbf{c}}^{33})\mathbf{A}^3\mathbf{x}^3 &= \hat{\mathbf{c}}^{31}\mathbf{f}^1 + \hat{\mathbf{c}}^{32}\mathbf{f}^2 + \hat{\mathbf{c}}^{33}\mathbf{f}^3 \end{aligned}$$

[Compare (3.16), for the three-region interregional model.] By appropriate extension of matrices $\mathbf{A}$, $\mathbf{C}$, $\mathbf{x}$, and $\mathbf{f}$ to incorporate three regions, the fundamental model is still $(\mathbf{I} - \mathbf{CA})\mathbf{x} = \mathbf{Cf}$, as in (3.22), with solution $\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf}$, as in (3.23).

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Table 3.5 Flow Data for a Hypothetical Two-Region Multiregional Case

	Purchasing Sector					
	Region r			Region s		
Selling Sector	1	2	3	1	2	3
1	225	600	110	225	325	125
2	250	125	425	350	200	270
3	325	700	150	360	240	200

Finally, when there are p regions, let

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2 & \cdots & \mathbf{0} \\ \vdots & \vdots & & \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}^p \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{11} & \cdots & \hat{\mathbf{c}}^{1p} \\ \hat{\mathbf{c}}^{21} & \cdots & \hat{\mathbf{c}}^{2p} \\ \vdots & & \\ \hat{\mathbf{c}}^{p1} & \cdots & \hat{\mathbf{c}}^{pp} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^p \end{bmatrix}, \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^1 \\ \mathbf{f}^2 \\ \vdots \\ \mathbf{f}^p \end{bmatrix}$$

Then $(\mathbf{I} - \mathbf{CA})\mathbf{x} = \mathbf{Cf}$ and $\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf}$ still represents the system and its solution; only the dimensions of the matrices have changed.

3.4.4 Numerical Example: Hypothetical Two-Region Multiregional Case

Assume that we have the flow data in Table 3.5, representing total inputs purchased by producing sectors in each region, regardless of whether these are locally produced or imported from the other region. These are the $\mathbf{Z}^r = [z_{ij}^r]$ and $\mathbf{Z}^s = [z_{ij}^s]$ data.

Suppose, further, that $\mathbf{x}^r = \begin{bmatrix} 1000 \\ 2000 \\ 1000 \end{bmatrix}$ and $\mathbf{x}^s = \begin{bmatrix} 1200 \\ 800 \\ 1500 \end{bmatrix}$, so that the regional technical coefficients matrices, $\mathbf{A}^r = [a_{ij}^r]$ and $\mathbf{A}^s = [a_{ij}^s]$, are

$$\mathbf{A}^r = \begin{bmatrix} .225 & .300 & .110 \\ .250 & .063 & .425 \\ .325 & .350 & .150 \end{bmatrix}, \mathbf{A}^s = \begin{bmatrix} .188 & .406 & .083 \\ .292 & .250 & .180 \\ .300 & .300 & .133 \end{bmatrix}$$

For the trade proportions, we need measures of the total amount of each good, i, that is available in each region – T_i^r and T_i^s, in (3.18). Table 3.6 provides an example of these data. (Note that the row sums for each sector in each region must be the total output for that sector in that region, as recorded in the appropriate $\mathbf{x}$ vector.) The proportions – $c_i^{rs} = z_i^{rs}/T_i^s$ – are easily found. Here

$$\mathbf{c}^{rr} = \begin{bmatrix} .721 \\ .812 \\ .735 \end{bmatrix}, \mathbf{c}^{rs} = \begin{bmatrix} .183 \\ .583 \\ .078 \end{bmatrix}, \mathbf{c}^{sr} = \begin{bmatrix} .279 \\ .188 \\ .265 \end{bmatrix}, \text{ and } \mathbf{c}^{ss} = \begin{bmatrix} .817 \\ .417 \\ .922 \end{bmatrix}$$

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Table 3.6 Interregional Commodity Shipments for the Hypothetical Two-Region Multiregional Case

	Commodity 1		Commodity 2		Commodity 3	
	r	s	r	s	r	s
r	800	200	1300	700	900	100
s	310	890	300	500	325	1175
T	$T_1^r = 1110$	$T_1^s = 1090$	$T_2^r = 1600$	$T_2^s = 1200$	$T_3^r = 1225$	$T_3^s = 1275$

Thus the building blocks in this example for the two-region multiregional input–output model are

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^r & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^s \end{bmatrix} = \begin{bmatrix} .225 & .300 & .110 & 0 & 0 & 0 \\ .250 & .063 & .425 & 0 & 0 & 0 \\ .325 & .350 & .150 & 0 & 0 & 0 \\ 0 & 0 & 0 & .188 & .406 & .083 \\ 0 & 0 & 0 & .292 & .250 & .180 \\ 0 & 0 & 0 & .300 & .300 & .133 \end{bmatrix}$$

and

$$\mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{rr} & \hat{\mathbf{c}}^{rs} \\ \hat{\mathbf{c}}^{sr} & \hat{\mathbf{c}}^{ss} \end{bmatrix} = \begin{bmatrix} .721 & 0 & 0 & .183 & 0 & 0 \\ 0 & .812 & 0 & 0 & .583 & 0 \\ 0 & 0 & .735 & 0 & 0 & .078 \\ .279 & 0 & 0 & .817 & 0 & 0 \\ 0 & .188 & 0 & 0 & .417 & 0 \\ 0 & 0 & .265 & 0 & 0 & .922 \end{bmatrix}$$

Therefore

$$(\mathbf{I} - \mathbf{CA})^{-1}\mathbf{C} = \begin{bmatrix} 1.127 & .447 & .300 & .478 & .418 & .153 \\ .628 & 1.317 & .606 & .552 & 1.115 & .323 \\ .512 & .526 & 1.101 & .335 & .470 & .247 \\ .625 & .369 & .250 & 1.224 & .456 & .216 \\ .238 & .385 & .205 & .278 & .650 & .167 \\ .472 & .445 & .589 & .594 & .529 & 1.232 \end{bmatrix} \tag{3.24}$$

and, for example, the impacts of new final demands of 100 for sector 1 outputs by consumers in each region – that is, with $\mathbf{f}' = \begin{bmatrix} 100 & 0 & 0 & 100 & 0 & 0 \end{bmatrix}$ – are found,

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as in (3.23),

$$\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf} = \begin{bmatrix} 160.50 \\ 118.00 \\ 84.70 \\ \hline 184.90 \\ 51.60 \\ 106.60 \end{bmatrix}$$

So, $\mathbf{x}^r = \begin{bmatrix} 160.50 \\ 118.00 \\ 84.70 \end{bmatrix}$ and $\mathbf{x}^s = \begin{bmatrix} 184.90 \\ 51.60 \\ 106.60 \end{bmatrix}$.

Similarly, if $\mathbf{f}' = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, which represents new final demands of 100 for sector 1 output by consumers in region r only, we find

$$\mathbf{x} = \begin{bmatrix} 112.70 \\ 62.80 \\ 51.20 \\ \hline 62.50 \\ 23.80 \\ 47.20 \end{bmatrix}$$

Exactly as in an interregional model, $\mathbf{x}^s = \begin{bmatrix} 62.50 \\ 23.80 \\ 47.20 \end{bmatrix}$ reflects interregional spillovers in the multiregional system, in this case from region r (the location of the final demand change) to region s.

It is important to bear in mind, from the general statement of the multiregional input–output model in (3.22) or (3.23), that both intermediate demands, $\mathbf{Ax}$, and final demand, $\mathbf{f}$, are premultiplied by the matrix $\mathbf{C}$; this distributes these demands to supplying sectors across regions. Thus $\mathbf{f}^r$ and $\mathbf{f}^s$ represent demands by (shipments to) the final-demand sectors in regions r and s respectively, not final demands for the products of regions r and s (as in the interregional input–output model). The operation $\mathbf{Cf}$ converts these demands into a set of shipments by each region to contribute toward satisfaction of the final demands. In the two-region model here, $\mathbf{f}^r$ is satisfied in part by shipments from sectors in region r, $\hat{\mathbf{c}}^{rr}\mathbf{f}^r$ and in part by shipments from sectors in region s, $\hat{\mathbf{c}}^{sr}\mathbf{f}^r$. An example of a typical element in $\mathbf{f}^r$ might be new energy demands by a state government resulting from a new state office building in region r in that state. Depending upon the particular region, some or all of that energy demand will be met from within region r, the rest from outside the region. This is reflected in the appropriate elements in $\hat{\mathbf{c}}^{rr}$ and $\hat{\mathbf{c}}^{sr}$.

Thus, if one wants to assess the impacts of new *region-specific* final demands (such as from a foreign airline for Boeing airliners, as in the interregional example in section 3.3) it is necessary to replace $\mathbf{Cf}$ by, say, $\mathbf{f}^*$, which represents the new final demands

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already distributed appropriately to the region or regions of interest, and then to find

$$\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{f}^* \tag{3.25}$$

This is to be contrasted with (3.23). Continuing with the data for this example,

$$(\mathbf{I} - \mathbf{CA})^{-1} = \begin{bmatrix} 1.463 & .471 & .359 & .258 & .345 & .135 \\ .668 & 1.483 & .720 & .526 & .600 & .290 \\ .604 & .572 & 1.445 & .274 & .327 & .145 \\ .314 & .298 & .263 & 1.428 & .676 & .212 \\ .216 & .167 & .221 & .292 & 1.326 & .162 \\ .409 & .376 & .329 & .636 & .734 & 1.308 \end{bmatrix} \tag{3.26}$$

If $(f^*)_1^r = 100$ represents the value of new foreign airline orders for aircraft produced in region r, we would find, using (3.25)

$$\mathbf{x} = \begin{bmatrix} 146.30 \\ 66.80 \\ 60.40 \\ 31.40 \\ 21.60 \\ 40.90 \end{bmatrix}$$

3.4.5 The US MRIO Models

The first large-scale implementation of the MRIO framework was initiated at the Harvard Economic Research Project (HERP) and was further developed by Professor Karen Polenske and her associates at MIT. In its most detailed form, this is a model for 1963 with 51 regions (the 50 states and Washington, DC) and 79 sectors in each region. A thorough description of the model and its construction is provided in Polenske (1980). There was a second estimation and implementation of the MRIO framework for the 1977 US economy involving researchers at MIT and also Jack Faucett Associates, Inc., an economics consulting firm (see Jack Faucett Associates, Inc., 1981–1983). Since then there have been some additional attempts at creating multiregional input–output models for the USA. Because of widespread use, this system is viewed as an *alternative* to the IRIO model; as we will see below, it could as well be seen as an approach to *estimating* the intra- and interregional elements of an IRIO framework.[17]

Most implementations of interregional/multiregional input–output structures in recent decades have been generated through a combination of techniques and estimating procedures, all designed to estimate the numbers (especially the interregional transactions/coefficients) needed for the MRIO framework. These are generally known as "hybrid" techniques; they are a blend of some survey information, expert opinion and mechanical approaches. Some of these are explored in more detail in Chapter 8.

[17] An early comparison of the MRIO and IRIO models is provided in Hartwick, 1971.

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3.4.6 *Numerical Example: The Chinese Multiregional Model for 2000*

In 2003 the Institute of Developing Economies (Tokyo) in conjunction with the Japanese External Trade Organization published an ambitious set of multiregional input–output data for China in 2000, with 30 sectors and eight regions. (See Okamoto and Ihara, 2005, for detailed discussions of table construction and a number of comparative regional economic analyses that use the Chinese multiregional input–output framework.)

Tables 3.7–3.9 contain data for a highly aggregated version of the Chinese work, with three sectors and three regions (this is for illustration purposes only).[18] The transactions are denominated in 10,000 yuan (CYN) [also known as renminbi, meaning "people's currency" (RMB)].[19] We can easily trace the effects of hypothesized changes in final demands throughout the sectors and regions of the Chinese economy in this three-regional illustration. For example, assume that there is an increase of ¥100,000 in export demand for manufactured goods from the North. We would use

$$(\Delta \mathbf{f}^N)' = \begin{bmatrix} 0 & 100 & 0 & \quad 0 & 0 & 0 & \quad 0 & 0 & 0 \end{bmatrix}$$

in conjunction with the total requirements matrix in Table 3.9 to assess the impacts of this final demand change throughout the economy. We can examine similar implications of the same amount of increased export demand for manufactured goods in each of the other regions, using in turn $(\Delta \mathbf{f}^S)' = \begin{bmatrix} 0 & 0 & 0 & \quad 0 & 100 & 0 & \quad 0 & 0 & 0 \end{bmatrix}$ for export demands in the South and $(\Delta \mathbf{f}^R)' = \begin{bmatrix} 0 & 0 & 0 & \quad 0 & 0 & 0 & \quad 0 & 100 & 0 \end{bmatrix}$ for export demands in the Rest of China.

Premultiplying each of these vectors, in turn, by the total requirements matrix in Table 3.9 produces the results shown in Table 3.10. The new export demand generates differing own-region economic effects, depending on the region in which the manufacturing sector experiences the new export demand. When the demand is for manufactured goods made in the North, the total output of all sectors in that region increases by ¥215,300. If the demand is for Southern manufactured goods, the total value of new outputs in that region is ¥236,100, and when the new demand is for manufactured goods from the rest of China, output of all sectors there increases by ¥203,900. Interregional spillovers to each of the other regions are indicated by the other entries in the bottom row of Table 3.10. Adding spillovers to own-region impacts, we see that total national effects of the ¥100,000 stimulus for manufactures are ¥259,800, ¥268,500, and ¥240,200, respectively, when the stimulus is in the North, the South, and the Rest of China, respectively.

Many other observations can be made with the aid of results like these in Table 3.10. For example, in terms of interregional spillovers, it is clear that the largest external effect occurs when the demand is in the North; the ¥40,700 increase in Southern outputs is the largest effect of any in the bottom row of the table. In this highly aggregated example from China, it is clear from both the within-South effects (¥268,500) and the North-to-South spillover effect (¥40,700) that Southern manufacturing occupies a dominant

[18] These data are from the Institute of Developing Economies-Japan External Trade Organization (IDE-JETRO), 2003. Details of the regional and sectoral aggregations can be found in Appendix 3.2.

[19] The symbol usually seen is ¥, although sometimes with just one horizontal stroke. With two lines it is the same as the symbol for the Japanese yen.

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Table 3.7 Chinese Interregional and Intraregional Transactions, 2000 (in ¥10,000)

	North			South			Rest of China		
	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
North									
Natural Resources	1, 724	6,312	406	188	1,206	86	14	49	4
Manuf. & Const.	2, 381	18,458	2, 987	301	3,331	460	39	234	57
Services	709	3,883	1,811	64	432	138	5	23	5
South									
Natural Resources	149	656	42	3, 564	8,828	806	103	178	15
Manuf. & Const.	463	3,834	571	3, 757	34,931	5, 186	202	1,140	268
Services	49	297	99	1, 099	6,613	2, 969	31	163	62
ROC									
Natural Resources	9	51	3	33	254	18	1, 581	3,154	293
Manuf. & Const.	32	272	41	123	1,062	170	1, 225	6,704	1, 733
Services	4	25	7	25	168	47	425	2,145	1, 000
Total Output	16, 651	49,563	15,011	27, 866	81,253	23, 667	11, 661	21,107	8, 910

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Table 3.8 Direct Input Coefficients for the Chinese Multiregional Economy, 2000

	North			South			Rest of China		
	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
North									
Natural Resources	0.1035	0.1273	0.0270	0.0067	0.0148	0.0036	0.0012	0.0023	0.0005
Manuf. & Const.	0.1430	0.3724	0.1990	0.0108	0.0410	0.0194	0.0034	0.0111	0.0064
Services	0.0426	0.0783	0.1206	0.0023	0.0053	0.0058	0.0004	0.0011	0.0006
South									
Natural Resources	0.0089	0.0132	0.0028	0.1279	0.1087	0.0340	0.0089	0.0084	0.0017
Manuf. & Const.	0.0278	0.0774	0.0381	0.1348	0.4299	0.2191	0.0173	0.0540	0.0301
Services	0.0029	0.0060	0.0066	0.0394	0.0814	0.1255	0.0026	0.0077	0.0070
ROC									
Natural Resources	0.0006	0.0010	0.0002	0.0012	0.0031	0.0008	0.1356	0.1494	0.0329
Manuf. & Const.	0.0019	0.0055	0.0027	0.0044	0.0131	0.0072	0.1050	0.3176	0.1945
Services	0.0002	0.0005	0.0004	0.0009	0.0021	0.0020	0.0364	0.1016	0.1122

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Table 3.9 Leontief Inverse Matrix for the Chinese Multiregional Economy, 2000

	North			South			Rest of China		
	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
North									
Natural Resources	1.1631	0.2561	0.0965	0.0227	0.0582	0.0268	0.0064	0.0161	0.0085
Manuf. & Const.	0.3008	1.7275	0.4080	0.0537	0.1596	0.0849	0.0191	0.0529	0.0314
Services	0.0840	0.1686	1.1794	0.0115	0.0306	0.0202	0.0035	0.0093	0.0054
South									
Natural Resources	0.0325	0.0681	0.0321	1.1919	0.2504	0.1114	0.0245	0.0459	0.0232
Manuf. & Const.	0.1194	0.2943	0.1588	0.3258	1.9193	0.5036	0.0742	0.2010	0.1187
Services	0.0193	0.0447	0.0284	0.0848	0.1920	1.1965	0.0142	0.0375	0.0252
ROC									
Natural Resources	0.0034	0.0079	0.0039	0.0062	0.0164	0.0082	1.1958	0.2793	0.1061
Manuf. & Const.	0.0098	0.0245	0.0133	0.0176	0.0478	0.0272	0.2068	1.5681	0.3532
Services	0.0021	0.0051	0.0030	0.0045	0.0114	0.0075	0.0730	0.1916	1.1716

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Table 3.10 Region- and Sector-Specific Effects (in ¥1000) of a ¥100,000 Increase in Final Demand for Manufacturing Goods, China, 2000

	Produced in the North			Produced in the South			Produced in ROC		
Sector	North	South	ROC	North	South	ROC	North	South	ROC
Nat. Res.	25.6	6.8	0.8	5.8	25.0	1.6	1.6	4.6	27.9
Mfg. &Const.	172.8	29.4	2.5	16.0	191.9	4.8	5.3	20.1	156.8
Services	16.9	4.5	0.5	3.1	19.2	1.1	0.9	3.8	19.2
Total	215.3	40.7	3.8	24.9	236.1	7.5	7.8	28.5	203.9

position in the economy. We will explore measures of intra- and interregional impacts in more detail in Chapter 6.

3.5 The Balanced Regional Model

3.5.1 Structure of the Balanced Regional Model

A model that has a different sort of "regional" character was proposed in Leontief *et al.* (1953, Ch. 4) and has been implemented in specific applications, including an analysis of the effects in the US economy, on both sectors and regions, of a diversion of production away from military goods and to nonmilitary consumer goods (Leontief *et al.*, 1965). This has been called a *balanced regional model* (or *intranational* model). The basic mathematical structure of this model is identical to that of the interregional input–output model, but the interpretation of each of the components of the model is rather different. The entire analytical structure is based on the observation that in any national economy there are goods with different kinds of market areas. There are some goods for which production and consumption are equal ("balance") only at the national level. These are goods that have essentially a national (or, indeed, international) market area – sectors such as automobiles, aircraft (total airliner production in Washington $\neq$ total demand for aircraft in Washington), furniture, and agriculture. On the other hand, there are other sectors for which production and consumption tend to balance at a lower geographical level; they serve a regional or local rather than a national market. Examples might be electricity, real estate, warehousing, and personal and repair services (the number of shoeshines produced in an urban area equals the demand for shoeshines in that area). Clearly there is in reality an entire spectrum of possibilities, from sectors that serve extremely small local markets (shoe repair) to large national and international markets (aircraft). To illustrate the model structure with a simple example, we suppose that all sectors can be assigned to either a national (N) or a regional (R) category. (One possible criterion for classification of sectors would be the percentage of interregional as opposed to intraregional shipments of the products of that sector.)

Then, from a table of national input coefficients, one can rearrange the sectors so that, for example, all the regional sectors are listed first and all the national sectors follow.

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Let sectors $1, 2, \ldots, r$ represent the regionally balanced sectors and let sectors $r+1, \ldots, n$ represent nationally balanced sectors. Then, the rearranged table of national input coefficients will be

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RN} \\ \mathbf{A}^{NR} & \mathbf{A}^{NN} \end{bmatrix} \tag{3.27}$$

Let $\mathbf{x}^R$ and $\mathbf{f}^R$ (r-element column vectors) represent total output and final demand for the regional sectors, and let $\mathbf{x}^N$ and $\mathbf{f}^N$, which are $(n-r)$-element column vectors, represent output and final demand for the national sectors. Define

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^R \\ \mathbf{x}^N \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^R \\ \mathbf{f}^N \end{bmatrix}$$

Then, in exactly the same spirit as the two-region interregional input–output model, we have $(\mathbf{I}-\mathbf{A})\mathbf{x} = \mathbf{f}$. Here this is

$$\begin{aligned} (\mathbf{I}-\mathbf{A}^{RR})\mathbf{x}^R \; - \; \mathbf{A}^{RN}\mathbf{x}^N &= \mathbf{f}^R \\ -\; \mathbf{A}^{NR}\mathbf{x}^N + (\mathbf{I}-\mathbf{A}^{NN})\mathbf{x}^N &= \mathbf{f}^N \end{aligned} \tag{3.28}$$

It is important to notice that the R and N superscripts do not refer here to specific geographic *locations* of sectors, as in the interregional model. Rather, they serve to partition the sectors into two types – those whose market areas are national and those whose market areas are regional.[20] For example, a typical element $a_{ij}^{RN} x_j^N$ of the vector $\mathbf{A}^{RN}\mathbf{x}^N$ in (3.28) records inputs from sector i (in the regionally balanced set of sectors) to sector j (in the nationally balanced set of sectors). This will become clearer in the numerical example below.

More compactly, in partitioned matrix form,

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A}^{RR}) & -\mathbf{A}^{RN} \\ -\mathbf{A}^{NR} & (\mathbf{I}-\mathbf{A}^{NN}) \end{bmatrix} \begin{bmatrix} \mathbf{x}^R \\ \mathbf{x}^N \end{bmatrix} = \begin{bmatrix} \mathbf{f}^R \\ \mathbf{f}^N \end{bmatrix}$$

and so

$$\begin{bmatrix} \mathbf{x}^R \\ \mathbf{x}^N \end{bmatrix} = \begin{bmatrix} (\mathbf{I}-\mathbf{A}^{RR}) & -\mathbf{A}^{RN} \\ -\mathbf{A}^{NR} & (\mathbf{I}-\mathbf{A}^{NN}) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{f}^R \\ \mathbf{f}^N \end{bmatrix} \tag{3.29}$$

Using regular solution procedures, we find the total outputs of each sector in each of the two categories, due to an exogenous change in final demand for the outputs of one or more national sectors and/or one or more regional sectors. For example, in the arms-reduction study, there was assumed to be a 20 percent across-the-board decrease in government demand for the output of military-related goods, some of which were produced by national sectors (e.g., aircraft) and some of which were produced

[20] Partitioning of this sort can be done for a wide variety of purposes. For example, if one is particularly interested in energy-producing sectors, one might want to divide all sectors into two groups – those that produce energy and those that do not produce energy. Partitioned matrices will be employed frequently in the remainder of this book. Important results on inverses of partitioned matrices are presented in Appendix A.

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by regional sectors (e.g., warehousing), and an assumed across-the-board increase in nonmilitary final demands. Hence, elements in both $\mathbf{f}^R$ and $\mathbf{f}^N$ experienced change.

Thus far, there is nothing explicitly *spatial* in the model. The categorization of either nationally balanced or regionally balanced sectors deals only with the size of the market areas involved. For regional sectors, we need to have the new final demands, $\mathbf{f}^R$, distributed across regions. That is, we need to have $\mathbf{f}^{R(s)}$, the final demand for regionally balanced goods in region s, where $\sum_s \mathbf{f}^{R(s)} = \mathbf{f}^R$. In addition, we need, for each region, s, an estimate of the proportion of the output of each nationally balanced sector that is produced in region s, namely

$$\mathbf{p}^s = \begin{bmatrix} p^s_{r+1} \\ \vdots \\ p^s_n \end{bmatrix}$$

The vector $\hat{\mathbf{p}}^s\mathbf{x}^N$ indicates that part of the output of new national goods, $\mathbf{x}^N$, that must be produced by sectors $r+1$ through n in region s. Since the elements of $\mathbf{p}^s$ are the proportions of total national output that occur in region s, $\sum_s p^s_i = 1$ for $i = r+1, \ldots, n$, or $\sum_s \hat{\mathbf{p}}^s = \mathbf{I}$.

Total output in region s is an n-element vector

$$\mathbf{x}^{(s)} = \begin{bmatrix} \mathbf{x}^{R(s)} \\ \mathbf{x}^{N(s)} \end{bmatrix} \tag{3.30}$$

where $\mathbf{x}^{R(s)}$ contains the outputs of the r regionally balanced goods that are made in region s, and $\mathbf{x}^{N(s)} (= \hat{\mathbf{p}}^s\mathbf{x}^N)$ indicates production of nationally balanced goods that occurs in region s.

The $\mathbf{x}^{R(s)}$ term involves two components: (1) production in region s to meet region-specific final demand for regionally balanced goods, $\mathbf{f}^{R(s)}$ (e.g., production in Michigan to satisfy interindustry needs and new final demand in Michigan for electricity produced in that state) and (2) production in region s to turn out that region's share of nationally balanced goods, $\mathbf{x}^{N(s)}$ (e.g., Michigan electricity used as an input to Michigan production of automobiles to satisfy part of the nationwide demand for automobiles). That is,

$$\begin{aligned} \mathbf{x}^{R(s)} &= (\mathbf{I} - \mathbf{A}^{RR})^{-1}\mathbf{f}^{R(s)} + (\mathbf{I} - \mathbf{A}^{RR})^{-1}\mathbf{A}^{RN}\mathbf{x}^{N(s)} \\ &= (\mathbf{I} - \mathbf{A}^{RR})^{-1}\mathbf{f}^{R(s)} + (\mathbf{I} - \mathbf{A}^{RR})^{-1}\mathbf{A}^{RN}\hat{\mathbf{p}}^s\mathbf{x}^N \end{aligned} \tag{3.31}$$

Remember, from (3.27), that all the coefficients in the $\mathbf{A}$ matrices reflect *national* technology; the "R" and "N" serve to partition this national technology into two types of sectors. Production in each particular region is assumed to utilize this same technology, as reflected in the $(\mathbf{I} - \mathbf{A}^{RR})$ matrix and its inverse. In Appendix 3.3, these results are derived directly from observations on the inverse of the partitioned matrix in (3.29).

For the allocation of region R's share of production of nationally balanced goods, found in (3.29), we have

$$\mathbf{x}^{N(s)} = \hat{\mathbf{p}}^s\mathbf{x}^N \tag{3.32}$$

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In this way, then, the balanced regional model allocates the impacts of new $\mathbf{f}^R$ and $\mathbf{f}^N$ demand to the various sectors in each region.

3.5.2 Numerical Example

An example will illustrate more exactly how this works. Let

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RN} \\ \mathbf{A}^{NR} & \mathbf{A}^{NN} \end{bmatrix} = \begin{bmatrix} .10 & .15 & .05 & .03 \\ .03 & .10 & .02 & .10 \\ .12 & .03 & .20 & .10 \\ .10 & .02 & .25 & .15 \end{bmatrix} \tag{3.33}$$

and

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}^R \\ \mathbf{f}^N \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 200 \\ 200 \end{bmatrix}$$

Then $\mathbf{x}$ is found as in (3.29)

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^R \\ \mathbf{x}^N \end{bmatrix} = \begin{bmatrix} 168.30 \\ 163.40 \\ 325.70 \\ 354.70 \end{bmatrix} \tag{3.34}$$

These figures represent total outputs, throughout the nation, of the four sectors.

Assume that there are three regions in the country and that the region-specific distribution of final demands $\mathbf{f}^R$ is

$$\mathbf{f}^{R(1)} = \begin{bmatrix} 40 \\ 30 \end{bmatrix}, \ \mathbf{f}^{R(2)} = \begin{bmatrix} 50 \\ 30 \end{bmatrix}, \ \mathbf{f}^{R(3)} = \begin{bmatrix} 10 \\ 40 \end{bmatrix}$$

and that $\mathbf{p}^1 = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}$, $\mathbf{p}^2 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$, and $\mathbf{p}^3 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}$. We find $(\mathbf{I} - \mathbf{A}^{RR})^{-1}$ from the data in $\mathbf{A}$;

$$(\mathbf{I} - \mathbf{A}^{RR})^{-1} = \begin{bmatrix} 1.117 & .186 \\ .037 & 1.117 \end{bmatrix}$$

Using (3.31),

$$\mathbf{x}^{R(1)} = \begin{bmatrix} 67.47 \\ 51.75 \end{bmatrix}, \ \mathbf{x}^{R(2)} = \begin{bmatrix} 72.73 \\ 52.97 \end{bmatrix}, \ \mathbf{x}^{R(3)} = \begin{bmatrix} 28.05 \\ 58.65 \end{bmatrix} \tag{3.35}$$

[Note, as must be the case in a consistent model, that $\mathbf{x}^R$, in (3.34), is indeed $\mathbf{x}^{R(1)} + \mathbf{x}^{R(2)} + \mathbf{x}^{R(3)}$.] Using $\hat{\mathbf{p}}^1$, $\hat{\mathbf{p}}^2$, and $\hat{\mathbf{p}}^3$, the distribution of nationally balanced goods

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across regions is found as

$$\mathbf{x}^{N(1)} = \hat{\mathbf{p}}^1\mathbf{x}^N = \begin{bmatrix}195.40\\106.40\end{bmatrix},\ \mathbf{x}^{N(2)} = \hat{\mathbf{p}}^2\mathbf{x}^N = \begin{bmatrix}65.14\\141.90\end{bmatrix},\ \mathbf{x}^{N(3)} = \hat{\mathbf{p}}^3\mathbf{x}^N = \begin{bmatrix}65.14\\106.40\end{bmatrix} \quad (3.36)$$

where $\mathbf{x}^N$ must equal $\mathbf{x}^{N(1)} + \mathbf{x}^{N(2)} + \mathbf{x}^{N(3)}$, because of the way in which the $\mathbf{p}$ are defined.

Putting the results in (3.35) and (3.36) together, as in (3.30), we have

$$\mathbf{x}^{(1)} = \begin{bmatrix}67.47\\51.75\\ \\195.40\\106.40\end{bmatrix},\quad \mathbf{x}^{(2)} = \begin{bmatrix}72.73\\52.97\\ \\65.14\\141.90\end{bmatrix},\quad \mathbf{x}^{(3)} = \begin{bmatrix}28.05\\58.65\\ \\65.14\\106.40\end{bmatrix} \quad (3.37)$$

The entire outputs in (3.34) have been allocated across the three regions. As noted, production in each region is assumed to utilize the same technology, as reflected in $(\mathbf{I} - \mathbf{A}^{RR})$. But the model does recognize that production, whether of goods with a national market area or with a subnational market area, occurs in geographically specific locations, and the information in the distribution of the $\mathbf{f}^N$ elements and in the $\mathbf{p}^s$ vectors reflects this spatial distribution of production.

3.6 The Spatial Scale of Regional Models

To give the reader a feeling for the vast variety of geographic scales that have been modeled in "regional" input–output applications, we list a few (of very many) references, starting at the micro-spatial end of the spectrum.

- Cole (1987) describes a model for the city of Buffalo, New York, and Cole (1999) looks at an inner-city neighborhood in Buffalo.
- Robison and Miller (1988, 1991) consider small Idaho timber economies (logging/sawmills) – in the latter reference consisting of six communities (five containing sawmills; combined population around 20,000). They term these "community" input–output models. In Robison (1997) the model is for a rural two-county region in central Idaho (total population less than 12,000) which was disaggregated into seven community-centered sub-county regions.
- Hewings, Okuyama and Sonis (2001) present a four-region metropolitan area model. Three of the regions are sub-divisions of the City of Chicago, and the fourth is composed of the remaining counties making up the Chicago metropolitan area (six counties in all).
- Jackson *et al.* (2006) and Schwarm, Jackson and Okuyama (2006) suggest a new approach to generating data for the 51-state US model (as in the US MRIO model discussed above in section 3.4.5).

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- Richardson, Gordon and Moore (2007, and numerous other citations) create a 51-state US MRIO model.
- Boomsma and Oosterhaven (1992) describe a variety of two-region Dutch models made up of one region of interest and the rest of The Netherlands as the second region.
- West (1990) contains a summary of Australian input–output models in single-region and connected-region frameworks.
- Eurostat (2002), Hoen (2002). These references deal with the construction (and application) of a kind of many-region (or many-nation) model for the EC that lies between the IRIO and MRIO styles.
- IDE-JETRO (2006). Here the focus of attention is the Asian "multinational" or "multilateral" tables connecting ten countries (China, Indonesia, Japan, Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, and the USA). These are produced at five-year intervals.
- Leontief (1974), Leontief, Carter and Petri (1977), Fontana (2004) and Duchin (2004). These references discuss various aspects of what has come to be called the Leontief world model. Originally this was structured in terms of two "mega-regions" (developed and less developed countries). In Duchin and Lange (1994) the application uses a framework of 16 world regions (aggregations of countries) covering 189 countries.
- Inomata and Kuwamori (2007) and Development Studies Center, IDE-JETRO (2007). These references discuss a ten-sector model that combines a multinational character – China, Japan, ASEAN5 (Indonesia, Malaysia, the Philippines, Singapore, and Thailand), East Asia (Korea and Taiwan) and the USA – with *regional* disaggregations of China into seven regions and Japan into eight regions. Thus there are 18 geographic areas; some are true sub-national regions (the 15 in China and Japan), one is a nation (the USA) and two are multinational areas (ASEAN5, East Asia). The originators have called it a transnational interregional input–output (TIIO) model.

Many of these applications are discussed in Chapter 8.

3.7 Summary

In this chapter we have explored some of the most important modifications that need to be made to the basic input–output model (Chapter 2) when analysis is to be carried out at a regional level. We have seen that the input–output framework can be used either to study one single region in isolation, or it can be employed in studying one or more regions whose economic connections are made explicit in the model. While the representations of these connected regional models appear quite complicated, the models are logical extensions of the basic input–output structure that are designed to (1) reflect possibly differing production practices for the same sectors in different regions and (2) capture the trade relationships between sectors in different regions.

In more recent decades, work has been carried out with multi*national* input–output models, where "region" is replaced by "nation" in the framework. These have come

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about as a result of the increasing economic interdependence of nations – as exemplified, for example, in the European Union. We will explore some of these models in Chapter 8, because they generally involve "hybrid" approaches to estimation of the necessary data. Finally, a "global" model has been proposed as an interconnected set of broad groups of national economies. In this kind of framework, impacts of alternative development policies in less-developed countries can be studied for global impacts. (For example, Leontief, 1974; Leontief, Carter and Petri, 1977.) This will be explored briefly in Chapter 8 also.

Appendix 3.1 Basic Relationships in the Multiregional Input–Output Model

In standard input–output fashion, the total demand for commodity i in region s is given by

$$\sum_{j=1}^{n} a_{ij}^{s} x_j^{s} + f_j^{s} \tag{A3.1.1}$$

The total supply of commodity i in region s is the total that is shipped in from other regions,

$$\sum_{r=1}^{p} z_i^{rs} \ (r \neq s)$$

plus the amount that is supplied from within the region, z_i^{ss}. This is just T_i^s, the sum of the elements in column s in Table 3.8, as defined in (3.18). Since shipments (supplies) occur only to satisfy needs (demands), we have, for each commodity i

$$T_i^s = \sum_{j=1}^{n} a_{ij}^{s} x_j^{s} + f_i^{s} \tag{A3.1.2}$$

Total production of i in region r is equivalent to the total amount of i shipped from r, including that kept within the region

$$x_i^r = \sum_{s=1}^{p} z_i^{rs} \tag{A3.1.3}$$

From the definition of the interregional proportions in section 3.4.2, $c_i^{rs} = z_i^{rs}/T_i^s$, (A3.1.3) can be rewritten as

$$x_i^r = \sum_{s=1}^{p} c_i^{rs} T_i^s \tag{A3.1.4}$$

Putting T_i^s, as defined in (A3.1.2), into (A3.1.4)

$$x_i^r = \sum_{s=1}^{p} c_i^{rs} \left(\sum_{j=1}^{n} a_{ij}^{s} x_j^{s} + f_i^{s} \right) \quad (i = 1, \ldots, n) \tag{A3.1.5}$$

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Using familiar matrix notation, let

$$\mathbf{x}^r = \begin{bmatrix} x_1^r \\ \vdots \\ x_n^r \end{bmatrix},\ \mathbf{x}^s = \begin{bmatrix} x_1^s \\ \vdots \\ x_n^s \end{bmatrix},\ \mathbf{f}^s = \begin{bmatrix} f_1^s \\ \vdots \\ f_n^s \end{bmatrix}$$

$$\mathbf{A}^s = \begin{bmatrix} a_{11}^s & \cdots & a_{1n}^s \\ \vdots & & \vdots \\ a_{n1}^s & & a_{nn}^s \end{bmatrix},\ \hat{\mathbf{c}}^{rs} = \begin{bmatrix} c_1^{rs} & 0 & \cdots & 0 \\ 0 & c_2^{rs} & & \\ \vdots & & & \\ 0 & & & c_n^{rs} \end{bmatrix}$$

The reader should be convinced that the entire set of n equations for outputs of goods in region r can be expressed as

$$\mathbf{x}^r = \sum_{s=1}^{p} \hat{\mathbf{c}}^{rs}(\mathbf{A}^s\mathbf{x}^s + \mathbf{f}^s) = \sum_{s=1}^{p} \hat{\mathbf{c}}^{rs}\mathbf{A}^s\mathbf{x}^s + \sum_{s=1}^{p} \hat{\mathbf{c}}^{rs}\mathbf{f}^s \qquad \text{(A3.1.6)}$$

There will be p such matrix equations, one for each region r ($r = 1, \ldots, p$). Again using matrix notation, as in section 3.4, we can construct

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^1 \\ \vdots \\ \mathbf{x}^s \\ \vdots \\ \mathbf{x}^p \end{bmatrix},\ \mathbf{f} = \begin{bmatrix} \mathbf{f}^1 \\ \vdots \\ \mathbf{f}^s \\ \vdots \\ \mathbf{f}^p \end{bmatrix},\ \mathbf{A} = \begin{bmatrix} \mathbf{A}^1 & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & \vdots \\ \mathbf{0} & & \mathbf{A}^s & & \mathbf{0} \\ \vdots & & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{A}^p \end{bmatrix},$$

and

$$\mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{11} & \cdots & \hat{\mathbf{c}}^{1s} & \cdots & \hat{\mathbf{c}}^{1p} \\ \vdots & & \vdots & & \vdots \\ \hat{\mathbf{c}}^{r1} & \cdots & \hat{\mathbf{c}}^{rs} & \cdots & \hat{\mathbf{c}}^{rp} \\ \vdots & & \vdots & & \vdots \\ \hat{\mathbf{c}}^{p1} & \cdots & \hat{\mathbf{c}}^{ps} & \cdots & \hat{\mathbf{c}}^{pp} \end{bmatrix}$$

Then the p matrix equations in (A3.1.6) can be compactly expressed as

$$\mathbf{x} = \mathbf{C}(\mathbf{A}\mathbf{x} + \mathbf{f}) = \mathbf{C}\mathbf{A}\mathbf{x} + \mathbf{C}\mathbf{f}$$

from which

$$(\mathbf{I} - \mathbf{C}\mathbf{A})\mathbf{x} = \mathbf{C}\mathbf{f} \qquad \text{(A3.1.7)}$$

and

$$\mathbf{x} = (\mathbf{I} - \mathbf{C}\mathbf{A})^{-1}\mathbf{C}\mathbf{f} \qquad \text{(A3.1.8)}$$

as in (3.22) and (3.23) in the text.

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Appendix 3.2 Sectoral and Regional Aggregation in the 2000 Chinese Multiregional Model

Figure A3.2.1 Regional Aggregation in the 2000 Chinese Multiregional Model

Table A3.2.1 Regional Classifications in the 2000 Chinese Multiregional Model

3-Region Aggregation	Regions	Provinces and Municipalities
North	Northeast	Heilongjiang, Jilin, Liaoning
	North	Beijing, Tianjin, Hebei, Shandong
South	South	Hainan, Guangdong, Fujian
	Central	Hunan, Jiangxi, Hubei, Henan, Anhui, Shanxi
	East	Jiangsu, Shanghai, Zhejiang
Rest of China	Northwest	Xinjiang, Qinghai, Gansu, Ningxia, Shaanxi, Inner Mongolia
	Southwest	Tibet, Sichuan, Yunnan, Guizhou, Guangxi, Chongqing

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Table A3.2.2 Sectoral Aggregation in the 2000 Chinese Multiregional Model

3-Sector Aggregation	Industry Sectors
Natural Resources	agriculture
	mining & processing
Manufacturing & Construction	light industry
	energy industry
	heavy industry & chemical industry
	construction
Services & Other Sectors	transportation & telecommunications services
	commercial services
	other

Appendix 3.3 The Balanced Regional Model and the Inverse of a Partitioned $(\mathbf{I} - \mathbf{A})$ Matrix

We use the results from Appendix A on the inverse of a partitioned matrix. For the balanced regional model, let

$$(\mathbf{I}-\mathbf{A}) = \begin{bmatrix} (\mathbf{I}-\mathbf{A}^{RR}) & -\mathbf{A}^{RN} \\ -\mathbf{A}^{NR} & (\mathbf{I}-\mathbf{A}^{NN}) \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{F} \\ \mathbf{G} & \mathbf{H} \end{bmatrix} \text{ and } (\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} \mathbf{S} & \mathbf{T} \\ \mathbf{U} & \mathbf{V} \end{bmatrix}.$$

Then, from (3.29)

$$\begin{aligned} \mathbf{x}^R &= \mathbf{S}\mathbf{f}^R + \mathbf{T}\mathbf{f}^N \\ \mathbf{x}^N &= \mathbf{U}\mathbf{f}^R + \mathbf{V}\mathbf{f}^N \end{aligned} \tag{A3.3.1}$$

This generates total output throughout the nation of both regionally balanced goods $(\mathbf{x}^R)$ and nationally balanced goods $(\mathbf{x}^N)$.

In this case, using the partitioned inverse results above, we have

$$\begin{aligned} &\mathbf{S} = (\mathbf{I}-\mathbf{A}^{RR})^{-1}(\mathbf{I}+\mathbf{A}^{RN}\mathbf{U}) \quad \mathbf{T} = (\mathbf{I}-\mathbf{A}^{RR})^{-1}\mathbf{A}^{RN}\mathbf{V} \\ &\mathbf{U} = \mathbf{V}\mathbf{A}^{NR}(\mathbf{I}-\mathbf{A}^{RR})^{-1} \quad \mathbf{V} = [(\mathbf{I}-\mathbf{A}^{NN}) - \mathbf{A}^{NR}(\mathbf{I}-\mathbf{A}^{RR})^{-1}\mathbf{A}^{RN}]^{-1} \end{aligned} \tag{A3.3.2}$$

Substituting for $\mathbf{S}$ and $\mathbf{T}$ in (A3.3.2), from (A3.3.1),

$$\mathbf{x}^R = (\mathbf{I}-\mathbf{A}^{RR})^{-1}\mathbf{f}^R + (\mathbf{I}-\mathbf{A}^{RR})^{-1}\mathbf{A}^{RN}(\mathbf{U}\mathbf{f}^R + \mathbf{V}\mathbf{f}^N) \tag{A3.3.3}$$

But $\mathbf{x}^N$, as in (A3.3.2), is just the $(\mathbf{U}\mathbf{f}^R + \mathbf{V}\mathbf{f}^N)$ term on the right-hand side of (A3.3.3), so

$$\mathbf{x}^R = (\mathbf{I}-\mathbf{A}^{RR})^{-1}\mathbf{f}^R + (\mathbf{I}-\mathbf{A}^{RR})^{-1}\mathbf{A}^{RN}\mathbf{x}^N \tag{A3.3.4}$$

To distribute both $\mathbf{x}^R$ and $\mathbf{x}^N$ production to individual regions, we need the regional distribution of final demands for regional goods – $\mathbf{f}^{R(s)}$, for each region s – and we need

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the regional distribution of production of each of the nationally balanced goods – $\mathbf{p}^s$ – for each region. Then, to add the spatial dimension, for a specific region s, $\mathbf{f}^R$ becomes $\mathbf{f}^{R(s)}$ and $\mathbf{x}^N$ becomes $\mathbf{x}^{N(s)}$, which is $\hat{\mathbf{p}}^s\mathbf{x}^N$. Therefore

$$\mathbf{x}^{R(s)} = (\mathbf{I} - \mathbf{A}^{RR})^{-1}\mathbf{f}^{R(s)} + (\mathbf{I} - \mathbf{A}^{RR})^{-1}\mathbf{A}^{RN}\hat{\mathbf{p}}^s\mathbf{x}^N \tag{A3.3.5}$$

This is (3.31) in the text.

Problems

3.1 The data in problem 2.2 described a small national economy. Consider a region within that national economy that contains firms producing in each of the three sectors. Suppose that the technological structure of production of firms within the region is estimated to be the same as that reflected in the national data, but that there is need to import into the region (from producers elsewhere in the country) some of the inputs used in production in each of the regional sectors. In particular, the percentages of required inputs from sectors 1, 2, and 3 that come from within the region are 60, 90, and 75, respectively. If new final demands for the outputs of the regional producers are projected to be 1300, 100, and 200, what total outputs of the three regional sectors will be needed in order to meet this demand?

3.2 The following data represent sales (in dollars) between and among two sectors in regions r and s.

	r		s	
r	40	50	30	45
	60	10	70	45
s	50	60	50	80
	70	70	50	50

In addition, sales to final demand purchasers were $\mathbf{f}^r = \begin{bmatrix} 200 \\ 200 \end{bmatrix}$ and $\mathbf{f}^s = \begin{bmatrix} 300 \\ 400 \end{bmatrix}$. These data are sufficient to create a two-region interregional input–output model connecting regions r and s. If, because of a stimulated economy, household demand increased by \$280 for the output of sector 1 in region r and by \$360 for the output of sector 2 in region r, what are the new necessary gross outputs from each of the sectors in each of the two regions to satisfy this new final demand? That is, find $\Delta\mathbf{x} = \begin{bmatrix} \Delta\mathbf{x}^r \\ \Delta\mathbf{x}^s \end{bmatrix}$ associated with $\Delta\mathbf{f}$.

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3.3 Suppose that you have assembled the following information on the dollar values of purchases of each of two goods in each of two regions, and also on the shipments of each of the two goods between regions:

Purchases in Region r		*Purchases in Region s*	
$z_{11}^{r} = 40$	$z_{12}^{r} = 50$	$z_{11}^{s} = 30$	$z_{12}^{s} = 45$
$z_{21}^{r} = 60$	$z_{22}^{r} = 10$	$z_{21}^{s} = 70$	$z_{22}^{s} = 45$
Shipments of Good 1		*Shipments of Good 2*	
$z_{1}^{rr} = 50$	$z_{1}^{rs} = 60$	$z_{2}^{rr} = 50$	$z_{2}^{rs} = 80$
$z_{1}^{sr} = 70$	$z_{1}^{ss} = 70$	$z_{2}^{sr} = 50$	$z_{2}^{ss} = 50$

These data are sufficient to generate the necessary matrices for a two-region multiregional input–output model connecting regions r and s. There will be six necessary matrices – $\mathbf{A}^r$, $\mathbf{A}^s$, $\hat{\mathbf{c}}^{rr}$, $\hat{\mathbf{c}}^{rs}$, $\hat{\mathbf{c}}^{sr}$ and $\hat{\mathbf{c}}^{ss}$. All of these will be 2×2 matrices. If the projected demands for the coming period are $\mathbf{f}^r = \begin{bmatrix} 50 \\ 50 \end{bmatrix}$ and $\mathbf{f}^s = \begin{bmatrix} 40 \\ 60 \end{bmatrix}$, find the gross outputs for each sector in each region necessary to satisfy this new final demand; that is, find $\mathbf{x}^r$ and $\mathbf{x}^s$.

3.4 A federal government agency for a three-region country has collected the following data on input purchases for two sectors, (1) manufacturing and (2) agriculture, for last year, in dollars. These flows are not specific with respect to region of origin; that is, they are of the $z_{ij}^{\cdot s}$ sort. Denote the three regions by A, B, and C.

	Region A		Region B		Region C	
	1	2	1	2	1	2
1	200	100	700	400	100	0
2	100	100	100	200	50	0

Also, gross outputs for each of the two sectors in each of the three regions are known. They are:

$$\mathbf{x}^A = \begin{bmatrix} 600 \\ 300 \end{bmatrix}, \ \mathbf{x}^B = \begin{bmatrix} 1200 \\ 700 \end{bmatrix} \text{ and } \mathbf{x}^C = \begin{bmatrix} 200 \\ 0 \end{bmatrix}$$

The agency hires you to advise them on potential uses for this information.

a. Your first thought is to produce a regional technical coefficients table for each region. Is it possible to construct such tables? If so, do it; if not, why not?
b. You also consider putting the data together to generate a national technical coefficients table. Is this possible? If so, do it; if not, why not?
c. Why is it not possible to construct from the given data a three-region multiregional input–output model?

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d. If the federal government is considering spending \$5,000 on manufactured goods and \$4,500 on agricultural products next year, what would you estimate as the national gross outputs necessary to satisfy this government demand?
e. Compare the national gross outputs for sectors 1 and 2 found in d, above, with the original gross outputs, given in the data set from last year. What feature of the input–output model does this comparison illustrate?

3.5 Consider the following two-region interregional input–output transactions table:

	North			South			
	Agric. (1)	Mining (2)	Constr. & Manuf.(3)	Agric.(1)	Mining (2)	Const. & Manuf. (3)	Total Output
North							
Agriculture (1)	277, 757	3, 654	1, 710, 816	8, 293	26	179, 483	3, 633, 382
Mining (2)	319	2, 412	598, 591	15	112	30, 921	743, 965
Construction & Manufacturing (3)	342, 956	39, 593	6, 762, 703	45, 770	3, 499	1, 550, 298	10, 931, 024
South							
Agriculture (1)	7, 085	39	98, 386	255, 023	3, 821	1, 669, 107	3, 697, 202
Mining (2)	177	92	15, 966	365	3, 766	669, 710	766, 751
Construction & Manufacturing (3)	71, 798	7, 957	2, 017, 905	316, 256	36, 789	8, 386, 751	14, 449, 941

a. Find the final-demand vectors and the technical coefficients matrices for each region.
b. Assume that the rising price of imported oil (upon which the economy is 99 percent dependent) has forced the construction and manufacturing industry (sector 3) to reduce total output by 10 percent in the South and 5 percent in the North. What are the corresponding amounts of output available for final demand? (Assume interindustry relationships remain the same, that is, the technical coefficients matrix is unchanged.)
c. Assume that tough import quotas imposed in Western Europe and the USA on this country's goods have reduced the final demand for output from the country's construction and manufacturing industries by 15 percent in the North. What is the impact on the output vector for the North region? Use a full two-region interregional model.
d. Answer the question in part c, above, ignoring interregional linkages, that is, using the Leontief inverse for the North region only. What do you conclude about the importance of interregional linkages in this aggregated version of this economy?

3.6 Consider the MRIO transactions table for China given in Table 3.7. Suppose all of the inputs to the North region from the South region were replaced with corresponding industry production from the Rest of China region. How would you reflect such a situation in the MRIO model? What would be the impact on total outputs of all regions and sectors for a final demand of ¥100,000 on export demand for manufactured goods produced in the North?

3.7 A three-region, five-sector version of the US multiregional input–output economy is given in Table A4.1.3 in the next chapter. Suppose that a new government military

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project is initiated in the western United States which stimulates new final demand in that region of (in millions of dollars) $\Delta\mathbf{f}^W = \begin{bmatrix} 0 & 0 & 100 & 50 & 25 \end{bmatrix}'$. What is the impact on total production of all sectors in all three regions of the United States economy stimulated by this final demand in the West?

3.8 Consider the three-region, five-sector version of an interregional input–output economy of Japan for 1965 given in Table A4.1.1 of Appendix 4.1. Suppose the same final demand vector given in problem 3.7 is placed on goods and services produced in Japan's South region. What is the impact on total production of all sectors in all three regions of Japan of this final demand in the South?

3.9 Consider the year 2000 IRIO model for China, Japan, the United States and an aggregation of other Asian nations including Indonesia, Malaysia, the Philippines, Singapore, and Thailand provided in the table below. Assume that annual final demand growth in China is 8 percent, growth in the USA and Japan is 4 percent, and that of other Asian nations is 3 percent. Compute the percentage growth in total output corresponding to the growth in final demand.

	United States			Japan			China			Rest of Asia		
2000	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services	Nat. Res.	Manuf. & Const.	Services
USA												
Nat. Res.	75,382	290,016	17,829	351	4,764	473	174	403	17	103	2,740	83
Manuf. & Const.	68,424	1,667,042	960,671	160	21,902	3,775	587	8,865	1,710	383	48,066	4,391
Services	95,115	1,148,909	3,004,357	118	6,095	807	300	1,400	290	197	7,393	953
Japan												
Nat. Res.	7	52	53	8,721	78,956	11,206	13	66	2	14	180	27
Manuf. & Const.	859	41,484	11,337	28,068	1,414,078	484,802	764	20,145	2,809	462	72,258	4,108
Services	97	4,390	1,424	24,901	662,488	1,001,852	107	2,765	335	270	7,616	1,189
China												
Nat. Res.	72	343	147	50	2,316	229	49,496	183,509	15,138	102	2,430	99
Manuf. & Const.	331	15,657	6,442	93	10,199	1,989	89,384	892,227	181,932	157	15,093	1,237
Services	38	2,238	1,099	17	1,780	280	25,391	210,469	136,961	23	2,078	132
ROA												
Nat. Res.	322	1,068	203	64	11,906	266	64	1,475	14	12,153	92,647	6,402
Manuf. & Const.	503	56,287	18,129	278	35,418	3,562	1,141	41,496	4,085	23,022	506,274	144,417
Services	152	4,578	1,921	41	3,982	447	138	3,669	422	15,163	215,470	239,053
TOTAL OUTPUT	468,403	5,866,935	11,609,307	140,622	3,883,455	4,058,191	408,153	2,000,741	702,248	173,008	727,367	1,225,460

3.10 Assume that you have a very limited computer that can directly determine the inverse of matrices no larger than 2×2. Given this limited computer, explain how you could go about determining $\mathbf{L}$ for

$$\mathbf{A} = \begin{bmatrix} 0 & 0.1 & 0.3 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0 & 0 \\ 0.2 & 0 & 0.1 & 0.3 & 0.1 \\ 0.3 & 0 & 0 & 0.1 & 0.3 \\ 0.3 & 0.2 & 0.1 & 0.1 & 0.2 \end{bmatrix}$$

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a. Compute the Leontief inverse in this manner.
b. What implications does such a procedure have for the computation of very large matrices (e.g., $n > 1000$)?

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