7 Nonsurvey and Partial-Survey Methods: Fundamentals

7.1 Introduction

The heart of any input–output analysis is the table of input–output coefficients describing the relationships between inputs and outputs for a particular economy. To produce a table based on a survey of establishments in the economy is an expensive and time-consuming task, not only at a national level, but also for regions (states, counties, metropolitan areas, etc.). In this chapter we examine some approaches that attempt to adapt older tables to reflect more recent economic conditions or to borrow information in a table for one economy to use for a different economy. In a very general way, these may be thought of as modifications of tables over time or across space, respectively.

7.2 The Question of Stability of Input–Output Data

One of the most serious concerns of those who use input–output models in applied work is that the table of technical coefficients available to them for the economy that they are studying will generally reflect data from a much earlier year. For example, a survey-based or so-called benchmark US input–output table based upon 2002 transactions was not generally available until 2007. These time lags reflect the fact that when establishments in different industries are surveyed for information regarding their purchases of inputs and their sales of output, it takes a great deal of time to obtain the data, organize the information, and reconcile inconsistencies – for example, reported purchases of sector i goods by sector j establishments may differ from reported sales by sector i to sector j establishments. (We will return to this reconciliation problem in section 8.9.) This is a general and continuing problem with survey-based tables.

It is clear that techniques of production will and do change over time, for a variety of reasons. Among others:

1. There is technological change itself, whereby new techniques of production are introduced in a sector (e.g., replacement of some human labor with robots in automobile production).
2. If there is a large increase in demand for the products of a particular sector, output will increase (subject, of course, to capacity constraints), and the producer may

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experience economies of scale. For example, if the scale of operation of a firm was very small at the time it was surveyed, relatively large material inputs per dollar of its output might be recorded. Later, after the level of production is increased, economies of scale might be reflected in lower amounts of at least some inputs per dollar of output. (In terms of the usual production function geometry, as in Figure 2.1(b), such scale economies mean that each isoquant represents a higher level of output than under the original conditions of production.)

3. New products are invented (e.g., plastics) which means both that (a) there may be an entirely new sector – row and column – in a sufficiently disaggregated table or at least the product mix will change in an existing sector if the new product is classified there, and (b) it may be used to replace an older product as an input to production in other sectors (e.g., plastic bottles rather than glass for soft drinks).
4. Relative prices change, and this may cause substitution among inputs in a production process (e.g., a switch from oil to natural gas as an energy source after a sharp increase in oil prices).
5. The more aggregated the input–output table, the greater the number of distinct products that are encompassed under one sectoral classification. To recall an extreme example from Chapter 3, if the food and kindred products sector produces mostly tomato soup in one year, there will be a need for tin cans in which to package the output. If, in a later year, the output of the food and kindred products sector is primarily chocolate bars, paper will be required for wrapping the product, not tin. Thus the relative proportions of products that are mixed together in a sector will influence the aggregate production recipe (column of input coefficients) for that sector.
6. Changes from domestically produced to imported inputs – or from imported to domestically produced – will alter the economic interrelationships between sectors in the domestic economy. This is particularly noticeable in interregional and multiregional input–output models.

For reasons such as these, an economy's technical coefficients matrix will change over time. Attempts to quantify these changes are often termed studies of *structural change*. Many of the earliest studies were primarily concerned with *measurement* of this change, and we examine several of these in this section. A second avenue of inquiry has concentrated on the *decomposition* of changes into two or more components of the overall change. We explore a number of these studies later, in section 13.1.

7.2.1 Stability of National Coefficients

Leontief (1951, 1953) was the first to use a national input–output model to study structural change, specifically for the US economy over the period 1919–1939. Structural change in his view is a change in the technical coefficient matrix of the system. Leontief also introduced the idea of substituting one or more (ultimately, all) columns of old input coefficients into a new technical coefficients matrix. Kanemitsu and Ohnishi (1989) used a similar partial substitution method to study technological change in the Japanese economy for the 1970–1980 period.

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In the 1953 publication, Leontief examined the overall effects of structural change by forcing (1) the 1919 US economy to satisfy 1929 final demands (and comparing the result with actual 1929 outputs) and (2) the 1929 US economy to satisfy 1939 final demands (and comparing this result with actual 1939 outputs). This has become a standard approach to measuring the overall effects of technical change. An early application is in Rasmussen (1957, esp. Chapter 9), where changes were measured for the Danish economy over the 1947–1949 period.

Following this approach, Carter (1970) analyzed the changes in the US economy in some detail as they were reflected in the 1939, 1947, and 1958 US input–output data. With, say, a 50-sector classification, each year's table of technical coefficients would contain some 2500 a_{ij} coefficients, or there would be 2500 elements in each of the Leontief inverse matrices. It is not immediately obvious how best to compare three sets of 2500 coefficients in order to judge how "different" they are. In general, then, summary measures of comparison become necessary. We briefly explore two kinds of comparisons; one uses a_{ij} coefficients directly, and the other is based on the Leontief inverse.

Comparisons of Direct-Input Coefficients If one constructs two-dimensional plots, in which the horizontal axis is used to measure the size of particular coefficients in the earlier year (t_0) and the vertical axis measures the size of coefficients in the later year (t_1), where the scales along the two axes are the same, then a particular a_{ij} coefficient will have as its horizontal/vertical coordinates the value of that coefficient at time t_0 and at time t_1 – $a_{ij}(t_0)$ and $a_{ij}(t_1)$. For an n-sector economy, there will be n^2 points in such a figure.

If all coefficients remained unchanged over the period, then all the points would fall along a 45-degree line. On the other hand, for coefficients that have increased over time the points will fall above the 45-degree line. Similarly, if coefficients have decreased over time, the points will tend to fall below the 45-degree line. Carter examined figures of this sort for given sets of sectors as inputs (that is, the a_{ij} for specific i's) and found, for example, that input coefficients for the "general inputs" sectors (energy, transportation, trade, communications, and other services) tended to increase over time, while those for materials inputs did not. Industry-specific analyses showed, for example, that coefficients measuring iron and steel inputs to productive sectors (a_{ij}, where i = iron and steel) clustered generally below the 45-degree line, when $t_0 = 1947$ and $t_1 = 1958$; similarly, those for aluminum inputs (a_{ij}, where i = aluminum) tended to cluster above the 45-degree line, for the same time period. This clearly reflects decreased use of iron and steel and increased use of aluminum as inputs to productive processes over the 1947–1958 period.

Comparisons of Leontief Inverse Matrices One way to quantify in an aggregate way the effects of input–output coefficient change over time is to compare the total output vector that would be needed for a given set of final demands, using the Leontief inverses from various technical coefficients matrices. For example, Carter used actual US final demand in 1961, $\mathbf{f}(1961)$, in conjunction with $\mathbf{L}(1939) = [\mathbf{I} - \mathbf{A}(1939)]^{-1}$,

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$\mathbf{L}(1947)$ and $\mathbf{L}(1958)$ to calculate $\mathbf{x}(1961/1939)$, $\mathbf{x}(1961/1947)$ and $\mathbf{x}(1961/1958)$. For example,

$$\mathbf{x}(1961/1939) = \mathbf{L}(1939)\mathbf{f}(1961)$$

Here $\mathbf{x}(1961/1939)$ represents the gross output that would be needed from each sector of the economy to satisfy 1961 final demands if the structure of production were that of 1939. (In all cases, these were technical coefficients matrices that excluded households.) Representative results (Carter, 1970, Table 4.1, pp. 35–36) were as follows for total intermediate output – total output, $\mathbf{x}(1961/19xy)$, less final demand, $\mathbf{f}(1961)$ – to satisfy known 1961 final demands (in millions of 1947 dollars and for $xy = 39$, 47, or 58):

- Using 1939 coefficients – 324,288
- Using 1947 coefficients – 336,296
- Using 1958 coefficients – 336,941
- Actual 1961 output – 334,160

The implications are that, over time, intermediate input requirements are relatively stable. Carter suggests that the small increase in total intermediate input represents a slight increase in specialization within sectors and a relative decrease in the use of labor and capital in later years. Overall, while there were noteworthy changes in specific sectors, it appeared from this study that in most sectors structural change was very gradual. This, of course, supports the contention that input–output coefficient tables may remain useful for a number of years, even though the year in which they were constructed may appear to make them out of date.

A sampling of later studies following this same general approach includes:

- Vaccara (1970). The issue was US structural change over 1947, 1958, and 1961 using the 1947 and 1958 US input–output models, in this case focusing on both gross and intermediate output, the latter to remove the possibly dominating influence of sales to final demand.
- Bezdek (1978) looked at the same question of structural change and extended Vaccara's analysis to 1963 and 1966, using data based on somewhat different conventions (for example, regarding transfers).[1]
- Bezdek and Dunham (1978) also employed this line of inquiry. They used an aggregation of 80-order data sets (for 1947, 1958, and 1963) to 11 "functional industries" and made comparisons of their results on intermediate output change over 1947–1963 with the similar work (using other aggregations) by Carter (1970) for the USA. They also compared their 1958–1963 results with those reported by Stäglin and Wessels (1972) in a study with a similar purpose for (what was then) West Germany over 1958–1962.

[1] There is a good deal of other work, not all of it published, by Vaccara and/or Bezdek and others who were at one time associated with the US input–output projects in the Office of Business Economics (OBE) or, more recently, the Bureau of Economic Analysis (BEA) of the US Department of Commerce.

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In many of these studies that used data from several consecutive time periods, the objective was often to try to determine whether trends observed in earlier periods appeared to continue to later periods. To the extent that regularities could be uncovered, the hope was that they might suggest approaches that could be used to update or project interindustry data in the absence of complete surveys. In general, that goal proved elusive; as observed in one study, the changes seemed to be "highly erratic, uneven and unpatterned." (Bezdek, 1978, p. 224).

Blair and Wyckoff (1989) examined changes in the US economy over 1963–1980. They considered not only the endpoint years (the 1963 and 1980 tables, the latter an update of 1977) but also data from the intervening 1967, 1972, and 1977 survey-based input–output tables. To assess the effects of changes in final demand, they held production technology in its 1980 form and forced that structure to satisfy, in turn, the final demands for 1972, 1977, 1980, and 1984. In addition, they also fixed a vector of final demands (for 1984) and used it with the varying technical coefficients matrices for 1972, 1977, and 1980. From these experiments, they concluded that the two methods for assessing structural changes overall produce roughly similar results.

Other Summary Measures Column sums of **A** matrices (with, say, households exogenous) show how a given sector depends on other sectors for inputs. If $\sum_i a_{ij}(t_0) = 0.32$ and $\sum_i a_{ij}(t_1) = 0.54$, we would conclude that sector j became more dependent upon other sectors in the economy in the period from t_0 to t_1 and also that sector j depended less on primary inputs – labor, capital, imports. These represent kinds of sectoral "linkage" in an economy, as do column sums of Leontief inverse matrices (output multipliers, Chapter 6). These and other linkage concepts will be taken up in Chapter 12. The point here is simply to note that they provide alternative kinds of summary measures by which to examine coefficients over time.

Data for the US Economy Appendix B contains a representative set of historical input–output data for the US economy aggregated to seven sectors. Other data for the US and additional economies with more sectoral detail are on the website at www.cambridge.org/millerandblair.

7.2.2 Constant versus Current Prices

In studies such as Carter's that attempt to identify structural (technological) change it is appropriate to express the input–output relationships in constant dollars. Suppose that $z_{ij}(t_0) = \$40$, $x_j(t_0) = \$1000$, $z_{ij}(t_1) = \$160$, and $x_j(t_1) = \$2000$. Recall (Chapter 2) that a transaction in value terms, z_{ij}, is a physical flow from i to j, s_{ij}, multiplied by the price of input i, p_i. Then, in terms of current values (at time t_0 and at time t_1), $a_{ij}(t_0) = 0.04$ and $a_{ij}(t_1) = 0.08$. This doubling of the input coefficient from sector i to sector j might be interpreted as a reflection of technological change – a doubling of the

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importance of good i in industry j's production process. However, if the price of input i increased over the period, then the difference between $a_{ij}(t_0)$ and $a_{ij}(t_1)$ would be at least partly due to this price change, and to the extent that this was the case it would not reflect any changed technological relationships at all. To cite an extreme case, if the price of good i had doubled and if the same physical flow was used in time t_1, then the $z_{ij}(t_1) = s_{ij}(t_1)p_i(t_1) = \$160$ reflects entirely a change in the price of i. If this were reduced to the price level at t_0 – if $p_i(t_1)$ were divided by 2 – then in constant (t_0-level) not current (t_1-level) dollars $z_{ij}(t_1)$ is just \$80; thus, expressed in constant dollars, $a_{ij}(t_1) = \$80/\$2000 = 0.04$, and we would conclude that there has been no structural change at all in the way input i is used in production by sector j. This is why constant-dollar comparisons are generally used in studies that attempt to identify structural change in an economy.

However, in addressing the question of coefficient stability over time (which is, ultimately, the question of whether or not "old" tables can be used reasonably in "new" times), current values are appropriate. There are two reasons for this. In the first place, when input prices increase, the price of the output produced from them will tend to increase also. Recall that the denominator of a_{ij} is x_j, which is a physical output, s_j, multiplied by the price of j, p_j. In the example above, if good i were the only input to sector j whose price had increased, it is not likely that the price of j would have doubled also, but it might well have increased slightly in the period from t_0 to t_1. However, if prices of all (or most) inputs to j had increased over the period, then the price of j is almost certain to have gone up also, so there will be some compensating movement in the numerators and the denominators of the a_{ij}. Thus, coefficients using current prices are likely to exhibit more stability, since price changes will be reflected in both numerators and denominators. This has been noted repeatedly; early studies include Tilanus and Rey (1964) at a national level and Conway (1980) at a regional level. More recently, in a very large study Shishido *et al.* (2000) use 45 individual coefficient tables for 20 countries and one region in China (there were tables for several different years for many of the countries) to examine coefficient change as an economy develops.

Secondly, due to the necessity of dealing with aggregated classifications, sectors contain a wide variety of individual products. Suppose that products a and b are classified as belonging to sector i (for example, heating oil and natural gas in the energy sector). If the price of one of these products, say a, rises relative to the other, then in establishments in sector j where substitution is possible between a and b, there will tend to be replacement of the higher-priced input, a, by the lower-priced one, b. This substitution, in turn, will tend to stabilize the value of the transaction z_{ij}, when that value is measured in current dollars, even though the physical composition of the transaction may be quite different at t_1 from what it was at t_0. (For example, if the price of oil rises relative to that of natural gas, a transaction from the energy sector to sector j may contain relatively more natural gas than oil in t_1 as compared with t_0.)

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7.2.3 *Stability of Regional Coefficients*

In Chapter 3 we saw that a regional technical coefficient, a_{ij}^{r}, can be broken down to the sum of the regional input coefficient, a_{ij}^{rr}, and the coefficient representing the amount of good i produced in other regions that is used per dollar's worth of output of sector j in region r, $a_{ij}^{\tilde{r}r}$ (where $\tilde{r}$ indicates regions other than r); $a_{ij}^{rr} = a_{ij}^{r} - a_{ij}^{\tilde{r}r}$. (For studies that concentrate on a specific region, where it is not necessary to use a superscript to designate the particular region, the simpler notation $r_{ij} = a_{ij} - m_{ij}$ is often used for regional input coefficients, technical coefficients, and "import" coefficients.)[2] Both the technical coefficients and the import coefficients, which represent trade patterns, are likely to be subject to variations over time. This has led to the speculation that regional coefficients are likely to be more unstable than technical coefficients, since they are made up of two unstable components – technical coefficients and import coefficients. For example, suppose $a_{ij}(t_0) = 0.1$, $a_{ij}(t_1) = 0.2$, $m_{ij}(t_0) = 0.05$, and $m_{ij}(t_1) = 0.1$. Then $r_{ij}(t_0) = 0.05$, $r_{ij}(t_1) = 0.1$, and the percentage increases in a_{ij}, m_{ij} and r_{ij} from t_0 to t_1 are all 100. On the other hand, if $m_{ij}(t_1) = 0.08$, then $r_{ij}(t_0) = 0.05$, $r_{ij}(t_1) = 0.12$, and the percentage increases are 100, 60, and 140, for a_{ij}, m_{ij}, and r_{ij}, respectively. Thus, in this case, the regional input coefficient is more unstable than either the technical coefficient or the import coefficient, even though the latter two moved in the same direction over t_0 to t_1.

An early study of coefficient stability at the regional level can be found in Beyers (1972), who used three survey-based input–output tables for the state of Washington, for 1963, 1967, and 1972 (Bourque and Weeks, 1969; Beyers *et al.*, 1970; and Bourque and Conway, 1977, respectively). Results of an examination of the regional input coefficients for the 1963 and 1967 Washington survey-based tables, in current dollars, are not conclusive (Beyers, 1972, Table 4, p. 372). For example, examination of the 888 coefficients for which a_{ij} experienced a change over the 1963–1967 period revealed that in 21.3 percent of the cases there was no change in m_{ij}; the change in r_{ij} was the same as in a_{ij}. In 16.2 percent of the cases, a_{ij} and m_{ij} moved in the same direction and there was no change in r_{ij}; in these cases the presence of both a_{ij} and m_{ij} in the definition of r_{ij} was "compensating." In 10.4 percent of the cases, a_{ij} and m_{ij} moved in opposite directions and hence led to a more unstable r_{ij}. However, in the remaining 52.1 percent of the cases, the effects were ambiguous – either a_{ij}, m_{ij}, and r_{ij} all moved in the same direction or a_{ij} and m_{ij} moved in the opposite direction (both of these kinds of movements may or may not lead to more instability in r_{ij} than in either a_{ij} or m_{ij}). For example, if $a_{ij}(t_0) = 0.2$, $a_{ij}(t_1) = 0.19$, $m_{ij}(t_0) = 0.05$, $m_{ij}(t_1) = 0.01$, then $r_{ij}(t_0) = 0.15$ and $r_{ij}(t_1) = 0.18$. While both a_{ij} and m_{ij} have decreased over time, r_{ij} has increased, and the percentage change in r_{ij} (in absolute terms) is larger than the percentage change in a_{ij} – a 20 percent increase versus a 5 percent decrease, respectively.

[2] In interregional and multiregional models we generally distinguish between inputs that come from other regions in the national economy and those that are imported from outside the nation. In the general discussion of this chapter, "import" means "not produced in the region."

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Examination of the Leontief inverses for the regional input coefficients and regional technical coefficients for the two years showed "the regional [input coefficients] matrix appears somewhat less stable than the [regional] technical requirements matrix" (Beyers, 1972, p. 372). However, the amount of change was relatively unimportant for overall impact analysis. For example, in an analysis of the Leontief–Carter type, total 1967 output calculated by using the 1963 coefficients matrix and the 1967 final demand was found to be only 2.3 percent larger than total actual 1967 output; intermediate output was 10.5 percent larger. However, the usual caveat applies; namely, some individual sectoral outputs were badly estimated using the 1963 matrix (the worst being overestimated by 77 percent). Further analyses of the Washington survey-based data (Conway, 1977, 1980) arrive at similar conclusions.

Another early study using survey-based state-level data is to be found in Emerson (1976), based on tables for Kansas for 1965 and 1970, including full import and export matrices. The results are, like those for Washington, not terribly conclusive. Although there were some changes in the import coefficients, and consequently in the Kansas regional input coefficients, the problem was judged to be "not acute but . . . of sufficient importance to warrant concern" (Emerson, 1976, p. 275). Also, Baster (1980) supplied some evidence on relative stability of trade coefficients in a study for the Strathclyde region in Scotland. At the level of the individual firm or establishment, 79 percent of the coefficients showing imports from the rest of Scotland were constant over the 1974–1976 period, and an additional 13.5 percent of the coefficients varied by no more than 10 percent over the period. At the sectoral level (that is, aggregating establishments), over 90 percent of the import coefficients were stable.

7.2.4 Summary

There is no question but that coefficients change over time, at both national and at regional levels. It is also apparent that for aggregate kinds of measures, such as total economy-wide output associated with a specific vector of final demand, the error introduced by using an "old" table may not be large. On the other hand, there are other much simpler methods for forecasting total output that are not much worse. As an example, Conway (1975) estimates total Washington 1967 output, $\mathbf{i}'\mathbf{x}^W(1967)$, using known total final demands for 1963 and 1967 – $\mathbf{i}'\mathbf{f}^W(1963)$ and $\mathbf{i}'\mathbf{f}^W(1967)$ – and total 1963 output, $\mathbf{i}'\mathbf{x}^W(1963)$. His estimate is simply

$$\mathbf{i}'x^W(1967) = [\mathbf{i}'\mathbf{x}^W(1963)]\left[\frac{\mathbf{i}'\mathbf{f}^W(1967)}{\mathbf{i}'\mathbf{f}^W(1963)}\right]$$

This is known as a "final-demand blowup" approach; in the Washington case it led to an overestimate of 3.1 percent (Conway, 1975, p. 67), as opposed to the input–output-generated error of 2.3 percent noted above (Beyers, 1972, p. 368). That is, there are much simpler ways to be not much worse off, at this very aggregate level. Of course, the main point of the input–output model is precisely that it generates results at the sectoral level, and for this kind of detail out-of-date tables can produce considerable

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error. For this reason, there is ongoing concern with improving techniques for updating or projecting input–output data. We examine some of these below and in the following chapter.

7.3 Updating and Projecting Coefficients: Trends, Marginal Coefficients, and Best Practice Methods

7.3.1 Trends and Extrapolation

Early in the history of input–output models it was thought that analysis of the trends in input–output coefficients might be a tempting approach to the problem of estimating probable changes in input–output coefficients over time. Given two or more coefficient matrices defined for an economy over the same set of sectors, linear (or nonlinear) trends could be established for each particular coefficient, and then extrapolations could be made to the year in question (with negative coefficients set equal to zero). For example, if a particular a_{ij} at time t_0 equals 0.2 and if the coefficient for the same i and j is 0.15 three years later (t_0+ 3), then a linear trend extrapolation would suggest that at t_0+ 6, a_{ij} would be equal to 0.10. This is of course a very elementary kind of "analysis." Two early studies found, not surprisingly, that such extrapolations generated worse results than simply using the most recent coefficients table; see Tilanus (1966) for the Netherlands and Barker (Allen and Gossling, 1975, Ch. 2) for UK tables. This approach is no longer given much attention.

7.3.2 Marginal Input Coefficients

Suppose that one is forecasting into the future from the current year, t, to some future year, $t + s$. Given $\mathbf{A}(t)$ and a forecast of $\mathbf{f}(t + s)$, one would then estimate $\mathbf{x}(t + s)$ as

$$\mathbf{x}(t+s) = \mathbf{L}(t)\mathbf{f}(t+s) \tag{7.1}$$

where $\mathbf{L}(t) = [\mathbf{I} - \mathbf{A}(t)]^{-1}$. Suppose that, in addition to the current-year data, there is a set of input–output data for a previous year, $t - r$. Then one could generate a set of marginal input coefficients, a^*_{ij}, defined as

$$a^*_{ij}(t) = \frac{z_{ij}(t) - z_{ij}(t-r)}{x_j(t) - x_j(t-r)} = \frac{\Delta z_{ij}}{\Delta x_j}$$

These coefficients relate the *change* (from year $t - r$ to year t) in the amount of input i purchased by industry j to the *change* (over the same period) in the total amount of j produced. To the extent that the average and marginal coefficients differ, the latter may reflect scale effects. The argument can be made that the marginal coefficient better reflects the inputs from i to j that would be used when the output of sector j changes, due to new (forecast) final demands.

For example, let $z_{ij}(t-r) = \$500$, $z_{ij}(t) = \$560$, $x_j(t-r) = \$5000$, and $x_j(t) = \$6000$, so that $a_{ij}(t) = \$560/\$6000 = 0.0933$ and $a^*_{ij}(t) = \$60/\$1000 = 0.06$. Putting ourselves back to year $t - r$, $a_{ij}(t - r) = \$500/\$5000 = 0.1$. If at time $t - r$ we had "forecast" $x_j(t)$ to be \$6000, our estimate of $z_{ij}(t)$, based on the usual *average* input coefficient,

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would have been $a_{ij}(t-r)x_j(t) = (0.1)(\$6000) = \$600$. However, if we had had a marginal coefficient at $t-r$, $a^*_{ij}(t-r)$, we could have made a forecast of $z_{ij}(t)$ as $z_{ij}(t) = z_{ij}(t-r) + \Delta z_{ij} = z_{ij}(t-r) + a^*_{ij}(t-r)\Delta x_j = \$500 + a^*_{ij}(t-r)(\$1000)$. In particular, if our estimate of $a^*_{ij}(t-r)$ had been 0.06 [which is our $a^*_{ij}(t)$], our estimate of $z_{ij}(t)$would have been perfect, at \$560. This is the basic idea behind the use of marginal coefficients for forecasting. The alternative to estimating directly the level of new output, at time $t+s$, as in (7.1), is to forecast the change in output, using marginal coefficients, and add it to the current level; that is

$$\mathbf{x}(t+s) = \mathbf{x}(t) + \Delta\mathbf{x} = \mathbf{L}(t)\mathbf{f}(t) + \mathbf{L}^*(t)\Delta\mathbf{f} \tag{7.2}$$

where $\Delta\mathbf{f} = \mathbf{f}(t+s) - \mathbf{f}(t)$, $\mathbf{L}^*(t) = [\mathbf{I} - \mathbf{A}^*(t)]^{-1}$ and $\mathbf{A}^*(t)$ is the matrix of marginal input coefficients. Since the elements in $\mathbf{x}(t+s)$ are found using a combination of current average coefficients, $\mathbf{A}(t)$, and marginal coefficients, $\mathbf{A}^*(t)$, this is in effect a way of introducing changing coefficients over time into the analysis.

Although the idea of using marginal coefficients to reflect changes in input–output structure has a certain logical appeal, early experiments by Tilanus (1967) on a series of Dutch national input–output tables for 13 consecutive years (1948 through 1960) were not encouraging. For $r = 5$ (that is, calculating marginal coefficients over the previous five-year period) and $s = 1, 2, 3, 4^1/_2$ and $6^1/_2$ (years of projection), using marginal coefficients in this way gave results that were not as good as when the most recent table of average coefficients was used – the approach in (7.1) turned out to be better than that in (7.2).

7.3.3 "Best Practice" Firms

An alternative approach for projecting the technology in an input–output table in the future is the "best practice" firm idea pioneered by Miernyk (for example, in Miernyk, 1965). In constructing a table for short-term forecasting into the future – say, three to six years – Miernyk suggested that one not gather current information from *all* firms in each sector, or even from some random sample of firms. Rather, one should obtain data only from the "best practice" firms in a sector – those that are technologically most advanced at present. Such firms can be defined as those for which the ratios of employment or wage payments to total gross output are relatively low ("low labor intensity") or those with relatively high ratios of profits to total gross output. Firms could be identified as belonging to the best practice group if they satisfied any one or only if they satisfied several of these (or similar) criteria simultaneously.

The logic is that these firms, which are somewhat unusual currently (in the sense of being "better than average" for their sector), probably represent the technology that will be generally in use in the future – the best of today will be the average of the future. There are many obvious objections to this idea – why should "best" today be "average" in five years for *all* sectors? Is this approach valid for three years, or five years, or seven years in the future? And so on. But in its favor is the fact that it is a workable, feasible way of constructing technical coefficients matrices that are more likely to represent the

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future structure of production than would a table that was constructed to represent the average structure in each sector today.

7.4 Updating and Projecting Coefficients: The RAS Approach and Hybrid Methods

7.4.1 The RAS Technique

Early work at updating input–output information, done under Stone's direction, is reported in Stone (1961); Stone and Brown (1962); Cambridge University, Department of Applied Economics (1963); and Bacharach (1970). Because this technique requires less information than is usually obtained in a survey of the sort that underlies survey-based input–output tables, it is often referred to as a partial-survey, or a nonsurvey method. It is now recognized that full surveys are generally impractical and that a "hybrid" approach is called for, in which some kinds of superior information (from small, focused surveys, expert opinion, etc.) are incorporated into an otherwise "nonsurvey" procedure.[3] In this section we examine the widely-used "RAS" procedure (also known as a "biproportional" matrix balancing technique); the origin of the name will become clear in what follows. There have been numerous variations – attempts at refinement and improvement of this procedure – and research continues to be active.[4] Later we will see how additional information can be incorporated into the basic RAS procedure, producing an example of a hybrid technique.

To begin, assume that we have an input–output direct input coefficients table for an n-sector economy for a given year in the past (in what follows, we will designate this as year "0") and that we would like to update those coefficients to a more recent year (for example, the current year, which we will designate year "1"). Using obvious notation, we have $\mathbf{A}(0)$ and we want $\mathbf{A}(1)$, the n^2 coefficients for the n sectors in the economy for the more recent or current year.[5]

The RAS technique generates an estimate of these coefficients from $3n$ pieces of information for the year of interest (year 1). These are: (1) total gross outputs, x_j (which are also needed with survey-based transactions information); (2) total interindustry (intermediate) sales, by sector – for sector i this is $\sum_{j=1}^{n} z_{ij}$, which is the same as total output of sector i less sector i's sales to final demand (since $x_i = \sum_{j=1}^{n} z_{ij} + f_i$) and (3) total interindustry purchases, by sector – for sector j this is $\sum_{i=1}^{n} z_{ij}$, which is the same

[3] See Lahr (1993) for a thorough discussion and an extensive set of references. As noted by Richardson (1985, p. 624): "If survey-based models are too expensive, conversion of national coefficients too mechanical, and short cuts too unreliable, the hybrid approaches are the wave of the future."

[4] An excellent overview of RAS and similar matrix adjustment techniques is to be found in several of the chapters in Allen and Gossling (1975), which also contains a good list of early references. See also Polenske (1997) for a thorough critical review. An important newer reference is the June, 2004, issue of *Economic Systems Research*, a special issue on "Biproportional Techniques in Input–Output Analysis," edited by Lahr and de Mesnard. See especially the lead article by the editors (Lahr and de Mesnard, 2004).

[5] The RAS approach is usually presented in the context of updating *coefficients*, and we maintain that viewpoint in this section. As we will later see, one can equally well use RAS to update *transactions* and then derive updated coefficients from those updated transactions.

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as $x_j - v_j$ (total output of sector j less total purchases by j from the payments sector – labor inputs to sector j, imported inputs to sector j, taxes paid for government services, interest paid on capital loans, rental payments for land, etc.)

It has become conventional in the RAS literature to define $u_i = \sum_{j=1}^{n} z_{ij}$ and $v_j = \sum_{i=1}^{n} z_{ij}$; as vectors, these are $\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$. Since these need to be known for year 1, they will be designated $\mathbf{u}(1)$ and $\mathbf{v}(1)$. (In using $\mathbf{u}$ and $\mathbf{v}$, we are following established convention in the literature on nonsurvey techniques. Throughout this text we have also, following convention, used $\mathbf{v}'$ for the value-added (row) vector. Also, in Chapter 5 we followed another convention in using $\mathbf{U}$ and $\mathbf{V}$ for the Use and Make *matrices*, respectively, in a commodity-by-industry input–output accounting framework. The context should always make clear what is intended.)

Thus, the problem that the RAS procedure addresses is: given an $n \times n$ matrix $\mathbf{A}(0)$ and given three n-element vectors for a more recent year – $\mathbf{x}(1)$, $\mathbf{u}(1)$, and $\mathbf{v}(1)$ – estimate $\mathbf{A}(1)$. We denote this estimate as $\tilde{\mathbf{A}}(1)$. If we are dealing with, say, a 25-sector economy, we are estimating the 625 coefficients in $\tilde{\mathbf{A}}(1)$ from 75 pieces of information. These are: (1) the 25 row sums of the unknown transactions matrix, $\mathbf{Z}(1)$, namely $\mathbf{u}(1) = \mathbf{Z}(1)\mathbf{i}$; (2) the 25 column sums of the same matrix, $\mathbf{v}(1)' = \mathbf{i}'\mathbf{Z}(1)$ [or $\mathbf{v}(1) = \mathbf{Z}(1)'\mathbf{i}$]; and (3) the 25 year-1 gross outputs, $\mathbf{x}(1)$, which are needed to convert an estimate of a $z_{ij}(1)$ into an estimate of a technical coefficient $a_{ij}(1)$.

We develop the procedure for the general 3×3 case, and then present a 3×3 numerical example. The potential usefulness of the technique is in real-world applications, where the number of sectors is much larger than three and hence the difference between n^2 and $3n$ is large. For example, with an 80-sector table, $n^2 = 6400$, whereas $3n = 240$. For the general 3×3 case, we assume that base-year coefficients are known,

$$\mathbf{A}(0) = \begin{bmatrix} a_{11}(0) & a_{12}(0) & a_{13}(0) \\ a_{21}(0) & a_{22}(0) & a_{23}(0) \\ a_{31}(0) & a_{32}(0) & a_{33}(0) \end{bmatrix} \tag{7.3}$$

and for the "target" year we have

$$\mathbf{x}(1) = \begin{bmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \end{bmatrix}, \ \mathbf{u}(1) = \begin{bmatrix} u_1(1) \\ u_2(1) \\ u_3(1) \end{bmatrix}, \ \mathbf{v}(1) = \begin{bmatrix} v_1(1) \\ v_2(1) \\ v_3(1) \end{bmatrix} \tag{7.4}$$

Initially, assume $\mathbf{A}(0) = \mathbf{A}(1)$, namely that the technical coefficients have remained stable over time. To test the credibility of this hypothesis, we investigate whether or not it is consistent with year-1 information on intermediate sales and purchases. These are row and column sums of the transactions matrix, so it is necessary to convert coefficients into transactions – here this means that our *initial* estimate of the target transactions

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matrix is $\mathbf{Z}^0 = \mathbf{A}(0)\hat{\mathbf{x}}(1)$.[6] Here this is

$$\mathbf{Z}^0 = \mathbf{A}(0)\hat{\mathbf{x}}(1) = \begin{bmatrix} a_{11}(0) & a_{12}(0) & a_{13}(0) \\ a_{21}(0) & a_{22}(0) & a_{23}(0) \\ a_{31}(0) & a_{32}(0) & a_{33}(0) \end{bmatrix} \begin{bmatrix} x_1(1) & 0 & 0 \\ 0 & x_2(1) & 0 \\ 0 & 0 & x_3(1) \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(0)x_1(1) & a_{12}(0)x_2(1) & a_{13}(0)x_3(1) \\ a_{21}(0)x_1(1) & a_{22}(0)x_2(1) & a_{23}(0)x_3(1) \\ a_{31}(0)x_1(1) & a_{32}(0)x_2(1) & a_{33}(0)x_3(1) \end{bmatrix} \tag{7.5}$$

The issue is whether (or how well) the row sums and the column sums of the matrix in (7.5) correspond to our information about the target year economy – $\mathbf{u}(1)$ and $\mathbf{v}(1)$. Starting with row sums, we need to compare $\mathbf{u}^0 = \mathbf{Z}^0\mathbf{i} = [\mathbf{A}(0)\hat{\mathbf{x}}(1)]\mathbf{i}$ with $\mathbf{u}(1)$.[7]

If $\mathbf{u}^0 = \mathbf{u}(1)$, $\mathbf{Z}^0$ has the correct row sums. It then remains to be seen whether the column sums of $\mathbf{Z}^0$ match the known interindustry purchases given in $\mathbf{v}(1)$. If $\mathbf{i}'\mathbf{Z}^0 = \mathbf{v}(1)$, our work is finished, since the old technical coefficient matrix, $\mathbf{A}(0)$, in conjunction with the new gross outputs, $\mathbf{x}(1)$, generates the proper target year interindustry sales and purchases. Since the $\mathbf{u}(1)$ and $\mathbf{v}(1)$ are row and column sums of the (unknown) $\mathbf{Z}(1)$ matrix, they are sometimes referred to as "marginals" or "row and column margins" of $\mathbf{Z}(1)$.

It is much more likely that the no-change hypothesis fails – that $\mathbf{u}^0 \neq \mathbf{u}(1)$ and/or $\mathbf{v}^0 \neq \mathbf{v}(1)$. Specifically, suppose that row sums of the matrix in (7.5) are unsatisfactory;

$$\begin{aligned} a_{11}(0)x_1(1) + a_{12}(0)x_2(1) + a_{13}(0)x_3(1) &= u_1^0 \neq u_1(1) \\ a_{21}(0)x_1(1) + a_{22}(0)x_2(1) + a_{23}(0)x_3(1) &= u_2^0 \neq u_2(1) \\ a_{31}(0)x_1(1) + a_{32}(0)x_2(1) + a_{33}(0)x_3(1) &= u_3^0 \neq u_3(1) \end{aligned} \tag{7.6}$$

If a particular $u_i^0 > u_i(1)$, the elements in row i – $a_{i1}(0)$, $a_{i2}(0)$, $a_{i3}(0)$, in the example – are larger than they should be, since the $x_1(1)$, $x_2(1)$, and $x_3(1)$ contain "updated" (target year) information. [Similarly, if $u_k^0 < u_k(1)$, the elements of row k in $\mathbf{A}(0)$ are smaller than they should be.]

Let $u_i(1)/u_i^0 = r_i^1$ (the first of what will be a series of adjustment terms); when $u_i^0 > u_i(1)$, $r_i^1 < 1$. Let $i = 1$ for illustration. If each element in row 1 of $\mathbf{A}(0)$ is multiplied by r_i^1, each of those elements will be reduced. In particular, this operation generates a new set of coefficients in that row which, when multiplied by the $\mathbf{x}(1)$, will sum to $u_1(1)$ exactly, which is what we want.[8] Letting $r_1^1 a_{11}(0) = a_{11}^1$, $r_1^1 a_{12}(0) = a_{12}^1$,

[6] We use the notation $\mathbf{Z}^0$ because this represents an estimate of $\mathbf{Z}(1)$ based on no change in $\mathbf{A}(0)$. Subsequent estimates of the true $\mathbf{Z}(1)$ will be denoted $\mathbf{Z}^1, \mathbf{Z}^2, \ldots, \mathbf{Z}^k$.

[7] Similarly, $\mathbf{u}^0 = \mathbf{Z}^0\mathbf{i}$ will be the first of a series of estimates of the true $\mathbf{u}(1)$, again based on the no-change hypothesis.

[8] From (7.6) we have $a_{11}(0)x_1(1) + a_{12}(0)x_2(1) + a_{13}(0)x_3(1) = u_1^0$ where $u_1^0 > u_1(1)$. Letting $r_1^1 = u_1(1)/u_1^0$, and multiplying through by r_1^1, we have $r_1^1 a_{11}(0)x_1(1) + r_1^1 a_{12}(0)x_2(1) + r_1^1 a_{13}(0)x_3(1) = r_1^1 u_1^0 = \left(\frac{u_1(1)}{u_1^0}\right) u_1^0 = u_1(1)$.

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and $r_1^1 a_{13}(0) = a_{13}^1$, row 1 of $\mathbf{A}(0)$ has been altered to produce a new set of coefficients that constitute our first estimate of a better set of values, in the sense that they satisfy the target year information in $u_1(1)$ exactly.

Similarly, if $u_2^0 < u_2(1)$, we form $r_2^1 = u_2(1)/u_2^0 > 1$. Multiplying the elements in row 2 of $\mathbf{A}(0)$ by r_2^1 has the effect of *increasing* each of them sufficiently so that the new second row sum will equal the known $u_2(1)$. (The demonstration follows exactly the same argument as in footnote 8.) Letting $r_2^1 a_{21}(0) = a_{21}^1, r_2^1 a_{22}(0) = a_{22}^1$ and $r_2^1 a_{23}(0) = a_{23}^1$, we find a modified second row of $\mathbf{A}(0)$, where in this example all elements in this row have been increased. These are our first estimates of a better set of values for row 2 of $\mathbf{A}(0)$. Similarly, for row 3, since $u_3^0 \neq u_3(1)$ in (7.6), we use $r_3^1 = u_3(1)/u_3^0$ to multiply each coefficient in the third row of $\mathbf{A}(0)$ – reducing them if $u_3(1) < u_3^0$ and expanding if $u_3(1) > u_3^0$ – producing the known target year row sum $u_3(1)$.

This is the logic of the row adjustments. Algebraically, we want to multiply row 1 of $\mathbf{A}(0)$ by r_1^1, row 2 of $\mathbf{A}(0)$ by r_2^1 and row 3 of $\mathbf{A}(0)$ by r_3^1, and this is accomplished using a diagonal matrix made up of the r^1. (As we have seen earlier in this book, premultiplication of any matrix, $\mathbf{M}$, by a diagonal matrix, $\mathbf{D} = [d_i]$, has the effect of multiplying row i of $\mathbf{M}$ by the element d_i.) Thus a first estimate of a target-year $\mathbf{A}$ matrix, denoted $\mathbf{A}^1$, is given by

$$\mathbf{A}^1 = \begin{bmatrix} r_1^1 & 0 & 0 \\ 0 & r_2^1 & 0 \\ 0 & 0 & r_3^1 \end{bmatrix} \mathbf{A}(0) \tag{7.7}$$

The superscripts (at present, 1) in the description of the RAS technique refer to the "step" in the procedure; $\mathbf{A}^1$ is our first estimate, which means our estimate after the first step of the procedure; $\mathbf{A}^2$ will be our second estimate (and not "**A** squared"), and so on. This may appear cumbersome at first, but it turns out to be useful notation, as we will see. Letting $\mathbf{r}^1 = [r_1^1, r_2^1, r_3^1]$,

$$\hat{\mathbf{r}}^1 = \begin{bmatrix} r_1^1 & 0 & 0 \\ 0 & r_2^1 & 0 \\ 0 & 0 & r_3^1 \end{bmatrix}$$

the result in (7.7) can be expressed as

$$\mathbf{A}^1 = \hat{\mathbf{r}}^1 \mathbf{A}(0) \tag{7.8}$$

The composition of $\hat{\mathbf{r}}^1$ can easily be described using the "hat" notation once again to convert a vector into a diagonal matrix and recalling that the inverse of a diagonal matrix is another diagonal matrix whose elements are the reciprocals of those in the original matrix. Therefore

$$\hat{\mathbf{r}}^1 = [\hat{\mathbf{u}}(1)](\hat{\mathbf{u}}^0)^{-1} \tag{7.9}$$

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Following this first adjustment of $\mathbf{A}(0)$ we have a better estimate of $\mathbf{Z}(1)$, namely $\mathbf{Z}^1 = \mathbf{A}^1\hat{\mathbf{x}}(1) = \hat{\mathbf{r}}^1\mathbf{A}(0)\hat{\mathbf{x}}(1)$, with a set of row sums, $\mathbf{u}^1$, that correspond exactly to $\mathbf{u}(1)$. From (7.9) and $\mathbf{A}(0)\hat{\mathbf{x}}(1) = \hat{\mathbf{u}}^0$, we know that

$$\mathbf{u}^1 = \mathbf{Z}^1\mathbf{i} = [\hat{\mathbf{r}}^1\mathbf{A}(0)\hat{\mathbf{x}}(1)]\mathbf{i} = [[\hat{\mathbf{u}}(1)](\hat{\mathbf{u}}^0)^{-1}\hat{\mathbf{u}}^0]\mathbf{i} = \mathbf{u}(1) \tag{7.10}$$

(It was to ensure this equality that the modification of $\mathbf{A}(0)$ to $\mathbf{A}^1$ was made; this was illustrated in the previous footnote.)

The next issue, then, is whether or not the *column* sum information for the target year is captured in the improved matrix, $\mathbf{A}^1$. For that question we need to compare $\mathbf{v}(1)$ and $(\mathbf{Z}^1)'\mathbf{i} = \mathbf{v}^1 = \begin{bmatrix} v_1^1 & v_2^1 & v_3^1 \end{bmatrix}'$, the new column sums. These are

$$\begin{aligned} a_{11}^1x_1(1) + a_{21}^1x_1(1) + a_{31}^1x_1(1) &= (a_{11}^1 + a_{21}^1 + a_{31}^1)x_1(1) = v_1^1 \\ a_{12}^1x_2(1) + a_{22}^1x_2(1) + a_{32}^1x_2(1) &= (a_{12}^1 + a_{22}^1 + a_{32}^1)x_2(1) = v_2^1 \\ a_{13}^1x_3(1) + a_{23}^1x_3(1) + a_{33}^1x_3(1) &= (a_{13}^1 + a_{23}^1 + a_{33}^1)x_3(1) = v_3^1 \end{aligned} \tag{7.11}$$

If $v_1^1 = v_1(1)$, $v_2^1 = v_2(1)$, and $v_3^1 = v_3(1)$, then $\mathbf{A}^1 = \tilde{\mathbf{A}}(1)$, since it generates row and column sums that correspond to the observed $\mathbf{u}(1)$ and $\mathbf{v}(1)$.

In most cases, however, $\mathbf{v}^1 \neq \mathbf{v}(1)$, and so it is now necessary to modify the elements in $\mathbf{A}^1$ column by column. For example, if $v_1^1 > v_1(1)$ – the first $\mathbf{A}^1$ column sum in (7.11) is larger than it should be – let $v_1(1)/v_1^1 = s_1^1$ and multiply through the first equation in (7.11).[9] The superscript on s_1^1 indicates that this is our *first* modification of coefficients in order to meet column sum information. The modified coefficients in column 1 are then $s_1^1a_{11}^1$, $s_1^1a_{21}^1$, and $s_1^1a_{31}^1$; we denote these a_{11}^2, a_{21}^2, and a_{31}^2. The superscript 2 on the coefficients denotes that this is our *second* modification of elements from the original $\mathbf{A}(0)$ matrix.

Similarly, let $s_2^1 = v_2(1)/v_2^1$ and $s_3^1 = v_3(1)/v_3^1$. If a particular $v_j(1) > v_j^1$, the associated $s_j^1 > 1$ and the elements in the jth column of $\mathbf{A}^1$ are all increased when multiplied by s_j^1. On the other hand, if $v_k(1) < v_k^1$, then $s_k^1 < 1$, and each element in the kth column of $\mathbf{A}^1$ is reduced when it is multiplied by s_k^1. When a particular $v_m(1) = v_m^1$, the corresponding $s_m^1 = 1$, and the elements in column m of $\mathbf{A}^1$ will not be changed.

Algebraically, we now want to multiply column 1 of $\mathbf{A}^1$ by s_1^1, column 2 by s_2^1, and column 3 by s_3^1. Postmultiplication of $\mathbf{M}$ by a diagonal matrix has the effect of multiplying column j of $\mathbf{M}$ by the element d_j, so we form a second estimate, $\mathbf{A}^2$, as

$$\mathbf{A}^2 = \mathbf{A}^1 \begin{bmatrix} s_1^1 & 0 & 0 \\ 0 & s_2^1 & 0 \\ 0 & 0 & s_3^1 \end{bmatrix} \tag{7.12}$$

Letting $\mathbf{s}^1 = [s_1^1, s_2^1, s_3^1]$, this is

$$\mathbf{A}^2 = \mathbf{A}^1\hat{\mathbf{s}}^1 \tag{7.13}$$

[9] This gives $s_1^1(a_{11}^1 + a_{21}^1 + a_{31}^1)x_1(1) = s_1^1v_1^1 = [v_1(1)/v_1^1]v_1^1 = v_1(1)$, which is what we want.

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Given $\mathbf{v}(1)$ and $\mathbf{v}^1$, we see that

$$\hat{\mathbf{s}}^1 = [\hat{\mathbf{v}}(1)](\hat{\mathbf{v}}^1)^{-1} \tag{7.14}$$

[Compare $\hat{\mathbf{r}}^1$ in (7.9).] With this set of adjustments, we know that the column sums are correct; $\mathbf{Z}^2 = \mathbf{A}^2[\hat{\mathbf{x}}(1)]$, and

$$(\mathbf{Z}^2)'\mathbf{i} = [\mathbf{A}^2\hat{\mathbf{x}}(1)]'\mathbf{i} = \mathbf{v}(1) \tag{7.15}$$

precisely, since it was to ensure this equality that the change of $\mathbf{A}^1$ to $\mathbf{A}^2$ was made.

Note, from (7.8) and (7.13),

$$\mathbf{A}^2 = \hat{\mathbf{r}}^1\mathbf{A}(0)\hat{\mathbf{s}}^1 \tag{7.16}$$

Ignoring superscripts, hats, lower-case letters, and the (0), denoting base-year information, we have "**RAS**" on the right-hand side of (7.16). This is the origin of the name of the technique. The point here is that the **R** is seen to refer to a diagonal matrix of elements modifying rows, the **A** to the coefficient matrix being modified, and the **S** to a diagonal matrix of column modifiers.

While $\mathbf{A}^2$ in (7.13) now contains elements that, in conjunction with $\mathbf{x}(1)$, satisfy the $\mathbf{v}(1)$ margins [as in (7.15)], it will generally be the case that in modifying $\mathbf{A}^1$ to $\mathbf{A}^2$ we will have disturbed the row sum property of $\mathbf{A}^1$, given in (7.10). [Except in the case where $\hat{\mathbf{s}}^1 = \mathbf{I}$, meaning that $\mathbf{A}^1$ also satisfies *all* of the column margins exactly, and then $\mathbf{A}^1$ is our desired $\tilde{\mathbf{A}}(1)$.] Therefore, we must now test $\mathbf{A}^2$ for row sum conformability, in the same way that we tested $\mathbf{A}(0)$, originally, and the reader can see where this is going. Each subsequent row modification will generally upset the previous column modification, and vice versa – a column modification will upset the previous row modification. We explore one more iteration – a row and then a column modification.

Thus, we now find $\mathbf{Z}^2\mathbf{i}$; that is

$$\begin{bmatrix} a_{11}^2 & a_{12}^2 & a_{13}^2 \\ a_{21}^2 & a_{22}^2 & a_{23}^2 \\ a_{31}^2 & a_{32}^2 & a_{33}^2 \end{bmatrix} \begin{bmatrix} x_1(1) & 0 & 0 \\ 0 & x_2(1) & 0 \\ 0 & 0 & x_3(1) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} u_1^2 \\ u_2^2 \\ u_3^2 \end{bmatrix} \tag{7.17}$$

and let $\mathbf{u}^2 = \begin{bmatrix} u_1^2 \\ u_2^2 \\ u_3^2 \end{bmatrix}$. (The superscript on $\mathbf{u}$ indicates our *second* set of row sum estimates.) If, as is likely, $\mathbf{u}^2 \neq \mathbf{u}(1)$, we repeat the steps used in forming the diagonal row-modifying matrix – $r_1^2 = u_1(1)/u_1^2$, $r_2^2 = u_2(1)/u_2^2$ and $r_3^2 = u_3(1)/u_3^2$ – and define

$$\hat{\mathbf{r}}^2 = \begin{bmatrix} r_1^2 & 0 & 0 \\ 0 & r_2^2 & 0 \\ 0 & 0 & r_3^2 \end{bmatrix} = [\hat{\mathbf{u}}(1)](\hat{\mathbf{u}}^2)^{-1} \tag{7.18}$$

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[Compare $\hat{\mathbf{r}}^1$ in (7.9).] Note that the numerators in the r_i ratios are always the same, namely $u_i(1)$ – the number that we want. The denominators change, since they represent the "latest" estimates – here u_i^2 instead of u_i^1.

The entire procedure now follows the pattern that we have already established. If $\hat{\mathbf{r}}^2 = \mathbf{I}$, then $\mathbf{A}^2$ contains elements that satisfy both column and row margins, and we use it as $\tilde{\mathbf{A}}(1)$. If not – if $\mathbf{u}^2 \neq \mathbf{u}(1)$ – then we generate a further estimate of $\mathbf{A}(0)$ as

$$\mathbf{A}^3 = \hat{\mathbf{r}}^2\mathbf{A}^2 \tag{7.19}$$

The construction of $\hat{\mathbf{r}}^2$ assures that the row margins are now met.

The issue then (again) is whether the column sum properties of $\mathbf{A}^3$ satisfy the known target-year information in $\mathbf{v}(1)$. Thus v_1^2, v_2^2, and v_3^2 are generated, as in (7.11), with a_{ij}^3 here replacing a_{ij}^1 in that equation. Let $\mathbf{v}^2 = \begin{bmatrix} v_1^2 \\ v_2^2 \\ v_3^2 \end{bmatrix}$; if $\mathbf{v}^2 = \mathbf{v}(1)$, then we have in $\mathbf{A}^3$ a matrix that satisfies both row and column margins, and we use it for $\tilde{\mathbf{A}}(1)$. If $\mathbf{v}^2 \neq \mathbf{v}(1)$, we form

$$\hat{\mathbf{s}}^2 = [\hat{\mathbf{v}}(1)](\hat{\mathbf{v}}^2)^{-1} \tag{7.20}$$

exactly as in (7.14), but using the elements in $\mathbf{v}^2$ rather than those in $\mathbf{v}^1$. Then our next estimate of $\mathbf{A}(0)$ is given by

$$\mathbf{A}^4 = \mathbf{A}^3\hat{\mathbf{s}}^2 \tag{7.21}$$

Note, from (7.16) and (7.19), that

$$\mathbf{A}^3 = [\hat{\mathbf{r}}^2\hat{\mathbf{r}}^1]\mathbf{A}(0)[\hat{\mathbf{s}}^1] \tag{7.22}$$

and from (7.21)

$$\mathbf{A}^4 = [\hat{\mathbf{r}}^2\hat{\mathbf{r}}^1]\mathbf{A}(0)[\hat{\mathbf{s}}^1\hat{\mathbf{s}}^2] \tag{7.23}$$

Clearly, $\hat{\mathbf{r}}^1$, $\hat{\mathbf{r}}^2$, $\hat{\mathbf{s}}^1$, and $\hat{\mathbf{s}}^2$ are all diagonal matrices (3×3 in this example), so, for example,

$$[\hat{\mathbf{r}}^2\hat{\mathbf{r}}^1] = \begin{bmatrix} r_1^2 r_1^1 & 0 & 0 \\ 0 & r_2^2 r_2^1 & 0 \\ 0 & 0 & r_3^2 r_3^1 \end{bmatrix}$$

And similarly for $[\hat{\mathbf{s}}^1\hat{\mathbf{s}}^2]$. By repetition of these procedures, we would find

$$\begin{aligned} \mathbf{A}^5 &= [\hat{\mathbf{r}}^3\hat{\mathbf{r}}^2\hat{\mathbf{r}}^1]\mathbf{A}(0)[\hat{\mathbf{s}}^1\hat{\mathbf{s}}^2] \\ \mathbf{A}^6 &= [\hat{\mathbf{r}}^3\hat{\mathbf{r}}^2\hat{\mathbf{r}}^1]\mathbf{A}(0)[\hat{\mathbf{s}}^1\hat{\mathbf{s}}^2\hat{\mathbf{s}}^3] \\ &\vdots \\ \mathbf{A}^{2n} &= [\hat{\mathbf{r}}^n \cdots \hat{\mathbf{r}}^1]\mathbf{A}(0)[\hat{\mathbf{s}}^1 \cdots \hat{\mathbf{s}}^n] \end{aligned} \tag{7.24}$$

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Letting $\hat{\mathbf{r}} = [\hat{\mathbf{r}}^n \cdots \hat{\mathbf{r}}^1]$ and $\hat{\mathbf{s}} = [\hat{\mathbf{s}}^1 \cdots \hat{\mathbf{s}}^n]$, and, again, ignoring hats, lower-case letters and the (0), the right-hand side of (7.24) is "**RAS**." As mentioned earlier, this is the origin of the name of the procedure.

One may reasonably ask: how many alterations using row and column balancing factors *will* be needed until the adjusted matrix satisfies the row and column marginal totals for year 1? And, for that matter, do we know that, eventually, they *will* be satisfied, or may the sequence of adjustments make things continually worse instead of better? In general, it has been found that the RAS procedure in fact does converge. That is, after row adjustment $\hat{\mathbf{r}}^{k+1}$ we are closer to $\mathbf{u}(1)$ than we were after the previous adjustment, $\hat{\mathbf{r}}^k$, and after column adjustment $\hat{\mathbf{s}}^{k+1}$ we are closer to $\mathbf{v}(1)$ than we were after $\hat{\mathbf{s}}^k$.[10] The number of adjustments needed depends at least in part on how close one wants the row and column margins of the adjusted matrix to be to the known target-year values $\mathbf{u}(1)$ and $\mathbf{v}(1)$. One criterion is to continue the matrix adjustments until all elements in both $[|\mathbf{u}(1) - \mathbf{u}^k|]$ and $[|\mathbf{v}(1) - \mathbf{v}^k|]$ are no more than ε, where ε is some small positive number, say 0.001. This means that each u_i^k is within 0.001 of the desired $u_i(1)$, and also that each v_j^k is within 0.001 of its associated $v_j(1)$.

For cases in which one is interested in assessing impacts on an economy of some *future* event, a *projection* of an existing technical coefficients matrix is called for. One approach is again to use the RAS procedure, where now the values in the $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{x}$ vectors must be forecast into the future year τ; these estimates $\mathbf{u}(\tau)$, $\mathbf{v}(\tau)$, and $\mathbf{x}(\tau)$ will then be used along with the current or most recent base matrix, $\mathbf{A}(0)$.

7.4.2 *Example of the RAS Procedure*

We illustrate the mathematics with a 3×3 example. Let

$$\mathbf{A}(0) = \begin{bmatrix} .120 & .100 & .049 \\ .210 & .247 & .265 \\ .026 & .249 & .145 \end{bmatrix} \tag{7.25}$$

The information necessary for a full survey-based coefficients table for the target year, $\mathbf{A}(1)$, would be interindustry flows, $\mathbf{Z}(1)$, and total outputs, $\mathbf{x}(1)$. Suppose, in fact, that we have

$$\mathbf{Z}(1) = \begin{bmatrix} 98 & 72 & 75 \\ 65 & 8 & 63 \\ 88 & 27 & 44 \end{bmatrix} \tag{7.26}$$

and

$$\mathbf{x}(1) = \begin{bmatrix} 421 \\ 284 \\ 283 \end{bmatrix} \tag{7.27}$$

[10] These technical matters, dealing with properties of the RAS technique, including convergence, are beyond the scope of this book.

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Consequently,

$$\mathbf{u}(1) = \begin{bmatrix} 245 & 136 & 159 \end{bmatrix}' \tag{7.28}$$

and

$$\mathbf{v}(1) = \begin{bmatrix} 251 & 107 & 182 \end{bmatrix}' \tag{7.29}$$

and

$$\mathbf{A}(1) = [\mathbf{Z}(1)][\hat{\mathbf{x}}(1)]^{-1} = \begin{bmatrix} .2328 & .2535 & .2650 \\ .1544 & .0282 & .2226 \\ .2090 & .0951 & .1555 \end{bmatrix} \tag{7.30}$$

The point of partial-survey techniques is to develop reasonable estimates of the elements in $\mathbf{A}(1)$ in the absence of this kind of information on the full set of transactions in $\mathbf{Z}(1)$. To use the RAS approach, we need only the marginal information in $\mathbf{u}(1)$ and $\mathbf{v}(1)$, along with $\mathbf{x}(1)$ – as in (7.27), (7.28), and (7.29) – and the original or base year coefficients matrix, $\mathbf{A}(0)$, as in (7.25).

Beginning with the conjecture that the coefficients have not changed, we first examine the row sums of $\mathbf{A}(0)\hat{\mathbf{x}}(1)$, as in (7.5), in light of $\mathbf{u}(1)$. Here

$$\mathbf{Z}^1 = \mathbf{A}(0)\hat{\mathbf{x}}(1) = \begin{bmatrix} 50.520 & 28.400 & 13.867 \\ 88.410 & 70.148 & 74.995 \\ 10.946 & 70.716 & 41.035 \end{bmatrix}$$

and

$$\mathbf{u}^1 = \mathbf{Z}^1\mathbf{i} = \begin{bmatrix} 92.787 & 233.553 & 122.697 \end{bmatrix}'$$

Clearly, this is nowhere near to $\mathbf{u}(1)$ in (7.28) and adjustment is needed. To begin, then, $r_1^1 = u_1(1)/u_1^1 = 245/92.787 = 2.6405$, $r_2^1 = 0.5823$ and $r_3^1 = 1.2959$. Forming $\hat{\mathbf{r}}^1$ as in (7.9), we have

$$\hat{\mathbf{r}}^1 = [\hat{\mathbf{u}}(1)](\hat{\mathbf{u}}^1)^{-1} = \begin{bmatrix} 2.6405 & 0 & 0 \\ 0 & 0.5823 & 0 \\ 0 & 0 & 1.2959 \end{bmatrix}$$

and our first adjusted matrix, $\mathbf{A}^1$, is

$$\mathbf{A}^1 = \hat{\mathbf{r}}^1\mathbf{A}(0) = \begin{bmatrix} .3169 & .2640 & .1294 \\ .1223 & .1438 & .1543 \\ .0337 & .3227 & .1879 \end{bmatrix} \tag{7.31}$$

The elements in $\hat{\mathbf{r}}^1$ assure that the row sums of $\mathbf{A}^1\hat{\mathbf{x}}(1)$ will equal $\mathbf{u}(1)$, as in (7.10). Checking the column sums of $\mathbf{A}^1\hat{\mathbf{x}}(1)$ against $\mathbf{v}(1)$, we have

$$\mathbf{v}^1 = [\mathbf{A}^1\hat{\mathbf{x}}(1)]'\mathbf{i} = \begin{bmatrix} 199.06 & 207.48 & 133.46 \end{bmatrix}'$$

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and this is wide of the mark, since

$$\mathbf{v}(1) = \begin{bmatrix} 251 & 107 & 182 \end{bmatrix}$$

Then, as in (7.14),

$$\hat{\mathbf{s}}^1 = [\hat{\mathbf{v}}(1)](\hat{\mathbf{v}}^1)^{-1} = \begin{bmatrix} 1.2609 & 0 & 0 \\ 0 & 0.5157 & 0 \\ 0 & 0 & 1.3637 \end{bmatrix}$$

and, following (7.13),

$$\mathbf{A}^2 = \mathbf{A}^1\hat{\mathbf{s}}^1 = \begin{bmatrix} .3995 & .1219 & .1764 \\ .1542 & .0661 & .2104 \\ .0425 & .1664 & .2562 \end{bmatrix}$$

In this example, we arbitrarily set $\varepsilon = 0.005$, meaning that the alternating row and column adjustments would continue through the kth adjustment, when $|u_i(1) - u_i^k| \leq 0.005$ and $|v_j(1) - v_j^k| \leq 0.005$ for $i, j = 1, 2, 3$. For this example, $k = 12$ (six row adjustments and six column adjustments were needed). The final matrix, $\mathbf{A}^{12}$, is

$$\tilde{\mathbf{A}}(1) = \mathbf{A}^{12} = \begin{bmatrix} .3924 & .1219 & .1596 \\ .1509 & .0661 & .1897 \\ .0592 & .1887 & .2938 \end{bmatrix} \tag{7.32}$$

Rather than print all the present matrices, $\mathbf{A}^1$ through $\mathbf{A}^{11}$, Table 7.1 gives the successive values of two representative coefficients, a_{11} and a_{23}, beginning with the original $\mathbf{A}(0)$ matrix and continuing through each RAS iteration. In Table 7.2 we record the three elements in $[\mathbf{u}(1) - \mathbf{u}^k]$ and the three elements in $[\mathbf{v}(1) - \mathbf{v}^k]$ (transposed to make them row vectors, for ease of presentation), for $k = 0$ through 13. The $k = 0$ line shows the row and column differences using $\mathbf{A}(0)\hat{\mathbf{x}}(1)$ – that is, assuming $\mathbf{A}(0) = \mathbf{A}(1)$. As expected, at $k = 1$ the row margins, in $\mathbf{u}(1)$, are satisfied exactly – all zero elements in $[\mathbf{u}(1) - \mathbf{u}^1]$ – but the column margins, in $\mathbf{v}(1)$, are not. Then, step 2 adjusts for these column constraints – generating zeros in $[\mathbf{v}(1) - \mathbf{v}^2]$ – but throwing the row sums out of balance with $\mathbf{u}(1)$. Therefore, for odd values of k, the $\mathbf{u}$ differences are all zero; for even values of k, the $\mathbf{v}$ differences are all zero. At $k = 13$ (that is, following $k = 12$), all differences are less than 0.005 in absolute value (for the first time), and hence the RAS adjustment is terminated. Finally, in Table 7.3 we present the elements of each of the matrices, $\hat{\mathbf{r}}^1$ through $\hat{\mathbf{r}}^7$ and $\hat{\mathbf{s}}^1$ through $\hat{\mathbf{s}}^7$, as in $\mathbf{A}^{2n}$ in (7.24).

It is of interest to compare our RAS-generated target-year matrix, $\tilde{\mathbf{A}}(1)$ with $\mathbf{A}(1)$ in (7.30), which we would have available to us if the entire set of interindustry transactions in $\mathbf{Z}(1)$ had been known.

$$\tilde{\mathbf{A}}(1) = \begin{bmatrix} .3924 & .1219 & .1596 \\ .1509 & .0661 & .1897 \\ .0529 & .1887 & .2938 \end{bmatrix} \text{ and } \mathbf{A}(1) = \begin{bmatrix} .2328 & .2535 & .2650 \\ .1544 & .0282 & .2226 \\ .2090 & .0951 & .1555 \end{bmatrix}$$

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Table 7.1 Values of a_{11} and a_{23} at Each Step in the RAS Adjustment Procedure

k	a_{11}	a_{23}
0	.120	.265
1	.3169	.1543
2	.3995	.2104
3	.3812	.1966
4	.3957	.1913
5	.3902	.1912
6	.3931	.1900
7	.3920	.1900
8	.3926	.1898
9	.3923	.1898
10	.3925	.1897
11	.3924	.1897
12	.3924	.1897

Table 7.2 Differences from Row and Column Margins at Each Step in the RAS Adjustment Procedure

k	$[\mathbf{u}(1) - \mathbf{u}^k]'$			$[\mathbf{v}(1) - \mathbf{v}^k]'$		
0	152.2130	−97.5530	36.3030	101.1240	−62.2640	52.1030
1	0	0	0	51.9376	−100.4759	48.5383
2	−11.8055	−9.5328	21.3383	0	0	0
3	0	0	0	9.2120	−4.1679	−5.0441
4	−3.4458	−.0723	3.5181	0	0	0
5	0	0	0	1.8586	−.6862	−1.1724
6	−.7098	−.0024	.7122	0	0	0
7	0	0	0	.3798	−.1394	−.2404
8	−.1452	−.0007	.1459	0	0	0
9	0	0	0	.0778	−.0286	−.0492
10	−.0297	−.0002	.0299	0	0	0
11	0	0	0	.0159	−.0059	−.0101
12	−.0061	0	.0061	0	0	0
13	0	0	0	.0033	−.0012	−.0021

Even casual inspection shows that there are significant differences in most of the elements in these two matrices.

Define an error matrix, $\mathbf{E}(\mathbf{A})$, as $\mathbf{E}(\mathbf{A}) = \tilde{\mathbf{A}}(1) - \mathbf{A}(1)$. Here

$$\mathbf{E}(\mathbf{A}) = \begin{bmatrix} .1596 & -.1316 & -.1054 \\ -.0035 & .0379 & -.0329 \\ -.1561 & .0936 & .1383 \end{bmatrix}$$

Table 7.3 Elements in the Diagonal Matrices $\hat{\mathbf{r}}^k$ and $\hat{\mathbf{s}}^k$, for $k = 1, \ldots, 7$

k	$\hat{\mathbf{r}}^k$			$\hat{\mathbf{s}}^k$		
1	2.6405	.5823	1.2959	1.2609	.5157	1.3637
2	.9540	.9345	1.1550	1.0381	.9625	.9730
3	.9861	.9995	1.0226	1.0075	.9936	.9936
4	.9971	1.0000	1.0045	1.0015	.9987	.9987
5	.9994	1.0000	1.0009	1.0003	.9997	.9997
6	.9999	1.0000	1.0002	1.0001	.9999	.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Notice that column sums of $\mathbf{E}(\mathbf{A})$ are zero; this reflects the fact that column sums of $\tilde{\mathbf{A}}(1)$ and $\mathbf{A}(1)$ are equal, except for rounding in this small example.[11]

An alternative way to express the errors in each of the coefficients is to convert the elements in $\mathbf{E}(\mathbf{A})$ to percentages. Define $\mathbf{P}(\mathbf{A}) = [p(a)_{ij}]$ where

$$p(a)_{ij} = [|\tilde{a}_{ij} - a_{ij}(1)|/a_{ij}(1)] \times 100 = [|e(a)_{ij}|/a_{ij}(1)] \times 100$$

These are the absolute values of the errors as a percentage of the corresponding true coefficients in $\mathbf{A}(1)$. For this example,

$$\mathbf{P}(\mathbf{A}) = \begin{bmatrix} 68.6 & 51.9 & 39.8 \\ 2.3 & 134.4 & 14.8 \\ 74.7 & 98.4 & 88.9 \end{bmatrix}$$

Viewed in this way, also, it is clear that some of the RAS-estimated coefficients are wildly different from their survey counterparts. Six of the nine RAS-generated coefficients are in error by more than 50 percent – not a very successful estimate.

There are many measures available for quantifying the "difference" between two matrices. We illustrate several of them. *The mean absolute deviation* (MAD) simply averages the elements in $\mathbf{E}(\mathbf{A})$, ignoring sign:

$$\text{MAD} = (1/n^2) \sum_{i=1}^{n} \sum_{j=1}^{n} |e(a)_{ij}|$$

In our example, MAD $= (1/9)(0.8589) = 0.0954$. This represents the average amount (whether positive or negative) by which an estimated coefficient differs from the true coefficient. The *mean absolute percentage error* (MAPE) performs the same averaging

[11] The RAS marginal constraints assure that $\mathbf{i}'\tilde{\mathbf{Z}}(1) = \mathbf{i}'\mathbf{Z}(1)$. Since $\tilde{\mathbf{Z}}(1) = \tilde{\mathbf{A}}(1)\hat{\mathbf{x}}(1)$ and $\mathbf{Z}(1) = \mathbf{A}(1)\hat{\mathbf{x}}(1)$, $\mathbf{i}'\tilde{\mathbf{A}}(1)\hat{\mathbf{x}}(1) = \mathbf{i}'\mathbf{A}(1)\hat{\mathbf{x}}(1)$ and so (postmultiplying by $[\hat{\mathbf{x}}(1)]^{-1}$), $\mathbf{i}'\tilde{\mathbf{A}}(1) = \mathbf{i}'\mathbf{A}(1)$.

on the elements in $\mathbf{P}(\mathbf{A})$, namely

$$\text{MAPE} = (1/n^2)\sum_{i=1}^{n}\sum_{j=1}^{n} p(a)_{ij}$$

For this example, MAPE = (1/9)(575.38) = 63.76, which means that, on average, each coefficient will be either 63.8 percent larger or smaller than its true value; that is, it will be "in error" by 63.8 percent. [If the *direction* of error is thought to be important, then we could generate the elements in the $\mathbf{P}(\mathbf{A})$ matrix, retaining the signs. However, in that case, it is not very meaningful to find an average over all elements, since positive and negative errors would offset each other.] By these measures (and others, which we need not explore here), the matrix produced by the RAS procedure in this small example does not appear to be a particularly good reflection of $\mathbf{A}(1)$. At least this is the implication of these measures that examine the element-by-element accuracy of $\tilde{\mathbf{A}}(1)$ as compared with $\mathbf{A}(1)$. In larger examples, more representative of real-world input–output tables, there are more elements available for adjustment in any row or column and, in that sense, there is more flexibility in producing an estimate of the target-year matrix.

Another point of view is that while this individual cell accuracy (sometimes called *partitive* accuracy) may be important for some kinds of problems, the ultimate test of a set of input–output coefficients is how well they perform in practice (also sometimes known as *holistic* accuracy).[12] That is, perhaps we should be more concerned with the relative accuracy in the Leontief inverse matrices associated with $\tilde{\mathbf{A}}(1)$ and $\mathbf{A}(1)$. Here

$$\mathbf{L}(1) = \begin{bmatrix} 1.5651 & .4684 & .6146 \\ .3463 & 1.1599 & .4144 \\ .4264 & .2465 & 1.3829 \end{bmatrix} \tag{7.33}$$

and

$$\tilde{\mathbf{L}}(1) = [(\mathbf{I} - \tilde{\mathbf{A}}(1)]^{-1} = \begin{bmatrix} 1.7703 & .3298 & .4888 \\ .3310 & 1.1940 & .3955 \\ .2210 & .3438 & 1.5583 \end{bmatrix} \tag{7.34}$$

The associated error matrices are

$$\mathbf{E}(\mathbf{L}) = \begin{bmatrix} .2052 & -.1386 & -.1258 \\ -.0153 & .0341 & -.0189 \\ -.2054 & .0973 & .1754 \end{bmatrix}$$

and

$$\mathbf{P}(\mathbf{L}) = \begin{bmatrix} 13.1 & 29.6 & 20.5 \\ 4.4 & 2.9 & 4.6 \\ 48.2 & 39.5 & 12.7 \end{bmatrix}$$

[12] These terms are from Jensen (for example, Jensen, 1980).

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For this small example, percentage errors in $\mathbf{P}(\mathbf{L})$, associated with the Leontief inverse matrices, are generally considerably smaller than those in $\mathbf{P}(\mathbf{A})$.

Alternatively, consider the output multipliers associated with $\mathbf{L}(1)$ and $\tilde{\mathbf{L}}(1)$, $\mathbf{m}(o) = \begin{bmatrix} 2.3378 & 1.8748 & 2.4119 \end{bmatrix}$ and $\tilde{\mathbf{m}}(o) = \begin{bmatrix} 2.3223 & 1.8676 & 2.4426 \end{bmatrix}$. The vector of percentage errors, expressing each $[m(o)_j - \tilde{m}(o)_j]$ as a percentage of $m(o)_j$, is $\mathbf{p}(\mathbf{m}) = \begin{bmatrix} 0.66 & 0.38 & -1.27 \end{bmatrix}$. This indicates much closer correspondence between the estimated and true multipliers than might be expected from $\mathbf{E}(\mathbf{A})$ and $\mathbf{P}(\mathbf{A})$, and even from $\mathbf{E}(\mathbf{L})$ and $\mathbf{P}(\mathbf{L})$.

The power series expressions for L and $\tilde{\mathbf{L}}$ are helpful here, namely

$$\mathbf{L} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots \text{ and } \tilde{\mathbf{L}} = \mathbf{I} + \tilde{\mathbf{A}} + \tilde{\mathbf{A}}^2 + \tilde{\mathbf{A}}^3 + \cdots$$

From these, the (row) vector of multiplier differences can be expressed as

$$\mathbf{i}'\mathbf{L} - \mathbf{i}'\tilde{\mathbf{L}} = \mathbf{i}'(\mathbf{L} - \tilde{\mathbf{L}}) = \mathbf{i}'(\mathbf{I} - \mathbf{I}) + \mathbf{i}'(\mathbf{A} - \tilde{\mathbf{A}}) + \mathbf{i}'(\mathbf{A}^2 - \tilde{\mathbf{A}}^2) + \mathbf{i}'(\mathbf{A}^3 - \tilde{\mathbf{A}}^3) + \cdots$$

Clearly $\mathbf{i}'(\mathbf{I} - \mathbf{I}) = \mathbf{0}$, and also $\mathbf{i}'(\mathbf{A} - \tilde{\mathbf{A}}) = \mathbf{i}'\mathbf{E}(\mathbf{A}) = \mathbf{0}$, as noted above. Therefore,

$$\mathbf{i}'\mathbf{L} - \mathbf{i}'\tilde{\mathbf{L}} = \mathbf{0} + \mathbf{0} + \mathbf{i}'(\mathbf{A}^2 - \tilde{\mathbf{A}}^2) + \mathbf{i}'(\mathbf{A}^3 - \tilde{\mathbf{A}}^3) + \cdots$$

We see that the first two terms in the expression for the output multiplier differences are zero. (In the example in Table 2.5 we saw that these two terms in the power series accounted for between 85 and 92 percent of the total output effect.)

Comparison of multipliers is a test of the model in use, with specific final-demand vectors – $[1, 0, 0]'$, $[0, 1, 0]'$, and $[0, 0, 1]'$, respectively. We can also compare the performance using any arbitrarily chosen $\mathbf{f}(1)$ vector. For example, let $\mathbf{f}(1) = \begin{bmatrix} 800 \\ 700 \\ 300 \end{bmatrix}$; then from the Leontief inverses in (7.33) and (7.34),

$$\mathbf{x}(1) = \begin{bmatrix} 1764.20 \\ 1213.29 \\ 928.54 \end{bmatrix} \text{ and } \tilde{\mathbf{x}}(1) = \begin{bmatrix} 1793.74 \\ 1219.25 \\ 884.95 \end{bmatrix}$$

Again, expressing the differences as a percentage of $x_i(1)$,

$$\mathbf{p}(\Delta\mathbf{x}) = \begin{bmatrix} 1.67 \\ 0.49 \\ -4.69 \end{bmatrix}$$

The effect on the gross output of sector 3 is underestimated by almost five percent while the other two outputs are much more accurately estimated. Of course, results of this kind depend on the arbitrary $\mathbf{f}(1)$vector used for the illustration.

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Conclusions suggested by this example are: (1) the RAS procedure may generate a technical coefficients matrix that does not look very much like an associated full-survey matrix, but (2) an **A** matrix estimated by RAS may perform relatively well in practice, that is, when converted to its associated Leontief inverse, in terms of the sectoral gross outputs that it produces in conjunction with a given $\mathbf{f}(1)$ vector. We will examine another holistic measure of performance in Chapter 8, when we explore differences in output multipliers in a regional input–output model.

7.4.3 Updating Coefficients vs. Transactions

Early discussions of the technique assumed that one begins with a base year **A**; this is explicit in the "**RAS**" name. It appears that Deming and Stephan (1940) first used the biproportional adjustment technique that later became known as RAS. Leontief (1941) suggested a similar pair of influences (on rows and on columns) to account jointly for coefficient change. Stone and his colleagues at Cambridge apparently were unaware of this work when they proposed it in 1962 (Bacharach, 1970, p. 4; see also Lahr and de Mesnard, 2004). The Cambridge work emphasized operations on a base-year *coefficient* matrix, even though Bacharach (1970, p. 20) suggests that the ultimate interest was in a target-year *transactions* matrix.

In fact, this biproportional matrix balancing approach can be equally well applied directly to a base-year transactions matrix, $\mathbf{Z}(0)$, in conjunction with the required marginal information, $\mathbf{x}(1)$, $\mathbf{u}(1)$, and $\mathbf{v}(1)$. In this case, there is no need to convert the coefficients at each step, $\mathbf{A}^k$, to transactions, $\mathbf{Z}^k$, in order to check the degree of conformity with $\mathbf{u}(1)$ and $\mathbf{v}(1)$. There seems to have been some uncertainty in the literature on whether or not the end results of the two exercises – directly altering **A** vs. directly altering **Z** – are the same.[13]

In the former case (updating **A**), denote $\tilde{\mathbf{A}}^A(1) = \hat{\mathbf{r}}^A\mathbf{A}(0)\hat{\mathbf{s}}^A$, leading to $\tilde{\mathbf{Z}}^A(1) = \tilde{\mathbf{A}}^A(1)\hat{\mathbf{x}}(1)$ and in the latter case (updating **Z**), let $\tilde{\mathbf{Z}}^Z(1) = \hat{\mathbf{r}}^Z\mathbf{Z}(0)\hat{\mathbf{s}}^Z$, leading to $\tilde{\mathbf{A}}^Z(1) = \tilde{\mathbf{Z}}^Z(1)\hat{\mathbf{x}}(1)^{-1}$. The question is whether or not $\tilde{\mathbf{A}}^A(1) = \tilde{\mathbf{A}}^Z(1)$ or $\tilde{\mathbf{Z}}^A(1) = \tilde{\mathbf{Z}}^Z(1)$ (where superscripts indicate which matrix was used in the updating procedure). The answer is that it makes no difference which kind of matrix is used as the base for the updating procedure (coefficients or transactions); the results from the two approaches are the same. (See Dietzenbacher and Miller, 2009, for a proof).

Numerical Illustration This is the data set for the closed model in Chapter 2. Call this year 0 data:

$$\mathbf{Z}(0) = \begin{bmatrix} 150 & 500 & 50 \\ 200 & 100 & 400 \\ 300 & 500 & 50 \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} 1000 \\ 2000 \\ 1000 \end{bmatrix}, \quad \mathbf{A}(0) = \begin{bmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{bmatrix}$$

[13] See for example, the sequence of opinions expressed in Okuyama *et al.* (2002), Jackson and Murray (2004), and Oosterhaven (2005).

Assume that we have the following year 1 information (necessary for RAS):

$$\mathbf{x}(1) = \begin{bmatrix} 1200 \\ 2500 \\ 1400 \end{bmatrix}, \quad \mathbf{u}(1) = \begin{bmatrix} 780 \\ 810 \\ 1050 \end{bmatrix}, \quad \mathbf{v}(1) = \begin{bmatrix} 740 \\ 1270 \\ 630 \end{bmatrix}$$

Coefficient updating. Start with $\mathbf{A}(0)$. In this case we find

$$\tilde{\mathbf{A}}^A(1) = \hat{\mathbf{r}}^A \mathbf{A}(0) \hat{\mathbf{s}}^A = \begin{bmatrix} 0.1370 & 0.2205 & 0.0460 \\ 0.1752 & 0.0423 & 0.3529 \\ 0.3046 & 0.2452 & 0.0511 \end{bmatrix}$$

For ease of presentation, we have rounded all coefficients to four digits and all transactions to whole numbers. In this case, the associated transactions matrix is

$$\tilde{\mathbf{Z}}(1)^A = \tilde{\mathbf{A}}^A \hat{\mathbf{x}}(1) = \begin{bmatrix} 164 & 551 & 64 \\ 210 & 106 & 494 \\ 365 & 613 & 72 \end{bmatrix}$$

Transaction updating. Start with $\mathbf{Z}(0)$. RAS provides the update

$$\tilde{\mathbf{Z}}^Z(1) = \hat{\mathbf{r}}^Z \mathbf{Z}(0) \hat{\mathbf{s}}^Z = \begin{bmatrix} 164 & 551 & 64 \\ 210 & 106 & 494 \\ 365 & 613 & 72 \end{bmatrix}$$

illustrating that $\tilde{\mathbf{Z}}^A(1) = \tilde{\mathbf{Z}}^Z(1)$. Also, from this,

$$\tilde{\mathbf{A}}^Z(1) = \tilde{\mathbf{Z}}^Z(1)[\hat{\mathbf{x}}(1)]^{-1} = \begin{bmatrix} 0.1370 & 0.2205 & 0.0460 \\ 0.1752 & 0.0423 & 0.3529 \\ 0.3046 & 0.2452 & 0.0511 \end{bmatrix}$$

and $\tilde{\mathbf{A}}^A(1) = \tilde{\mathbf{A}}^Z(1)$. This does not prove but illustrates what is a general result.

7.4.4 An Economic Interpretation of the RAS Procedure

In the preceding sections, we have illustrated the mathematics of the RAS procedure for sequentially adjusting rows and columns of a given coefficient matrix, $\mathbf{A}(0)$, in order to generate an estimate of a more recent matrix, $\mathbf{A}(1)$, where only $\mathbf{x}(1)$, $\mathbf{u}(1)$, and $\mathbf{v}(1)$ are assumed known for the target year, 1. When the adjustment process is terminated – because the row and column margins are within the prespecified level of error, ε, from the elements in $\mathbf{u}(1)$ and $\mathbf{v}(1)$ – we have

$$\mathbf{A}(1) = \hat{\mathbf{r}} A(0) \hat{\mathbf{s}} \tag{7.35}$$

As we have seen, each element r_i in $\hat{\mathbf{r}}$ multiplies each element in row i of $\mathbf{A}(0)$ and each element s_j of $\hat{\mathbf{s}}$ multiplies each element in column j of $\mathbf{A}(0)$ – for $i, j = 1, \ldots, n$.

In this "updating" procedure, one might well ask why this kind of uniform proportional change should be expected for the elements in rows or columns of $\mathbf{A}(0)$.

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In the early development of the RAS procedure, Stone (1961) described the uniform changes along any row and down any column in **A** as reflecting what he termed the economic phenomena of *substitution effects* and *fabrication effects*, respectively. The former refers to the emergence of substitutes as production inputs; that is, the substitution of one input for another – for example, the use (throughout industrial processes) of plastic products in place of metal ones. The implication is that all a_{ij} in the plastics row (i) would increase (for example, be multiplied by 1.4) and all a_{kj} in the metals row (k) would decrease (for example, be multiplied by 0.82). The term fabrication effect refers to the altered proportion of value-added items in a sector's total purchases. For example, over time, the product of a particular sector may come to depend more on high-technology capital equipment and/or skilled labor. Thus, a dollar's worth of the product embodies proportionately less of interindustrial inputs and proportionately more of value-added inputs, and the a_{ij} in the column representing the industry in question would decrease (for example, be multiplied by 0.79).

To the extent that technological change in the style of production may be reflected in such substitution and fabrication effects, the RAS procedure has a logical economic basis. However, many researchers discount this oversimplified view of the way in which such change is distributed throughout an economy. Instead, they view RAS as a purely mathematical procedure. It can be shown that the RAS technique in fact emerges as the solution to a constrained optimization problem in which, subject to the row and column margins given in $\mathbf{u}(1)$ and $\mathbf{v}(1)$, we want to generate a new coefficient matrix, $\mathbf{A}(1)$, that "differs" as little as possible from our previous observation, $\mathbf{A}(0)$. The underlying logic is simply that, in the absence of any new information, we would assume that $\mathbf{A}(0)$ is still the best representation of interindustrial relationships. However, given some updated information – in $\mathbf{x}(1)$, $\mathbf{u}(1)$, and $\mathbf{v}(1)$ – a modified matrix, $\mathbf{A}(1)$, will usually be called for.

Two properties of the RAS procedure bear noting. Signs are preserved in the sense that no $a_{ij}(0) > 0$ will ever be changed to a negative-valued coefficient. As the fundamental definitions of $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ make clear, all the r_i and s_j modifiers of $\mathbf{A}(0)$ are non-negative. Thus, no matter how much a particular $a_{ij}(0)$ is modified, it will remain non-negative. Secondly, any $a_{ij}(0)$ that equals zero will remain zero throughout the RAS procedure, since all that happens to it is that it is multiplied by non-negative numbers. Suppose that sector i represents potatoes and sector j is automobiles; if $a_{ij}(0) = 0$, this represents the (believable) fact that potatoes were not purchased as direct inputs to automobile manufacturing in year 0. The RAS technique assures us that in the updated matrix $a_{ij}(1)$ will still be zero. This feature is a mixed blessing. In some cases, such as potatoes and automobiles, it is probably good that a zero-valued coefficient is preserved; potatoes were not used as a direct input to automobiles in year 0 and most probably not in year 1, either. On the other hand, if sector k is plastics and sector j is automobiles, it may be (if year 0 was long enough ago) that $a_{kj}(0) = 0$, but we know that for our more recent year 1, $a_{kj}(1) \neq 0$. Nevertheless, the RAS procedure by itself will predict $a_{kj}(1) = 0$.

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7.4.5 Incorporating Additional Exogenous Information in an RAS Calculation

The RAS technique, as discussed above, assumes only target-year information on $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{v}$. Often one may have particular information about specific transactions or specific coefficients. If a particular $z_{ij}(1)$is exogenously known, then since $x_j(1)$ is also known, so is $a_{ij}(1)$. Such information may come from a survey of an "important" industry in the economy, from an independent forecast of a particular sector's sales to one or more sectors, from expert opinions about production practices in a particular sector, and so on.

Suppose that a particular $z_{ij}(1)$ is known. Then one can subtract $z_{ij}(1)$ from both $u_i(1)$ and $v_j(1)$; this is equivalent to inserting a zero in the i, jth cell of $\mathbf{Z}(0)$ and hence of $\mathbf{A}(0)$. Continuing with our general 3×3 example, suppose $z_{31}(1)$ is known. Since $x_1(1)$ is also known, $a_{31}(1)$ is known as well.

Define $\bar{\mathbf{A}}(0)$ to be the same as $\mathbf{A}(0)$ except that $a_{31}(0)$ has been replaced with a zero. Define a 3×3 matrix $\mathbf{K}$ as

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{31}(1) & 0 & 0 \end{bmatrix}$$

This is just the null matrix with k_{31} replaced by the known target-year coefficient, $a_{31}(1)$. Then $\mathbf{A}(0) = \bar{\mathbf{A}}(0) + \mathbf{K}$. Denote by $\bar{\mathbf{u}}(1)$ and $\bar{\mathbf{v}}(1)$ the vectors that remain after $z_{31}(1)$ is subtracted from $u_3(1)$ and $v_1(1)$. These become the relevant new margins, and the RAS procedure is utilized, as usual, but with $\bar{\mathbf{A}}(0)$ as the base-year matrix, to be modified according to the (altered) row and column sum information for the target year, $\bar{\mathbf{u}}(1)$ and $\bar{\mathbf{v}}(1)$. The RAS technique will leave the new zero element, $a_{31}(0)$, unchanged. When the approximating technique is completed we construct our estimate of $\mathbf{A}(1)$ as[14]

$$\tilde{\mathbf{A}}(1)_{31} = \mathbf{K} + \hat{\mathbf{r}}\bar{\mathbf{A}}(0)\hat{\mathbf{s}} \tag{7.36}$$

Clearly, in an economy represented by a larger number of sectors, we may have estimates of several $z_{ij}(1)$ and hence of several of the target-year coefficients, $a_{ij}(1)$. In fact, if there is a "key" sector that is known to play a particularly important role in the economy, an entire column (intermediate inputs to the key sector) and/or an entire row (intermediate sales by the key sector) may be known or somehow independently determined. And indeed there may be more than one key sector. In all of these cases, there is no difference in the approach outlined. Of course, the matrix $\mathbf{K}$ will contain more nonzero (known) elements, the matrix $\bar{\mathbf{A}}(0)$ will contain more zeros, and the adjustments to $\mathbf{u}(1)$ and $\mathbf{v}(1)$ – to generate $\bar{\mathbf{u}}(1)$ and $\bar{\mathbf{v}}(1)$ – will be more extensive.[15]

[14] We use the "31" subscript to indicate which element was replaced by its true value. This does not generalize easily to cases in which more than one element is replaced by exogenous information, but it serves adequately for present purposes.

[15] See section 7.4.8, below, on the role of zeros in creating infeasible problems – where RAS fails to generate a solution.

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7.4.6 *Modified Example: One Coefficient Known in Advance*

Here is an illustration. Suppose a_{31} is known in advance for the example in section 7.4.2; from (7.30), $a_{31}(1) = 0.209$, and therefore

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .209 & 0 & 0 \end{bmatrix}$$

and so

$$\bar{\mathbf{A}}(0) = \begin{bmatrix} .120 & .100 & .049 \\ .210 & .247 & .265 \\ 0 & .249 & .145 \end{bmatrix}$$

This is $\mathbf{A}(0)$ in (7.25) with $a_{31}(1)$ replaced by 0.

To employ the RAS procedure on $\bar{\mathbf{A}}(0)$ we find $\bar{\mathbf{u}}(1)$ and $\bar{\mathbf{v}}(1)$. The (known) interindustry flow in the target year, from sector 3 to sector 1, is $z_{31}(1) = a_{31}(1)x_1(1) = (0.209)(421) = 87.989$; this therefore must be netted out of both $u_3(1)$ and $v_1(1)$, leading to $\bar{\mathbf{u}}(1) = \begin{bmatrix} 245 & 136 & 71.011 \end{bmatrix}'$ and $\bar{\mathbf{v}}(1) = \begin{bmatrix} 163.011 & 107 & 182 \end{bmatrix}'$. Following (7.36) we find

$$\tilde{\mathbf{A}}(1)_{31} = \begin{bmatrix} .2909 & .1892 & .2431 \\ .0963 & .0884 & .2486 \\ .2090 & .0992 & .1514 \end{bmatrix} \tag{7.37}$$

Recall, from (7.30), that

$$\mathbf{A}(1) = \begin{bmatrix} .2328 & .2535 & .2650 \\ .1544 & .0282 & .2226 \\ .2090 & .0951 & .1555 \end{bmatrix}$$

and the error matrix for this estimate, $\mathbf{E}(\mathbf{A}) = \tilde{\mathbf{A}}(1)_{31} - \mathbf{A}(1)$, is

$$\mathbf{E}(\mathbf{A}) = \begin{bmatrix} -.0581 & -.0643 & -.0219 \\ .0581 & .0602 & .0260 \\ 0 & .0041 & -.0041 \end{bmatrix}$$

In this case, the reader can easily find that MAD $= (1/9)(0.2968) = 0.0330$ and MAPE $= 36.5$; in the original example, without any prior information on coefficient values, we found MAD $= 0.0954$ and MAPE $= 63.8$. By these measures, then, the RAS estimate in (7.37), which includes exogenous information on $a_{31}(1)$ in the target year, is more accurate than was $\tilde{\mathbf{A}}(1)$ in (7.32).

It turns out, however, that assessment of the performance of modified RAS estimates depends very much on the measure used to measure the differences between matrices, specifically $\tilde{\mathbf{A}}(1) - \mathbf{A}(1)$ (no exogenous information) and $\tilde{\mathbf{A}}(1)_{ij} - \mathbf{A}(1)$ [substitution of the true $a_{ij}(1)$]. Table 7.4 illustrates this sensitivity for the numerical example begun

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Table 7.4 MAD and MAPE when One Coefficient is Known in Advance in an RAS Estimate

Element known in advance	MAD (×100)	MAPE
None	9.55	63.8
a_{11}	5.52	31.6
a_{12}	7.24	36.6
a_{13}	8.53	62.1
a_{21}	9.49	63.0
a_{22}	8.80	48.6
a_{23}	9.45	60.8
a_{31}	3.30	36.5
a_{32}	9.17	**69.4**
a_{33}	7.48	47.7

in section 7.4.2 and continued above. This table presents the MAD (multiplied by 100 for easier reading) and MAPE measures associated with each of the $\tilde{\mathbf{A}}(1)_{ij}$ matrices generated using prior information on a single $a_{ij}(1)$ cell in $\mathbf{A}(1)$. In this small example, there is improvement (over the no-prior-information case) as measured by MAD, but using the MAPE measure we find that correct prior information on a_{32} (in bold type) makes the overall estimate worse. (This sensitivity to alternative "metrics" for comparing closeness of matrices is discussed, with numerical examples, in de Mesnard and Miller, 2006.)

This result (worse results with better information) has been discussed before in the literature, although the importance of the measure of distance between matrices was not emphasized. In an early example that was frequently cited, Miernyk (1977) presented this counterintuitive result, using "mean percentage difference" as the measure of distance between the predicted and true target-year matrix. The idea was later taken up by Miller and Blair (1985) in the first edition of this text, where a further example appeared to illustrate the same point. In fact, both of these results have been shown to be flawed – there were errors with the RAS procedures (improper computer programs, stopping criteria that were too loose, etc.).[16] Nonetheless, later experiments with data sets that are much larger and more reflective of real-world applications have identified examples in which additional (correct) information generates poorer RAS estimates, under several fairly common distance measures. (Examples can be found in Szyrmer, 1989, and Lahr, 2001.) Nonetheless, the overwhelming majority of the evidence suggests the contrary. As a general rule, introduction of accurate exogenous information in RAS improves the resulting estimates. This is what hybrid models are designed to do.

[16] These are taken up in detail in de Mesnard and Miller (2006).

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7.4.7 *Hybrid Models: RAS with Additional Information*

In the decades since RAS was first proposed, there have been many applications at both national and regional levels. These have led to numerous variations, modifications, and extensions of the technique. An examination of the tables of contents or annual indexes of many journals in the field – especially *Economic Systems Research* and *Journal of Regional Science* – will reveal a large number of articles with "RAS," "partial-survey methods," "nonsurvey methods," "biproportional methods" or "hybrid models" in the title. Among the modifications are methods labeled "TRAS" (identified by its originators as a "three-stage RAS" or as a "two-stage RAS algorithm"; see Gilchrist and St. Louis, 1999, p. 186 and Gilchrist and St. Louis, 2004, p. 150, respectively), "GRAS" (for "generalized" RAS, for matrices that include negative numbers; see Junius and Oosterhaven, 2003) or "ERAS" (for "extended" RAS; see Israilevich, 1986).

Indeed, the preponderance of tables that are currently (beginning of the twenty-first century) being produced employ the "hybrid" notion of combining some kind of balancing of tables (usually using RAS or a variant) after "superior" information has been introduced, in the style of the example in sections 7.4.4 and 7.4.5. For example, the Bureau of Economic Analysis at the US Department of Commerce uses an adjusted RAS procedure to generate annual input–output tables for non-benchmark-table years in the USA.[17] In Europe, Eurostat is the agency that oversees collection and compilation of input–output data for the European Union member countries. Tables for non-benchmark years are produced using the Eurostat method, a modified and expanded RAS approach. (See Eurostat, 2002, esp. Chapter 14.)

A major trick in these kinds of applications is establishing which sectors (columns, rows or even individual cells) are most "important" to the economy, since these are the elements for which superior information would be preferred. In Chapter 12 we examine some of the approaches to identifying "important" sectors in an economy on the basis of their input–output data. As noted, this kind of exploration also identifies important (sets of) coefficients for which one would ideally like to have superior data to combine with RAS or some similar procedure for the remaining cells. There is an immense literature on this subject, and we will explore some of it in Chapter 12. Some of the approaches are essentially *mathematical* in nature – for example, those that are concerned with the influence of errors in one or more elements in a matrix on the resulting elements in the associated inverse matrix – and others are more *economic* in nature, in which attempts are made to identify important, or "key," sectors in an economy. In actuality this distinction tends to blur, since influential elements often belong to what turn out to be important sectors.

[17] This is described in Planting and Guo, 2004. The authors speak of "… [the] new automated updating and balancing method …" (p. 157).

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7.4.8 The Constrained Optimization Context

The notion of the "difference" between two matrices is a subtle one; there are many alternative measures. The RAS procedure can be shown to minimize

$$D[\mathbf{A}(0) : \tilde{\mathbf{A}}(1)] = \sum_i \sum_i \left\{ \tilde{a}_{ij}(1) \ln \left[\frac{\tilde{a}_{ij}(1)}{a_{ij}(0)} \right] \right\}$$

subject to the constraints on row and column sums given by $\mathbf{u}(1) = [(\tilde{\mathbf{A}})(1)\hat{\mathbf{x}}(1)]\mathbf{i}$ and $\mathbf{v}(1) = \mathbf{i}'[(\tilde{\mathbf{A}})(1)\hat{\mathbf{x}}(1)]$. (This is explored in Appendix 7.2 to this chapter.) The objective function, $D[\mathbf{A}(0) : \tilde{\mathbf{A}}(1)]$, has an interpretation as the "information" measure of distance between $\mathbf{A}(0)$ and $\tilde{\mathbf{A}}(1)$. In a sense, it generates the $\tilde{\mathbf{A}}(1)$ which, given $\mathbf{A}(0)$ and the information in $\mathbf{x}(1)$, $\mathbf{u}(1)$, and $\mathbf{v}(1)$, generates the least "surprise."

Many other potentially attractive measures have been proposed to represent the difference (or distance) between the estimated matrix and the base-year matrix. These become an objective function in an associated constrained optimization problem. The constraints continue to be the row and column margins, as in RAS. However, $\tilde{a}_{ij} \geq 0$ for all i and j must be added as an additional n^2 constraints because, unlike the RAS procedure, non-negativity of the solutions to these programming problems cannot be assured. Sometimes, also, bounds have been set on the sizes of relative change allowed for the elements. For example, $(0.5)a_{ij}(0) \leq \tilde{a}_{ij}(1) \leq (1.5)a_{ij}(0)$ would assure that each original coefficient did not increase or decrease by more than 50 percent.

Some of the objectives that have been proposed in this input–output updating context are:[18]

- Total absolute deviation: $\sum_i \sum_j \left| a_{ij}(0) - \tilde{a}_{ij}(1) \right|$. Divided by n^2, this is known as the mean absolute deviation (MAD). This and the following two objectives can be converted to a linear form, thus creating a linear program which is easily solved. (Jackson and Murray, 2004.)
- Weighted absolute deviation: $\sum_i \sum_j a_{ij}(0) \left| a_{ij}(0) - \tilde{a}_{ij}(1) \right|$. (Lahr, 2001.)
- Relative deviation: $\sum_i \sum_j \frac{|a_{ij}(0) - \tilde{a}_{ij}(1)|}{a_{ij}(0)}$. (Matuszewski, Pitts and Sawyer, 1964.) Multiplied by 100 and divided by n^2, this is known as the mean absolute percentage error (MAPE).
- Squared (or quadratic) deviation: $\sum_i \sum_j [a_{ij}(0) - \tilde{a}_{ij}(1)]^2$. (Almon, 1968.) This and the next two objectives require solution of a nonlinear program, which may be problematic.
- Weighted squared deviation: $\sum_i \sum_j a_{ij}(0)[a_{ij}(0) - \tilde{a}_{ij}(1)]^2$. [Canning and Wang, 2005, use a weighted quadratic penalty function in a program designed to estimate the z_{ij}^r and z_i^{rs} components of a multiregional input–output model (Chapter 3).]

[18] Constraints always include non-negativity of the $\tilde{a}_{ij}(1)$, along with the row and column margins, namely $\sum_j \tilde{a}_{ij}(1)x_j = u_i$ and $\sum_i \tilde{a}_{ij}(1)x_j = v_j$, respectively.

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- Relative squared deviation: $\sum_i \sum_j \frac{[a_{ij}(0)-\tilde{a}_{ij}(1)]^2}{a_{ij}(0)}$. (Friedlander, 1961.) This is Pearson's Chi-square measure, used early by Deming and Stephan (1940).
- Sign-preserving absolute differences: $\sum_i \sum_j \left|a_{ij}(0) - y_{ij}a_{ij}(0)\right|$, where $y_{ij}a_{ij}(0) = \tilde{a}_{ij}(1)$. (Junius and Oosterhaven, 2003.)[19]

The nonlinear alternatives require solution of possibly large and complex nonlinear programs, with their attendant difficulties, including computational issues (despite powerful computer programs and software), local rather than global optima, etc. Early overviews of some of these alternative minimization objectives can be found in Lecomber (Allen and Gossling, 1975, Ch. 1) and Hewings and Janson (1980, Appendix). Many recent proposals and extensive discussions are contained in Lahr and de Mesnard (2004), de Mesnard (2004) and Jackson and Murray (2004). In particular, Jackson and Murray present extensive results for applications of a total of 10 model formulations (including those listed above) to the problem of estimating the 1972 23-sector US industry-by-industry data from a 1967 matrix and 1972 margins. They found that, generally, RAS produced the best results. Canning and Wang (2005) contains a discussion of the advantages of a mathematical programming approach to constrained matrix-balancing problems and reviews some of the important contributions in the literature.

7.4.9 *Infeasible Problems*

In general, the RAS procedure converges to within acceptable tolerance in a reasonable number of iterations – often less than 50. However, examples of nonconvergence have appeared in the literature. The usual explanation is that the matrix being adjusted is too sparse – contains too many zeros. A very disaggregated transactions matrix (hundreds of sectors) or interregional trade-flow matrices would have more zeros than, say, a highly aggregated national table.[20] Intuitively, the problem with zeros is that the entire burden of change is forced onto the remaining, nonzero elements, and they may be inadequate to the task (depending in large part on the locations of the zeros relative to the nonzeros).

Here is a very simple illustration of the issue:[21] Let

$$\mathbf{Z}(0) = \begin{bmatrix} 5 & 0 \\ 4 & 3 \end{bmatrix}, \ \mathbf{u}(1) = \begin{bmatrix} 10 \\ 2 \end{bmatrix}, \text{ and } \mathbf{v}(1) = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

[19] In this case, the constraints are non-negativity of the y_{ij} and margin constraints of $\sum_j y_{ij}a_{ij}(0)x_j = u_i$ and $\sum_i y_{ij}a_{ij}(0)x_j = v_j$. Linearization is possible, as in the first three cases.

[20] For example, nonconvergence occurred while working with inter-state trade tables in developing the US 1967 multiregional model (see Möhr, Crown and Polenske, 1987).

[21] From de Mesnard (2003).

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The difficulty with the problem is clear when we look at the required new margins in relation to the structure that $\tilde{\mathbf{Z}}(1)$ must have, namely

$$\begin{array}{cc} \begin{bmatrix} \tilde{z}_{11}(1) & 0 \\ \tilde{z}_{21}(1) & \tilde{z}_{22}(1) \end{bmatrix} & \begin{bmatrix} 10 \\ 2 \end{bmatrix} \\ [\ 7 \qquad 5\ ] & \end{array}$$

To satisfy $u_1(1) = 10$, it is clear that $\tilde{z}_{11}(1) = 10$, since $\tilde{z}_{12}(1) = 0$ because zeros are perpetuated in RAS. Clearly, if $u_1(1) = 10$ then $\tilde{z}_{21}(1)$ would need to be -3 in order to satisfy $v_1(1) = 7$, but this is impossible since RAS does not generate negative elements from those that are positive. One straightforward way out of the problem is to assign a small positive number to zero-valued cells in the base matrix.[22] In this small illustration, changing $z_{12}(0)$ from zero to, say, 0.5, introduces exactly the flexibility that is needed, and as a consequence RAS will produce (rounded)[23]

$$\tilde{\mathbf{Z}}(1) = \begin{bmatrix} 6.5911 & 3.4089 \\ 0.4099 & 1.5901 \end{bmatrix}$$

An argument made in defense of this approach is that the original zero-valued elements could be the result of rounding; that is, these elements were actually very small flows that fell below the "reduce to zero" threshold in recording the data. On the other hand, some zeros represent true technological facts – as above in the illustration of a zero flow from potatoes to automobiles, which should be maintained in the target matrix. Moreover, in a large problem it may not be necessary to change all zeros into small positives, and then the issue is to decide which zeros should be altered. One approach uses a linear programming problem to select subsets of elements for augmentation (from zero to positive numbers); see Möhr, Crown and Polenske, 1987 for a discussion and illustration of this approach.

7.5 Summary

In this chapter we have examined approaches to estimating tables of input–output coefficients when a full matrix of interindustry transactions is not available. No nonsurvey or partial-survey technique can be expected to generate a table that is a perfect copy of what could be obtained if a complete survey were undertaken. On the other hand, errors and compromises of many sorts enter into the production of even the best survey-based table, so it can be argued that even a survey-based table is not a completely accurate snapshot of an economy. The updating problem has given rise to a number of approaches, usually including an RAS adjustment at some point, often combined with either survey data or expert opinion on certain key elements – sometimes individual coefficients, sometimes entire rows or columns. This hybrid strategy is an attempt to capture the best of several approaches – selective survey information, expert opinion, and the attractive mathematical features of the RAS technique.

[22] Apparently this was first done by Hewings, 1969, in his dissertation. (Cited in de Mesnard, 2003.)
[23] After nine iterations, using $|u_i(1) - u_i^k| \leq 0.001$ and $|v_i(1) - v_i^k| \leq 0.001$ for all i as the stopping criterion.

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Appendix 7.1 RAS as a Solution to the Constrained Minimum Information Distance Problem

The problem is to choose the elements of $\tilde{\mathbf{A}}(1)$ so as to minimize the information measure of distance between $\mathbf{A}(0)$ and $\tilde{\mathbf{A}}(1)$, namely

$$D[\mathbf{A}(0) : \tilde{\mathbf{A}}(1)] = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{a}_{ij}(1) \ln\left[\frac{\tilde{a}_{ij}(1)}{a_{ij}(0)}\right] \tag{A7.1.1}$$

subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}(1)x_j(1) = u_i(1) \ (i = 1, \ldots, n) \tag{A7.1.2}$$

$$\sum_{i=1}^{n} \tilde{a}_{ij}(1)x_j(1) = v_j(1) \ (j = 1, \ldots, n) \tag{A7.1.3}$$

Notice that the expression in (A7.1.1) is only defined for $a_{ij}(0) \neq 0$. The associated Lagrangian expression is

$$\begin{aligned} L = &\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{a}_{ij}(1) \ln\left[\frac{\tilde{a}_{ij}(1)}{a_{ij}(0)}\right] \\ &- \sum_{i=1}^{n} \lambda_i \left[\sum_{j=1}^{n} \tilde{a}_{ij}(1)x_j(1) - u_i(1)\right] - \sum_{j=1}^{n} \mu_j \left[\sum_{i=1}^{n} \tilde{a}_{ij}(1)x_j(1) - v_j(1)\right] \end{aligned} \tag{A7.1.4}$$

and the appropriate first-partial derivatives are

$$\partial L/\partial \tilde{a}_{ij}(1) = 1 + \ln \tilde{a}_{ij}(1) - \ln a_{ij}(0) - \lambda_i x_j(1) - \mu_j x_j(1) \tag{A7.1.5}$$

Setting $\partial L/\partial \tilde{a}_{ij}(1) = 0$ yields

$$\ln \tilde{a}_{ij}(1) = \ln a_{ij}(0) - 1 + \lambda_i x_j(1) + \mu_j x_j(1)$$

and, taking antilogarithms,

$$\tilde{a}_{ij}(1) = a_{ij}(0)\mathrm{e}^{[-1+\lambda_i x_j(1)+\mu_j x_j(1)]}$$

or, rearranging,

$$\tilde{a}_{ij}(1) = \mathrm{e}^{[\lambda_i x_j(1)-1/2]} a_{ij}(0) \mathrm{e}^{[\mu_j x_j(1)-1/2]} \tag{A7.1.6}$$

Let

$$r_i = \mathrm{e}^{[\lambda_i x_j(1)-1/2]} \tag{A7.1.7}$$

which is a function of λ_i only (that is, a row constraint), and let

$$s_j = \mathrm{e}^{[\mu_j x_j(1)-1/2]} \tag{A7.1.8}$$

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which is a function of μ_j only (that is, a column constraint). Then the right-hand side of (A7.1.6) can be shown as

$$\tilde{a}_{ij}(1) = r_i a_{ij}(0) s_j \tag{A7.1.9}$$

The new coefficient, $\tilde{a}_{ij}(1)$, is derived as the old coefficient, $a_{ij}(0)$, modified by a row-constraint term, r_i, and a column-constraint term, s_j.

The constraints of the problem, (A7.1.2) and (A7.1.3), are reproduced in the remaining first-order conditions, as usual, when we set $\partial L/\partial \lambda_i = 0$ $(i = 1, \ldots, n)$ and $\partial L/\partial \mu_j = 0$ $(j = 1, \ldots, n)$. Inserting (A7.1.9) into these two constraints gives

$$r_i = u_i(1) / \sum_{j=1}^{n} a_{ij}(0) s_j x_j(1)$$

and

$$s_j = v_j(1) / \sum_{j=1}^{n} r_i a_{ij}(0) x_j(1)$$

The values of r_i and s_j are found through iterative solution of these two equations. This is what the RAS procedure accomplishes. (See Macgill, 1977 or Bacharach, 1970 for details.)

The matrix equivalent of (A7.1.9) is

$$\tilde{\mathbf{A}}(1) = \hat{\mathbf{r}}\mathbf{A}(0)\hat{\mathbf{s}} \tag{A7.1.10}$$

as in (7.35) in the text, where

$$\hat{\mathbf{r}} = \begin{bmatrix} r_1 & 0 & \cdots & 0 \\ 0 & r_2 & & 0 \\ \vdots & & & \vdots \\ 0 & & & r_n \end{bmatrix} \text{ and } \hat{\mathbf{s}} = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ 0 & s_2 & & 0 \\ \vdots & & & \vdots \\ 0 & & & s_n \end{bmatrix}$$

Examining second-partial derivatives, we find

$$\partial^2 L / \partial \tilde{a}_{ij}(1)^2 = 1/\tilde{a}_{ij}(1) \tag{A7.1.11}$$

This is strictly positive for all $\tilde{a}_{ij}(1) > 0$. From (A7.1.9), this means for all $a_{ij}(0) > 0$, since $r_i > 0$ and $s_j > 0$ [(A7.1.7) and (A7.1.8)]. Thus the RAS solution minimizes $D[\mathbf{A}(0) : \tilde{\mathbf{A}}(1)]$ in (A7.1.1).

Problems

7.1 Consider the following US input–output tables for 1997[24], 2003, and 2005 (in \$ millons).

Produce industry-by-industry transactions tables using the assumption of industry-based technology for these three years. Suppose historical price indices for these tables

[24] The tables for 1997 differ from those provided in Appendix B in that they reflect data assembled "before redefinitions," as discussed in Chapter 4.

US Use 1997	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Imports*
1 Agriculture	74,938	15	1,121	150,341	2,752	13,400	11	(23,123)
2 Mining	370	19,461	4,281	112,513	53,778	5,189	30	(64,216)
3 Construction	1,122	29	832	7,499	11,758	50,631	27	–
4 Manufacturing	49,806	19,275	178,903	1,362,660	169,915	418,412	1,914	(765,454)
5 Trade, Transport & Utilities	21,650	11,125	76,056	380,272	199,004	224,271	612	6,337
6 Services	32,941	45,234	107,723	483,686	545,779	1,592,426	3,801	(16,942)
7 Other	63	781	422	33,905	19,771	26,730	–	(126,350)
US Make 1997	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Industry Output*
1 Agriculture	284,511	–	65	356	455	1,152	–	286,539
2 Mining	–	158,239	109	9,752	295	258	–	168,653
3 Construction	–	–	670,210	–	–	–	–	670,210
4 Manufacturing	–	727	1,258	3,703,275	39,720	36,034	3,669	3,784,683
5 Trade, Transport & Utilities	556	381	21,393	15,239	2,201,532	141,674	–	2,380,776
6 Services	–	410	54,850	1,306	109,292	6,444,098	1,821	6,611,778
7 Other	–	–	6,206	–	–	7,010	947,023	960,238
Commodity Output	285,067	159,757	754,091	3,729,928	2,351,295	6,630,226	952,513	14,862,876
US Use 2003	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Imports*
1 Agriculture	61,946	1	1,270	147,559	231	18,453	2,093	(26,769)
2 Mining	441	33,299	6,927	174,235	89,246	1,058	11,507	(125,508)
3 Construction	942	47	1,278	8,128	10,047	65,053	48,460	–
4 Manufacturing	47,511	22,931	265,115	1,249,629	132,673	516,730	226,689	(1,075,128)
5 Trade, Transport & Utilities	24,325	13,211	100,510	382,630	190,185	297,537	123,523	8,065
6 Services	25,765	42,276	147,876	509,084	490,982	2,587,543	442,674	(44,060)
7 Other	239	1,349	2,039	48,835	35,110	83,322	36,277	(177,578)

Table (*cont.*)

US Make 2003		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Output*
1	Agriculture	273,244	–	–	67	–	1,748	–	275,058
2	Mining	–	232,387	–	10,843	–	–	–	243,231
3	Construction	–	–	1,063,285	–	–	–	–	1,063,285
4	Manufacturing	–	–	–	3,856,583	–	30,555	3,278	3,890,416
5	Trade, Transport & Utilities	–	570	–	–	2,855,126	41	957	2,856,693
6	Services	–	475	–	–	133	9,136,001	3,278	9,139,886
7	Other	3,359	896	–	3,936	104,957	323,996	1,827,119	2,264,263
Commodity Output		276,602	234,328	1,063,285	3,871,429	2,960,216	9,492,341	1,834,631	19,732,832
US Use 2005		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Imports*
1	Agriculture	71,682	1	1,969	174,897	335	18,047	1,671	(31,248)
2	Mining	524	57,042	8,045	297,601	123,095	1,290	16,570	(226,059)
3	Construction	1,597	74	1,329	7,886	12,449	74,678	54,282	–
4	Manufacturing	61,461	34,860	339,047	1,452,738	183,135	589,452	255,456	(1,372,424)
5	Trade, Transport & Utilities	26,501	17,197	136,193	460,348	244,153	362,324	127,266	6,790
6	Services	27,274	52,297	165,179	543,690	610,978	3,017,728	529,779	(50,588)
7	Other	240	1,323	2,021	61,316	44,561	90,071	39,656	(208,971)
US Make 2005		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Output*
1	Agriculture	310,868	–	–	65	–	1,821	–	312,754
2	Mining	–	373,811	–	22,752	–	–	–	396,563
3	Construction	–	–	1,302,388	–	–	–	–	1,302,388
4	Manufacturing	–	–	–	4,454,957	–	26,106	4,467	4,485,529
5	Trade, Transport & Utilities	–	808	–	–	3,354,043	47	1,046	3,355,944
6	Services	–	556	–	–	152	10,473,161	3,771	10,477,640
7	Other	4,657	1,410	–	4,111	115,428	339,582	2,061,136	2,526,325
Commodity Output		315,525	376,586	1,302,388	4,481,885	3,469,622	10,840,717	2,070,419	22,857,143

are given in the following table (price indices in percent relative to some arbitrary earlier year):

	1997	2003	2005
Agriculture	100	113.5	122.7
Mining	96.6	131.3	201
Construction	181.6	188.9	209.9
Manufactuirng	133.7	150.8	156.9
Trade, Transport & Utilities	200.4	205.7	217.1
Services	129.3	151.6	219.8
Other	140	144.7	161.4

Produce a set of constant price input–output tables for the same years using 2005 as the base year for prices.

7.2 For the constant price tables constructed in problem 7.1, suppose we measure year-to-year change as the average of the absolute value of differences between the column sums of A for the same industry sectors in two different years. Which three sectors exhibited the most change from 1997 to 2005? How does that compare with the three most changed sectors measured in nominal dollars rather than constant dollars? Why are they different?

7.3 Using the current price tables constructed in problem 7.1, compute the marginal input coefficients between the years 1997 and 2005.

7.4 Consider the following interindustry transactions and total outputs two-sector input–output economy for the year 2000:

2000	*A*	*B*	Total Output
A	1	2	10
B	3	4	10
VA	6	4	

Suppose estimates are generated for the year 2010 for the vectors of total final demand, total value-added, and total output in the following table.

2010	Final Demand	Value Added	Total Output
A	12	10	25
B	6	8	20

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Using the 2000 table as a base and using the 2010 projections for final demand, value-added and total output, compute an estimate of the 2010 technical coefficients table using the RAS technique.

7.5 Using the 1997 input–output table expressed in 1997 dollars constructed in problem 7.1 and the vectors of intermediate inputs, intermediate outputs, and total outputs from the corresponding input–output table for 2005, compute an RAS estimate of the 2005 table using the 1997 table as a base. Compute the mean absolute percentage error (MAPE) of the RAS-estimated table for 2005 compared with the "real" 2005 table.

7.6 Suppose we have a baseline transactions matrix defined as $\mathbf{Z}(0) = \begin{bmatrix} 100 & 55 & 25 \\ 50 & 75 & 45 \\ 25 & 10 & 110 \end{bmatrix}$.

We are provided with estimates of intermediate inputs and outputs, $\mathbf{v}(1) = \begin{bmatrix} 265 \\ 225 \\ 325 \end{bmatrix}$ and $\mathbf{u}(1) = \begin{bmatrix} 325 \\ 235 \\ 255 \end{bmatrix}$, respectively.

a. Compute an estimate of the transactions table for the next year, $\tilde{\mathbf{Z}}^z(1)$ using $\mathbf{Z}(0)$, $\mathbf{v}(1)$ and $\mathbf{u}(1)$, using the RAS technique.

b. Suppose we know the vector of total outputs, $\mathbf{x}(1) = \begin{bmatrix} 750 \\ 500 \\ 1000 \end{bmatrix}$, corresponding to $\mathbf{Z}(0)$, and we also have an estimate of total outputs for next year, $\mathbf{x}(1) = \begin{bmatrix} 1000 \\ 750 \\ 1500 \end{bmatrix}$. Compute $\mathbf{A}(0)$ and use it along with $\mathbf{v}(0)$ and $\mathbf{u}(0)$ to generate an estimate of the technical coefficients matrix for next year $\tilde{\mathbf{A}}^A(1)$. Finally, compute $\tilde{\mathbf{A}}^z(1) = \tilde{\mathbf{Z}}^z(1)\hat{\mathbf{x}}(1)^{-1}$. Is $\tilde{\mathbf{A}}^A(1) = \tilde{\mathbf{A}}^z(1)$? Why or why not?

7.7 For the economy in problem 7.6, suppose we acquire a survey-based table of technical coefficients next year of $\mathbf{A}(1) = \begin{bmatrix} .2 & .1 & .033 \\ .035 & .167 & .05 \\ .03 & .033 & .133 \end{bmatrix}$. At the beginning of the survey we know only $a(1)_{32} = .033$ and we use that along with $\mathbf{A}(0)$, $\mathbf{v}(0)$, and $\mathbf{u}(0)$ to generate an intermediate estimate of the entire matrix of coefficients, $\tilde{\mathbf{A}}^*(1)$. If we measure difference between two matrices as MAPE, which estimate of $\mathbf{A}(1)$ is better – $\tilde{\mathbf{A}}(1)$ or $\tilde{\mathbf{A}}^*(1)$? Suppose early in the survey period we determine $a(1)_{11} = .2$ instead of knowing $a(1)_{32}$. Which estimate of $\mathbf{A}(1)$ is better – $\tilde{\mathbf{A}}(1)$ or $\tilde{\mathbf{A}}^*(1)$? How does this case differ from the case where $a(1)_{32}$ is known?

7.8 Consider the transactions matrix $\mathbf{Z}(0) = \begin{bmatrix} 100 & 55 & 25 \\ 0 & 75 & 25 \\ 25 & 10 & 110 \end{bmatrix}$ and projected vectors of

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intermediate inputs and outputs, $\mathbf{v}(1) = \begin{bmatrix} 125 \\ 140 \\ 160 \end{bmatrix}$ and $\mathbf{u}(1) = \begin{bmatrix} 180 \\ 100 \\ 145 \end{bmatrix}$, respectively. Compute the RAS estimate, $\tilde{\mathbf{Z}}(1)$. Suppose we learn that $v_1(0) = 100$ instead of 125. Is it possible to compute $\tilde{Z}(1)$ via the RAS technique? Why or why not?

7.9 For the US input–output tables for 1997 and 2005 (from problem 7.1, expressed in current year dollars rather than constant year dollars), compute the RAS estimate $\tilde{\mathbf{A}}(2005)$ using $\mathbf{A}(1997)$, $\mathbf{v}(2005)$, and $\mathbf{u}(2005)$. Compute the MAPE for $\tilde{\mathbf{A}}(2005)$ compared with $\mathbf{A}(2005)$. How does that error compare with the MAPE for $\tilde{\mathbf{L}}(2005) = [\mathbf{I} - \tilde{\mathbf{A}}(2005)]^{-1}$ when compared with $\mathbf{L}(2005)$?

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