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## ACCOUNTING AND FIXED PRICE MULTIPLIERS IN A SOCIAL ACCOUNTING MATRIX FRAMEWORK\*

This paper is concerned with the relationships between output, factor demands and income, and the decomposition of these relationships into separate effects as suggested by the structure of a social accounting matrix representation of the flows between them. Since output, factor and the non-government institutions sector (households and companies) are all disaggregated in the system to be examined, it follows that the analysis is concerned not only with output levels and the level of factor and household incomes, but also with the structure of production, the distribution of factor incomes, and the distribution of disposable income both among households and between them and the corporate sector. This is the first sense in which this paper is concerned with decomposition, and it makes the point that the distribution of income and the structure of production are inextricably interwoven.

The closed-loop character of the present formulation implies that the incomes of production activities, factors and institutions are all derived from injections into the economy via a multiplier process. This multiplier is a matrix  $\mathbf{M}$  which can be expressed as the product of three multiplier matrices  $\mathbf{M}_1$ ,  $\mathbf{M}_2$  and  $\mathbf{M}_3$ . The first of these captures the effects of transfers within the economy, for example, the distribution of profits from companies to households, and the transfers of goods between activities, which are the essence of input–output. The other matrices  $\mathbf{M}_2$  and  $\mathbf{M}_3$  capture the consequences of the circular flow of income within the economy. Matrix  $\mathbf{M}_3$  shows the full circular effects of an income injection going round the system and back to its point of origin in a series of repeated (and dampening) cycles. In contrast,  $\mathbf{M}_2$  captures the cross-effects of the multiplier process whereby an injection into one part of the system has repercussions on other parts. These cross-effects correspond to open-loop effects and hence to the recent class of models, such as that of Maton, Paukert and Skolka (1978), which trace the effects of some exogenous changes in income distribution on output and employment, with no allowance for the effects in the reverse direction of changes in output and employment on the distribution of income. The decomposition of  $\mathbf{M}$  into component parts is the second sense in which the paper is concerned with decomposition.

The first perspective on decomposition is illustrated in Section I by means of a simplified social accounting matrix (SAM) for Sri Lanka in 1970. This shows balanced accounts for factors, production activities, households and com-

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panies set in the broader framework of a full national accounting system. Section II then explores the structure of these accounting balances in terms of a multiplier matrix and its decomposition into transfer, open-loop and closed-loop effects.<sup>1</sup> An additive version of the decomposition due to Stone (1978*a*) is also presented.

The multipliers discussed in Section II of the paper are referred to as accounting multipliers. Their data base is the SAM observed for 1970 and their role is simply to represent the accounting balances of the SAM in a novel way which gives some insights into economic structure. With the accounting multipliers as a starting point, it is then possible to move on, in Section III of the paper, to consider the potentially more interesting case of multipliers due to income effects in a fixed price model. The argument shows that these fixed price multipliers are strictly analogous to the accounting multipliers. The only difference arises from extensive use of marginal expenditure propensities in the fixed price case, while the accounting multipliers are built up from the average expenditure propensities which can be calculated directly from the SAM. Thus the fixed price multipliers can be interpreted as having a data base which is the initial SAM now complemented by estimates of income elasticities when the latter differ from unity. Our pedagogic procedure of presenting accounting multipliers first, and then the fixed price multipliers, makes it possible to bring out the implications of income elasticity effects, such as Engel's law, within a fixed price system. Indeed, following the decomposition of the fixed price multiplier matrix in Section IV, we are able to show that the differences between this matrix and the accounting multiplier matrix can themselves be represented as a multiplicative matrix effect which is dependent on income elasticities which differ from unity.

The empirical results presented in Section V illustrate the various components of fixed price multipliers and alternative ways of deriving them. The results show how the estates sector in Sri Lanka is relatively isolated within the economy because its linkages with other sectors are slight. The results also show the extent to which input-output calculations underestimate the linkages between producing sectors in comparison with the case where the full circular flow of incomes is taken into account. More generally, the anatomy of the economy in terms of income and production structures, and their interdependence, is captured by the various multiplier matrices discussed in this paper.

The inclusion of different types of households in the present formulation distinguishes the approach from standard closed-loop Leontief systems and allows the distribution of income to be brought into the picture. The inclusion of factors as well as households implies that the present formulation extends the structure of accounting balances as set out by Quesnay (1758) and the previous closed-loop multiplier formulations which have been developed within

<sup>1</sup> This aspect has been treated previously by us in Pyatt, Roe *et al.* (1977), Chapter 4. However, there is an error in the exposition with respect to the treatment of indirect taxes, which is removed in the present paper. The multiplier decomposition has also been applied in Bell, Devarajan, Hazell and Slade (1976) and Stone (1978*a*, *b*).





his accounting framework.<sup>1</sup> While the data source for the present analysis is perhaps the first to give detailed accounts on a disaggregated basis for factors, households and activities,<sup>2</sup> subsequent studies have begun or been completed which achieve comparable tabulations.<sup>3</sup> In parallel with this effort, a great many studies are forthcoming which model simultaneously the behaviour of factor markets, production structure, and income distribution.<sup>4</sup> However, the published models do not have an explicit accounting structure and are heavily concerned with mechanisms for determining prices endogenously. The fixed price multipliers discussed here take prices as exogenous, while our accounting multipliers are clearly implicit in all closed models. By making them explicit, and then moving on to a simple fixed price model, we hope to provide some of the missing links between the sophisticated experimental models now being developed and the simple input-output, semi-input-output and macro-models which remain the basis of actual planning methods.

# I. THE SOCIAL ACCOUNTING MATRIX

Table 1 sets out a social accounting matrix (SAM) which provides the numerical base for subsequent empirical illustrations. In reading this table it is important to keep in mind the convention that entries are to be read as receipts for the row account in which they are located, and expenditures or outlays for their column account. The SAM is square because each account has both receipts and expenditures; and the row and column sums for a given account must be equal because all income must be accounted for by an outlay of one type or another. Eight groups of accounts are shown, some of which are further disaggregated. The partitioning of the eight groups into endogenous and exogenous accounts is discussed following an explanation of the flows depicted in Table 1.<sup>5</sup>

Factors of production receive income from domestic production (shown as the intersection of accounts in row block 1 with column block 4) which in turn is distributed to households and companies (rows 2, 3 intersecting with column 1), and as net factor income payments abroad (row 8 intersecting with column 1).

<sup>1</sup> The distinguishing feature of Quesnay's *Tableau Economique* from the present perspective is that value added in different production activities is paid directly to households of various types as opposed to being routed to them via a set of factor accounts. This simplified approach is also adopted in Desai (1961) and in a model of Iran (Pyatt *et al.* (1972)) which is of the fixed-price multiplier genre. In a recent note, Stone (1978*b*) has applied the analysis of accounting multipliers in this paper to Quesnay's *Tableau*.

<sup>2</sup> See Pyatt, Roe *et al.* (1977). This study distinguishes 18 types of labour and 3 of capital; 21 household groups; two types of companies and 48 different production activities.

<sup>3</sup> Pyatt and Round (1977) compare the Sri Lanka study with the social accounting basis of the Iran study referred to in a previous footnote and a subsequent investigation in Swaziland (Pyatt, Round *et al.* 1974). A recent conference reported on continuing work in the Philippines and Saudi Arabia, while results for Botswana (Greenfield, 1978), Malaysia (Pyatt and Round, 1978), and the United Kingdom (Stone, 1978*a*) were also presented.

<sup>4</sup> Recent publications include Adelman and Robinson (1978), Dervis and Robinson (1978), and Gupta (1977). Unpublished work sponsored by the World Bank's Research Program, includes that of Taylor, Bacha, Cardoso and Lysy on Brazil (forthcoming), and Ahluwalia and Lysy on Malaysia.

<sup>5</sup> See Pyatt, Roe *et al.* (1977) for a more detailed description of the accounting structure.

Factor incomes received by households include wages, unincorporated business profits, and rent on dwellings (row 2, column 1); but households also receive distributed profits from the corporate sector (2, 3), and transfers from government (2, 8), before arriving at total household income. Similarly, corporate enterprises receive factor incomes in the form of gross profits (3, 1), as well as current transfers from government (3, 5). Government income is derived from direct tax payments and other transfers by households (5, 2), corporate enterprises (5, 3) and from the rest of the world (5, 8), as well as intra-government transfers (5, 5), together with net indirect tax payments (5, 7) shown as a receipt from a special indirect tax account. The expenditures on domestically produced commodities are shown in the row of account 4. They include outlays by households (4, 2), government (4, 5), investment (4, 6), and the rest of the world (4, 8), as well as intermediate transactions between production activities (4, 4). Indirect taxes on all of these expenditures, and purchases of imported goods, are shown as separate outlays by the various spending units. They are received in row 7 by the account for (net) indirect taxes, and in row 8 by the rest of the world revenue account. Finally, outlays on domestic investment (column 6) are matched by domestic and foreign savings (row 6) where the latter (5, 8) is the final balancing item in the rest of the world accounts.

An important feature to note is that factors, institutions and activities are all disaggregated in Table 1, so that the SAM captures the distribution of factor incomes as well as their level. It also shows the distribution of income among household types.

To move from a SAM to a model structure requires that each account should be designated as endogenous or exogenous. The accounts in Table 1 have been ordered so that the endogenous accounts occupy the leading rows and columns of the SAM. This is shown schematically in Table 2. The notation to be used in subsequent discussion is given with this table, as are a number of accounting relationships, equations (1) to (11), which follow directly from the SAM structure. Equation (1) states that transactions between endogenous accounts, denoted by matrix  $\mathbf{N}$ , can be expressed as the product of a square matrix,  $\mathbf{A}_n$ , of average propensities to consume and a vector of endogenous incomes,  $\mathbf{y}_n$ . Similarly (2) equates leakages,  $\mathbf{L}$ , with the product of a non-square matrix,  $\mathbf{A}_l$ , of average propensities to leak and the endogenous incomes,  $\mathbf{y}_n$ . It is important to note that since  $\mathbf{N}$ ,  $\mathbf{L}$  and  $\mathbf{y}_n$  are observed in a SAM such as Table 1, the matrices  $\mathbf{A}_n$  and  $\mathbf{A}_l$  can be obtained directly. Equations (3) and (4) express the accounting relationship by which endogenous incomes are determined. Equations (5) and (6) have the same role with respect to incomes of the exogenous accounts,  $\mathbf{y}_x$ . Equation (7) sums expenditures (columns) of the endogenous accounts. It implies that, for these accounts, row and column sums will be equal provided equation (8) holds, i.e. provided column sums of  $\mathbf{A}_n$ , plus those of  $\mathbf{A}_l$ , add to unity in all cases. Equation (9) expresses column sums for exogenous accounts. The requirement that these be equal to row sums (equation (6)) yields equation (10). Finally, an implication of (10) is obtained in (11), which states that, in aggregate, injections into the system must equal leakages.

Table 2  
*Notation and Accounting Balances: Equations (1) to (11)*

Receipts	Expenditures		Totals
	Endogenous accounts	Exogenous accounts	
Endogenous accounts	$\mathbf{N} = \mathbf{A}_n \hat{\mathbf{y}}_n$ (1)	$\mathbf{X}$	$\mathbf{y}_n = \mathbf{n} + \mathbf{x}$ (3)
Exogenous accounts	$\mathbf{L} = \mathbf{A}_l \hat{\mathbf{y}}_n$ (2)	$\mathbf{R}$	$\mathbf{y}_x = \mathbf{l} + \mathbf{Ri}$ (5)
Totals	$\mathbf{y}'_n = (\mathbf{i}'\mathbf{A}_n + \mathbf{i}'\mathbf{A}_l)\hat{\mathbf{y}}_n$ (7)	$\mathbf{y}'_x = \mathbf{i}'\mathbf{X} + \mathbf{i}'\mathbf{R}$ (9)	$\lambda'_a \mathbf{y}_n = \mathbf{x}'\mathbf{i}$ (11)
	$\therefore \mathbf{i}' = \mathbf{i}'\mathbf{A}_n + \mathbf{i}'\mathbf{A}_l$ (8)	$\therefore \mathbf{A}_l \mathbf{y}_n - \mathbf{X}'\mathbf{i} = (\mathbf{R} - \mathbf{R}')\mathbf{i}$ (10)	

$\mathbf{A}_n = \mathbf{N}\hat{\mathbf{y}}_n^{-1}$  = matrix of average endogenous expenditure propensities.  
 $\mathbf{A}_l = \mathbf{L}\hat{\mathbf{y}}_n^{-1}$  = matrix of average propensities to leak.  
 $\mathbf{Ni} = \mathbf{n}$  = vector of row sums of  $\mathbf{N} = \mathbf{A}_n \mathbf{y}_n$ .  
 $\mathbf{Xi} = \mathbf{x}$  = vector of row sums of  $\mathbf{X}$ .  
 $\mathbf{Li} = \mathbf{l}$  = vector of row sums of  $\mathbf{L} = \mathbf{A}_l \mathbf{y}_n$ .  
 $\lambda'_a = \mathbf{i}'\mathbf{A}_l$  = vector of column sums of  $\mathbf{A}_l$ , i.e. the vector of aggregate average propensities to leak.  
 $\mathbf{N}$  = matrix of SAM transactions between endogenous accounts.  
 $\mathbf{X}$  = matrix of injections from exogenous into endogenous accounts.  
 $\mathbf{L}$  = matrix of leakages from endogenous into exogenous accounts.  
 $\mathbf{R}$  = matrix of SAM transactions between exogenous accounts.

From equation (4) and the definition of  $\mathbf{l}$  it follows that

$$\mathbf{y}_n = (\mathbf{I} - \mathbf{A}_n)^{-1} \mathbf{x} = \mathbf{M}_a \mathbf{x} \quad (12)$$

and

$$\mathbf{l} = \mathbf{A}_l (\mathbf{I} - \mathbf{A}_n)^{-1} \mathbf{x} = \mathbf{A}_l \mathbf{M}_a \mathbf{x} \quad (13)$$

provided that  $(\mathbf{I} - \mathbf{A}_n)^{-1}$  exists. This inverse is the accounting multiplier matrix  $\mathbf{M}_a$  which relates endogenous incomes  $\mathbf{y}_n$  to injections,  $\mathbf{x}$ . The existence and decomposition of  $\mathbf{M}_a$  are discussed in the next section.<sup>1</sup> Meanwhile it can be noted that the linkage between injections and leakages as given by equation (13) satisfies the requirement<sup>2</sup>

$$\mathbf{i}'\mathbf{A}_l \mathbf{M}_a = \lambda'_a \mathbf{M}_a = \mathbf{i}' \quad (14)$$

or, in words, that each injection is ultimately accounted for by one or more leakages.

In deriving the matrix  $\mathbf{M}_a$  it has been assumed that the accounts for factors, households, companies and production activities are endogenous. The corresponding exogenous accounts are therefore those for government current expenditure, investment,<sup>3</sup> indirect taxes and international transactions. Injections,  $\mathbf{X}$ , therefore include current transfers to households and companies both from government and the rest of the world, plus the demands placed on production activities through government consumption, investment and exports. Direct and indirect taxes, savings, imports and income transfers abroad constitute the leakages.<sup>4</sup>

<sup>1</sup> Tabular results of  $\mathbf{M}_a$  and  $\mathbf{A}_l \mathbf{M}_a$  are available on request from the authors.

<sup>2</sup> This follows from  $\mathbf{i}' = \mathbf{i}'(\mathbf{A}_n + \mathbf{A}_l) = \mathbf{i}'\mathbf{A}_n + \lambda'_a$   
 $= \lambda'_a(\mathbf{I} - \mathbf{A}_n)^{-1} = \lambda'_a \mathbf{M}_a$ .

<sup>3</sup> To obtain Tinbergen's semi-input-output model as a special case of our analysis, it would be necessary to endogenise investment in the non-traded goods sector(s).

<sup>4</sup> It can be noted that if model formulation was to specify the import and indirect tax content of government expenditure, investment and exports, then this would imply that some elements of  $\mathbf{R}$  were determined as a function of  $\mathbf{X}$ . The equation (11) would now be sufficient to determine the balance of trade, government savings, and the current account deficit on the balance of payments.



## II. DECOMPOSITION OF ACCOUNTING MULTIPLIERS

From equation (4) it follows that for any matrix  $\tilde{\mathbf{A}}_n$  of the same size as  $\mathbf{A}_n$  and such that  $(\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1}$  exists, we can write

$$\begin{aligned}\mathbf{y}_n &= \mathbf{A}_n \mathbf{y}_n + \mathbf{x} = (\mathbf{A}_n - \tilde{\mathbf{A}}_n) \mathbf{y}_n + \tilde{\mathbf{A}}_n \mathbf{y}_n + \mathbf{x} \\ &= (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1} (\mathbf{A}_n - \tilde{\mathbf{A}}_n) \mathbf{y}_n + (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1} \mathbf{x} \\ &= \mathbf{A}^* \mathbf{y}_n + (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1} \mathbf{x}.\end{aligned}\quad (15)$$

Multiplying throughout by  $\mathbf{A}^*$  and substituting for  $\mathbf{A}^* \mathbf{y}_n$  on the right-hand side of equation (15) now gives

$$\mathbf{y}_n = \mathbf{A}^{*2} \mathbf{y}_n + (\mathbf{I} + \mathbf{A}^*) (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1} \mathbf{x}.$$

Similarly, multiplying both sides of (15) by  $\mathbf{A}^{*2}$  and substituting for  $\mathbf{A}^{*2} \mathbf{y}_n$  in this last expression, we get

$$\begin{aligned}\mathbf{y}_n &= \mathbf{A}^{*3} \mathbf{y}_n + (\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2}) (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1} \mathbf{x} \\ &= (\mathbf{I} - \mathbf{A}^{*3})^{-1} (\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2}) (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1} \mathbf{x}\end{aligned}\quad (16)$$

provided that  $(\mathbf{I} - \mathbf{A}^{*3})^{-1}$  exists.

Comparison of (16) with (12) shows that the above algebra has decomposed the accounting multiplier matrix  $\mathbf{M}_a$  into the product of three separate matrices. This decomposition is quite general. It can become informative by reference to the structure of  $\mathbf{A}_n$  and choosing an  $\tilde{\mathbf{A}}_n$  accordingly. Specifically, we can write

$$\mathbf{A}_n = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{bmatrix} \quad \text{and} \quad \tilde{\mathbf{A}}_n = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{33} \end{bmatrix} \quad (17)$$

so that  $\mathbf{A}^*$  defined by equation (15) can be written

$$\mathbf{A}^* = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{A}_{13}^* \\ \mathbf{A}_{21}^* & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{32}^* & \mathbf{0} \end{bmatrix} \quad \text{where} \quad \begin{cases} \mathbf{A}_{13}^* = \mathbf{A}_{13} \\ \mathbf{A}_{21}^* = (\mathbf{I} - \mathbf{A}_{22})^{-1} \mathbf{A}_{21} \\ \mathbf{A}_{32}^* = (\mathbf{I} - \mathbf{A}_{33})^{-1} \mathbf{A}_{32} \end{cases} \quad (18)$$

and where the partitioning of  $\mathbf{A}_n$  (and of  $\tilde{\mathbf{A}}_n$  and  $\mathbf{A}^*$ ) corresponds to the separate accounts in the SAM for factors, the endogenous institutions (households and companies), and production activities.

At this point in the argument it is worth noting that the three-part decomposition of  $\mathbf{M}_a$  in (16) does not require the three-way partitioning of matrix  $\mathbf{A}_n$  as in (17):  $\mathbf{A}_n$  can be partitioned into as many (or few) sets of accounts as one wishes. Similarly, there is nothing special from a mathematical perspective in choosing to end the sequence of substitutions which leads to equation (16) after three steps. Further substitutions are possible, and the general result is

$$\mathbf{y}_n = (\mathbf{I} - \mathbf{A}^{*k})^{-1} (\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2} + \dots + \mathbf{A}^{*(k-1)}) (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1} \mathbf{x}.$$

Our choice of three partitions for  $\mathbf{A}_n$ , and the decision to end the chain of successive substitutions after three steps ( $k = 3$ ) derives from the structure of

the SAM in Table 1 and this structure derives in turn from the conceptual framework of economics. Thus the particular application of the mathematics which is illuminating in our context is to have three partitions of  $\mathbf{A}_n$ , corresponding to factors, endogenous institutions, and production activities; and to choose  $k = 3$  not simply because there are three partitions, but because with this particular trio of partitions, three steps in the sequence of substitutions corresponds to one complete cycle in the circular flow of income within the economy.

Further reference to the SAM (Table 1) shows that with the chosen partitioning of  $\mathbf{A}_n$ , its zero sub-matrices are indeed empty blocks within the accounts. The non-zero sub-matrix  $\mathbf{A}_{13}$  reflects payments from activities to factors;  $\mathbf{A}_{21}$  corresponds to the mapping of incomes from factors to households and companies; and the non-zero elements of  $\mathbf{A}_{32}$  record the average propensities with which different types of households consume the goods produced by the various production activities. Sub-matrix  $\mathbf{A}_{22}$  captures current transfers between endogenous institutions and in our example is restricted to the distribution of dividends and interest to households. Sub-matrix  $\mathbf{A}_{33}$  shows the transactions between activities, i.e. inter-industry flows.

With these conventions we now define

$$\mathbf{M}_{a1} = (\mathbf{I} - \tilde{\mathbf{A}}_n)^{-1}; \mathbf{M}_{a2} = (\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2}); \mathbf{M}_{a3} = (\mathbf{I} - \mathbf{A}^{*3})^{-1} \quad (19)$$

with the implication from (12) and (16) that

$$\mathbf{M}_a = \mathbf{M}_{a3} \mathbf{M}_{a2} \mathbf{M}_{a1}. \quad (20)$$

Equations (17)–(19) imply, first, that  $\mathbf{M}_{a1}$  is a block diagonal matrix with successive diagonal elements given by  $\mathbf{I}$ ,  $(\mathbf{I} - \mathbf{A}_{22})^{-1}$  and  $(\mathbf{I} - \mathbf{A}_{33})^{-1}$ . They also imply that

$$\mathbf{A}^{*2} = \begin{bmatrix} \mathbf{0} & \mathbf{A}_{13}^* \mathbf{A}_{32}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{21}^* \mathbf{A}_{13}^* \\ \mathbf{A}_{32}^* \mathbf{A}_{21}^* & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \text{so that} \quad \mathbf{M}_{a2} = \begin{bmatrix} \mathbf{I} & \mathbf{A}_{13}^* \mathbf{A}_{32}^* & \mathbf{A}_{13}^* \\ \mathbf{A}_{21}^* & \mathbf{I} & \mathbf{A}_{21}^* \mathbf{A}_{13}^* \\ \mathbf{A}_{32}^* \mathbf{A}_{21}^* & \mathbf{A}_{32}^* & \mathbf{I} \end{bmatrix} \quad (21)$$

and that  $\mathbf{M}_{a3}$  is also block diagonal with successive diagonal elements given by

$$(\mathbf{I} - \mathbf{A}_{13}^* \mathbf{A}_{32}^* \mathbf{A}_{21}^*)^{-1}, \quad (\mathbf{I} - \mathbf{A}_{21}^* \mathbf{A}_{13}^* \mathbf{A}_{32}^*)^{-1} \quad \text{and} \quad (\mathbf{I} - \mathbf{A}_{32}^* \mathbf{A}_{21}^* \mathbf{A}_{13}^*)^{-1}.$$

The structure of  $\mathbf{M}_{a2}$  and  $\mathbf{M}_{a3}$  derives from that of  $\mathbf{A}^*$ . From (18) it can be observed that the pattern of zero and non-zero cells of  $\mathbf{A}^*$  corresponds to a circular permutation matrix of size  $3 \times 3$ . Accordingly, if  $\mathbf{y}_n$  is partitioned compatibly with  $\mathbf{A}_n$ , then the structure of equation (15) implies that the partitions of  $\mathbf{y}_n$  are related to each other as points on a closed loop. In Fig. 1 these points are shown schematically as the corners of a triangle. Matrix  $\mathbf{A}^*$  represents the mapping from one partition of  $\mathbf{y}_n$  to another. Starting from any corner of the triangle, three steps in this mapping brings one back to the starting point. Hence the structure of  $\mathbf{A}^*$  implies that our formulation contains a closed-loop system which is the algebraic statement of the circular flow of income, e.g. from activities to factors to institutions; and then back to activities in the form

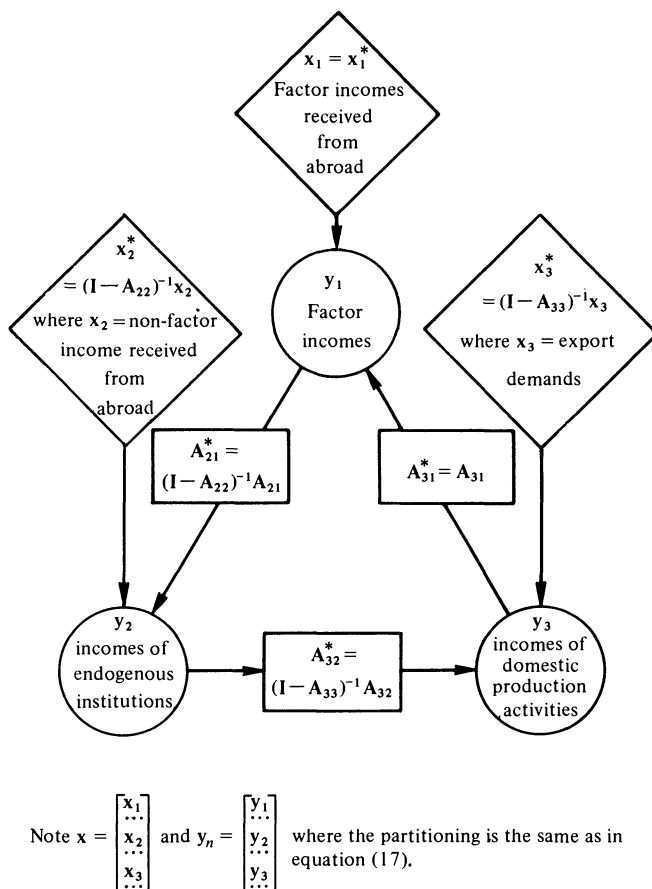


Fig. 1. The closed-loop structure of accounts as defined by equations (15), (17) and (18).

of consumption demand. This structure explains why  $\mathbf{M}_{a3}$  is block diagonal and justifies referring to this matrix as the closed-loop or circular multiplier matrix.

Matrix  $\mathbf{M}_{a1}$  is also block diagonal as previously noted. It captures the effects of one group of accounts on itself through direct transfers and is independent of the closed-loop nature of the system. Since there are no direct transfers between factors, the first diagonal block of  $\mathbf{M}_{a1}$  is simply an identity matrix. The second diagonal block captures the multiplier effect resulting from direct transfers between institutions  $(\mathbf{I} - \mathbf{A}_{22})^{-1}$ . The third diagonal block similarly refers to the multiplier effect of inter-industry transfers  $(\mathbf{I} - \mathbf{A}_{33})^{-1}$ , which is the Leontief inverse. Matrix  $\mathbf{M}_{a1}$  can be referred to as the transfers multiplier.

If  $\mathbf{M}_{a1}$  and  $\mathbf{M}_{a3}$  are block diagonal, all effects between partitions of  $\mathbf{y}_n$  must be captured by  $\mathbf{M}_{a2}$ . This matrix is therefore referred to as the cross-effects matrix or alternatively as the open-loop multiplier matrix. This terminology can be justified by considering the implications of one partition of  $\mathbf{y}_n$  for the others. Take as an example the effect of household and company incomes

on both factor incomes and production. This is an open-loop system and equivalent to breaking the closed loop by setting  $\mathbf{A}_{21} = \mathbf{0}$ , i.e. the effect of factor incomes on the incomes of institutions is ignored. From (18) it is apparent that  $\mathbf{A}_{21}^*$  is now zero, so that all terms in  $\mathbf{M}_{a3}$  and  $\mathbf{M}_{a2}$  which involve  $\mathbf{A}_{21}^*$  will be zero. This implies that  $\mathbf{M}_{a3}$  will now be an identity matrix. From (21), certain cells of  $\mathbf{M}_{a2}$  will also be zero. But the columns of  $\mathbf{M}_{a2}$  which refer to households and companies will be unaltered. These columns show the impact of incomes in the second partition of  $\mathbf{y}_n$  (endogenous institutions) on factor incomes (the first partition) and activity incomes (the third partition) in an open-loop system.

So far the discussion has assumed that the matrices  $\mathbf{M}_{a1}$ ,  $\mathbf{M}_{a2}$  and  $\mathbf{M}_{a3}$  exist and that it is legitimate to describe them as multiplier matrices in the sense that each has elements which are not less than the corresponding elements of an identity matrix. To justify this it can be noted that the matrix  $\mathbf{A}_n$  is semi-positive.<sup>1</sup> It follows that  $\mathbf{M}_a$  will be a multiplier matrix if it exists.

Mathematical conditions for the existence of  $\mathbf{M}_a$  can obviously be postulated.<sup>2</sup> If  $\mathbf{A}_n$  is a semi-positive indecomposable matrix, then  $\mathbf{M}_a$  will exist if no column sum exceeds unity and at least one column sum is strictly less than unity. Expression (8) supports the former conditions, and we have only to guarantee a leakage from some accounts for  $\mathbf{M}_a$  to exist, providing of course that  $\mathbf{A}_n$  is indecomposable. It is of interest to note that since  $(\mathbf{A}_n - \tilde{\mathbf{A}}_n)$  can be viewed as a circular permutation matrix then  $\mathbf{A}_n$  is certainly 'block' indecomposable of order 3. But this is not a sufficient condition for  $\mathbf{A}_n$  to be indecomposable in the general sense.

The existence of  $\mathbf{M}_a$  is enough to ensure the existence of  $\mathbf{M}_{a1}$ . This can be shown by first noting that  $\tilde{\mathbf{A}}_n$  is a semi-positive, completely decomposable matrix. If the conditions on the column sums hold for the existence of  $\mathbf{M}_a$  then they will hold for the existence of  $\mathbf{M}_{a1}$ , since  $\tilde{\mathbf{A}}_n$  is contained within  $\mathbf{A}_n$ . Furthermore,  $\mathbf{M}_{a1}$  will be a multiplier matrix. It also follows from (16) and (17) that  $\mathbf{A}^*$  will be semi-positive if  $\mathbf{M}_{a1}$  exists. Hence from (21)  $\mathbf{M}_{a2}$  will exist and will be a multiplier matrix. Finally, from (20), since  $\mathbf{M}_a$ ,  $\mathbf{M}_{a2}$  and  $\mathbf{M}_{a1}$  all exist, then  $\mathbf{M}_{a3}$  must also exist because it is bounded by finite matrices on both sides. Moreover,  $\mathbf{A}^{*3}$  is semi-positive, so that  $\mathbf{M}_{a3}$  is also a multiplier matrix.

A final remark on the existence of these multiplier matrices is to note that they essentially depend upon the designation of at least one exogenous account with at least some injection into, and hence some leakage from, the endogenous accounts which remain. This ensures at least one element of  $\lambda_a$  is positive.

To provide a useful way of presenting the results of our decomposition, Stone (1978a) has proposed an additive form of equation (20), namely.

$$\mathbf{M}_a = \mathbf{I} + (\mathbf{M}_{a1} - \mathbf{I}) + (\mathbf{M}_{a2} - \mathbf{I})\mathbf{M}_{a1} + (\mathbf{M}_{a3} - \mathbf{I})\mathbf{M}_{a2}\mathbf{M}_{a1} \quad (22)$$

so that elements of  $\mathbf{M}_a$  are accounted for by (i) the initial injection; (ii) the net contribution of transfer multiplier effects; (iii) the net contribution of open-

<sup>1</sup> This is always possible in a SAM since a negative element in the  $i$ th row,  $j$ th column can be set equal to zero and balance restored by adding a positive element of the required size in the  $j$ th row,  $i$ th column.

<sup>2</sup> These are generally discussed in a particularly relevant context by Lancaster (1968), pp. 94-5.

loop or cross-multiplier effects; and (iv) the net contribution of circular or closed-loop multiplier effects.<sup>1</sup> To illustrate this form of the decomposition requires results for the product matrix  $\mathbf{M}_{a2}\mathbf{M}_{a1}$  in addition to the details of  $\mathbf{M}_a$ .<sup>2</sup>

### III. FIXED-PRICE MULTIPLIERS

The accounting multipliers described in Section II are interesting for the information they contain on the structure of an economy as revealed by a SAM. However, because they are accounting multipliers they cannot be interpreted directly as measures of the effects of changes in injections into the economy on the levels of endogenous incomes. For this latter purpose we need to know how different economic agents behave in response to changes. In this and subsequent sections, we shall be concerned with the behaviour which generates the expenditure patterns of endogenous accounts under the assumption that prices remain fixed when income is altered. Since prices may in fact change, the multipliers obtained under this assumption are referred to as fixed-price multipliers.

Under the assumption that prices are fixed, it follows from the accounting balance equation (3) that

$$d\mathbf{y}_n = d\mathbf{n} + d\mathbf{x} \quad (23)$$

$$= \mathbf{C}_n d\mathbf{y}_n + d\mathbf{x} \quad (24)$$

$$= (\mathbf{I} - \mathbf{C}_n)^{-1} d\mathbf{x} = \mathbf{M}_c d\mathbf{x} \quad (25)$$

and similarly that

$$d\mathbf{l} = \mathbf{C}_l d\mathbf{y}_n \quad (26)$$

$$= \mathbf{C}_l (\mathbf{I} - \mathbf{C}_n)^{-1} d\mathbf{x} = \mathbf{C}_l \mathbf{M}_c d\mathbf{x}. \quad (27)$$

The result (23) is obtained by taking the total differential of (3). Equation (24) then follows from the fact that, if prices are fixed, the vector  $\mathbf{n}$  of incomes received by endogenous accounts, as a result of expenditures by these same accounts, can be a function of  $\mathbf{y}_n$  but otherwise is constant. Hence (24) follows from (23) if the  $(i, j)$ th element of matrix  $\mathbf{C}_n$  is the partial derivative of the  $i$ th element of  $\mathbf{n}$  with respect to the  $j$ th element of  $\mathbf{y}_n$ . In this sense  $\mathbf{C}_n$  is a matrix of marginal propensities to consume. If  $(\mathbf{I} - \mathbf{C}_n)^{-1}$  exists, then equation (25) shows how elements of  $\mathbf{y}_n$  change as a result of changes in injections. Similarly, the matrix  $\mathbf{C}_l$  in equation (26) is a matrix of marginal propensities to leak; and equation (27) shows how leakages change as a result of injections.

Equations (25) and (27) are analogous to equations (12) and (13). Consequently, under the condition that the matrix  $\mathbf{C}_n$  is non-negative,  $\mathbf{M}_c$  is a multiplier matrix, to be referred to as the fixed-price multiplier matrix. Matrices

<sup>1</sup> The arrangement (22) is applied in Stone (1978a) to a decomposition  $\mathbf{M}_a = \mathbf{M}_{a2}\mathbf{M}_{a3}\mathbf{M}_{a1}$  so that, in comparison with (20), the order of  $\mathbf{M}_{a2}$  and  $\mathbf{M}_{a3}$  is inverted. This alternative ordering was used in Pyatt, Roe *et al.* (1977), chapter 4. It is easily checked that both orderings are legitimate. However, the ordering adopted in (20) is perhaps to be preferred since it corresponds to the progression from transfer effects to open-loops to closed-loop models.

<sup>2</sup> Tabular results are available on request from the authors. It can be noted that, since  $\mathbf{M}_{a1}$  is block diagonal, it follows from the structure of  $\mathbf{M}_{a2}$  defined in (21) that setting off-diagonal blocks of  $\mathbf{M}_{a2}\mathbf{M}_{a1}$  equal to zero reduces this product to the matrix  $\mathbf{M}_{a1}$ .

Table 3  
*Estimates of the Matrices  $M_c$  and  $C_iM_c$  Derived from Tables 1 and 4*

Origin of injection																
(1) Factors of production					(2) Household curr. accts.		(3) Corporate curr. accts.		(4) Production activities: Commodity groups							
Labour		Capital			Urban	Rural	Estate	Private	State	Tea and rubber	Other agric.	Food process.	Other manuf.	Mining & constr.	Services	
Endogenous accounts (1) Factors of production	Labour															
	Urban	1.19	0.25	0.28	0.20	—	0.19	0.25	0.28	0.10	0.26	0.26	0.24	0.24	0.25	0.41
	Rural	0.34	1.47	0.58	0.37	—	0.34	0.47	0.58	0.18	0.58	0.74	0.60	0.40	0.45	0.62
	Estate	0.01	0.02	1.03	0.02	—	0.01	0.02	0.03	0.01	0.52	0.03	0.03	0.02	0.02	0.03
	Capital															
	Private	0.55	0.76	0.94	1.60	—	0.55	0.76	0.94	0.29	0.88	1.16	0.98	0.73	1.14	0.98
	Public	0.02	0.03	0.03	0.02	1.00	0.02	0.03	0.03	0.01	0.03	0.03	0.03	0.05	0.04	0.04
(2) Household curr. accts.	Sub-total	2.12	2.53	2.86	2.21	1.00	2.12	1.53	1.86	0.58	2.26	2.21	1.88	1.44	1.90	2.08
	Urban	1.31	0.41	0.49	0.54	—	1.31	0.41	0.49	0.47	0.45	0.51	0.45	0.40	0.49	0.62
	Rural	0.68	1.94	1.16	1.37	—	0.68	1.94	1.16	0.50	1.13	1.46	1.21	0.86	1.16	1.23
	Estate	0.02	0.03	1.04	0.04	—	0.02	0.03	1.04	0.02	0.53	0.04	0.04	0.02	0.03	0.04



$C_n$  and  $C_t$  will have column sums which add to unity and  $M_c$  will exist under conditions analogous to those for the existence of  $M_a$ . Hence, given estimates of the matrices  $C_n$  and  $C_t$ , both the fixed-price multiplier,  $M_c$ , and the matrix of marginal leakages,  $C_t M_c$ , can be calculated. These matrices are illustrated in Table 3 using data for Sri Lanka which are discussed below.

To go further, we need to consider data sources for  $C_n$  and  $C_t$ . This can be done with reference to Table 1, which shows that the outlays of factor incomes primarily generate incomes for the endogenous domestic institutions. The table shows that all urban labour income accrues to urban households. Thus the first column of  $C_t$  is zero, and all elements of the first column of  $C_n$  are also zero except the element in the row for urban households, which is one. Thus the sum of the first column of  $C_t$ , plus that for  $C_n$  also, is unity as it must be. The second, third and fifth columns of  $C_n$  and  $C_t$  are similarly obtained. For the fourth column, there are five different recipients of the income which accrues to private capital. The proportions in which they receive this income will depend on who owns private capital. And if the structure of ownership can be taken as given, then there is no reason to assume other than that increments of income will be distributed in the same proportions as the shares observed in the SAM. On these grounds, columns of  $C_n$  and  $C_t$  which refer to factor outlays are estimated by assuming that marginal and average propensities are the same.

For marginal and average propensities to be equal requires income elasticities of particular expenditures to be unity. This is clearly not true for household expenditures, and Table 4 sets out the marginal propensities which have been assumed. It is to be noted that the income elasticities of demand for imports are unusually low. This is partly because consumer imports in Sri Lanka include imports of the staple foods, rice and wheat, and partly because the observed cross-section elasticity has been lowered in recognition of the restrictions on imports which applied at that time.

For companies, marginal allocations of income have been assumed to be equal to the average allocations implied by Table 1. This is in default of any better basis for deciding how corporate taxation, savings and distribution policy might be responsive to changes in corporate income.

It has also been assumed that the allocation of total costs for production activities is the same at the margin as on average. The best way to justify this is as follows. First, the assumption of fixed prices would be reasonable if inter-industry technology follows Leontief assumptions so that there are no scale effects, and prices are fixed for given indirect tax rates if import prices are fixed and factor costs per unit of output are constant. These assumptions would make elements of  $A_t$  and  $C_t$  the same in the corresponding columns for production activities, with a similar equivalence of those elements of  $A_n$  and  $C_n$  which refer to inter-industry transactions. With respect to factor payments, profits will have a constant share if value-added price, i.e. value added per unit of output, is set as a constant mark-up over labour costs per unit of output. Labour costs per unit of output will be constant if labour is paid at fixed piece rates. Alternatively, it can be assumed that wage rates are fixed and the average product of labour is constant. This alternative assumption is necessary



Table 4  
*Household Average and Marginal Expenditure Propensities, Sri Lanka, 1970*

	Urban		Rural		Estate	
	Average	Marginal	Average	Marginal	Average	Marginal
Endogenous						
(4) Production activities: commodity groups						
Tea and rubber	0.005	0.002	0.008	0.006	0.008	0.006
Other agriculture	0.134	0.080	0.174	0.134	0.214	0.241
Food processing	0.092	0.059	0.154	0.149	0.174	0.246
Other manufactures	0.114	0.122	0.131	0.204	0.142	0.156
Mining and construction	0.001	0.001	0.001	0.001	0.001	0.001
Services	0.291	0.315	0.280	0.311	0.263	0.302
Sub-total	0.637	0.579	0.748	0.805	0.802	0.952
Exogenous						
(5) Government current account	0.111	0.164	0.017	0.022	—	0.010
(6) Institutions capital account	0.173	0.209	0.117	0.135	0.014	0.015
(7) Indirect taxes (net)	0.011	0.020	0.010	0.014	0.004	0.005
(8) Rest of world						
Commodity transactions	0.069	0.028	0.107	0.024	0.181	0.018
Other	—	—	—	—	—	—
Sub-total	0.364	0.421	0.251	0.195	0.199	0.048

Source: Pyatt, Roe *et al.* (1977), Chapter 5.

if labour incomes are to be assumed proportional to employment levels. It implies that the economy is working below capacity in all sectors. With these assumptions it is not unreasonable to assume prices fixed and that columns of  $\mathbf{C}_n$  which relate to activities can be estimated by columns of  $\mathbf{A}_n$ .

In aggregate, the above arguments imply that  $\mathbf{A}_n$  is equal to  $\mathbf{C}_n$  (and similarly for  $\mathbf{A}_i$  and  $\mathbf{C}_i$ ) except for the data in Table 4. These arguments also illustrate the fact that to estimate  $\mathbf{C}_n$ , and hence  $\mathbf{M}_c$ , it is only necessary to estimate a SAM and those income elasticities which are different from unity.

#### IV. DECOMPOSITIONS OF THE FIXED-PRICE MULTIPLIERS

A further implication of the discussion in the previous section is that the patterns of zero and non-zero entries in partitions of  $\mathbf{C}_n$  and  $\mathbf{A}_n$  are the same. Hence the fixed-price multiplier matrix can be decomposed into a transfer effects multiplier,  $\mathbf{M}_{c1}$ ; an open-loop multiplier matrix,  $\mathbf{M}_{c2}$ ; and a closed-loop multiplier matrix,  $\mathbf{M}_{c3}$ . Furthermore, these effects can be expressed multiplicatively as

$$\mathbf{M}_c = \mathbf{M}_{c3} \mathbf{M}_{c2} \mathbf{M}_{c1} \quad (28)$$

or in Stone's additive form<sup>1</sup>

$$\mathbf{M}_c = \mathbf{I} + (\mathbf{M}_{c1} - \mathbf{I}) + (\mathbf{M}_{c2} - \mathbf{I})\mathbf{M}_{c1} + (\mathbf{M}_{c3} - \mathbf{I})\mathbf{M}_{c2}\mathbf{M}_{c1}. \quad (29)$$

With prices fixed, the differences between corresponding elements of  $\mathbf{M}_a$  and  $\mathbf{M}_c$  must be due to income effects. This can be formalised by writing from (24)

$$d\mathbf{y}_n = \mathbf{C}_n d\mathbf{y}_n + d\mathbf{x} \quad (24)$$

$$= (\mathbf{C}_n - \mathbf{A}_n) d\mathbf{y}_n + \mathbf{A}_n d\mathbf{y}_n + d\mathbf{x} \quad (30)$$

$$= (\mathbf{I} - \mathbf{A}_n)^{-1} [(\mathbf{C}_n - \mathbf{A}_n) d\mathbf{y}_n + d\mathbf{x}]$$

$$= \mathbf{M}_a (\mathbf{C}_n - \mathbf{A}_n) d\mathbf{y}_n + \mathbf{M}_a d\mathbf{x}$$

$$= [\mathbf{I} - \mathbf{M}_a (\mathbf{C}_n - \mathbf{A}_n)]^{-1} \mathbf{M}_a d\mathbf{x}$$

$$= \mathbf{M}_y \mathbf{M}_a d\mathbf{x} \quad (31)$$

where

$$\mathbf{M}_y = [\mathbf{I} - \mathbf{M}_a (\mathbf{C}_n - \mathbf{A}_n)]^{-1} \quad (32)$$

and

$$\mathbf{M}_y \mathbf{M}_a = \mathbf{M}_c. \quad (33)$$

Thus the income effects can be captured in a matrix  $\mathbf{M}_y$  which transforms the accounting multiplier matrix  $\mathbf{M}_a$  into a fixed-price multiplier matrix  $\mathbf{M}_c$ . However,  $\mathbf{M}_y$  itself is not a multiplier matrix because, as can be seen from (32), elements of  $\mathbf{M}_y$  can be negative since elements of  $\mathbf{C}_n$  can be less than the corresponding elements of  $\mathbf{A}_n$ , i.e. income elasticities can be less than one. In our example, the matrix  $\mathbf{M}_y$  is particularly simple. Since only households have income elasticities which differ from one, it is only in the columns for households that  $\mathbf{M}_y$  differs from an identity matrix.

<sup>1</sup> Tabular results are available on request from the authors.

## V. EMPIRICAL RESULTS

A number of general points as well as particular features of the Sri Lanka economy in 1970 can be illustrated from the empirical results. Of the general features it can be noted from Table 3 that the columns for factors contain little information that is not included in the detail for institutions. With respect to labour, this is partly because there is a one-to-one relationship between types of labour and types of household, and partly because the basic SAM shown as Table 1 does not record any transfers between household types. Similarly, for public capital, all income goes directly to state corporations. From there it all leaks out immediately from the endogenous accounts, so that the columns for public capital and state corporations have a particularly simple structure.

Diagonal blocks of matrices  $\mathbf{M}_{a2}\mathbf{M}_{a1}$  and  $\mathbf{M}_{c2}\mathbf{M}_{c1}$  record the non-zero elements of the transfer matrices  $\mathbf{M}_{a1}$  and  $\mathbf{M}_{c1}$ . Given our assumptions, these are identical. Results for the simple Leontief inverse indicate that inter-industry linkages are weak in Sri Lanka except for the dependence of 'Other agriculture' on demands from 'Food processing'. In contrast, Table 3 shows that much stronger linkages are involved when the full circular flow illustrated in Fig. 1 is taken into account.

A general feature of Table 3 is the relative constancy of the multipliers along rows of the tables. For example, an injection of 100 units into any activity other than Services results in a fixed-price multiplier effect on Services which lies within the relatively narrow range of 58 to 83 units. The implication is that second- and third-order effects are largely independent of the structure of demand.<sup>1</sup>

This homogeneity of higher-order effects is important for the structure of employment and income distribution. Table 3 shows that whichever activity might be expanded, Urban labour income expands by 24 to 26 per cent of the size of the injection, unless the injection is into Services, where the multiplier is 0.41. Similarly, over the range of six activities, the multiplier for Rural wage income lies between a low of 0.40 (for Other Manufactures) and 0.74 (for Other Agriculture). For Estate Labour there is an exception to this rule: an injection into Tea and Rubber has a multiplier of 0.52 for Estate Labour but otherwise is 0.01, 0.02 or 0.03.

This general pattern of results is the consequence of linkages within the economy, or the lack of them, as seen through the original choice of SAM classifications in Table 1. The estate sector and the activity 'Tea and rubber' are largely independent of what goes on elsewhere in the economy, while other sectors are much more closely integrated.

Table 5 sets out some examples of the particular method of decomposition which is described in this paper. The format shows the additive decomposition of fixed price multipliers in the last four columns, and of the accounting multipliers in the first four columns. The central column then shows the income

<sup>1</sup> A similar phenomenon is observed by Stone (1978a) for the United Kingdom. It can be traced back through the structure of the multipliers to the fact that different household types have similar expenditure patterns.

Table 5  
*Illustrations of Multiplier Decomposition*

Account in which injection originates	Account affected by the injection	Accounting multiplier effects				Fixed-price multiplier effects						
		Initial Injection, $I$	Transfer effects, $M_{a1}-I$	O-L effects, $(M_{a2}-I) M_{a1}$	C-L effects, $(M_{a3}-I) M_{a2} M_{a1}$	Sub-total, $M_a$	Income effects, $(M_y-I) M_a$	Sub-total, $M_p$	C-L effects, $(M_{c3}-I) M_{c2} M_{c1}$	O-L effects, $(M_{c2}-I) M_{c1}$	Transfer effects, $M_{c1}-I$	Initial injection, $I$
(1) Tea and rubber	Tea and rubber	100				102	-	101	1			100
(2) Other agriculture	Other agriculture		2		59	61	1	62	60		1	
(3) Food processing	Food processing		1		32	33	5	38	37		1	
(4) Urban households	Urban households			8	31	39	9	45	37	8		
(5) Rural households	Rural households			25	78	103	10	113	88	25		
(6) Estate households	Estate households			50	3	53	*	53	3	50		
(7) Other agriculture	Other agriculture	100	5		57	162	-9	153	48		5	100
(8) Food processing	Food processing		1		31	32	-1	31	30		1	
(9) Other manufactures	Other manufactures		4		36	40	14	54	50		4	
(10) Urban households	Urban households			17	31	48	3	51	34	17		
(11) Rural households	Rural households			68	77	145	1	146	78	68		
(12) Estate households	Estate households			2	2	4	*	4	2			

(13) Services	—	2	—	52	54	-9	45	43	—	2	—
(14) Other agriculture	—	1	—	29	30	-2	28	27	—	1	—
(15) Food processing	—	5	—	34	39	11	50	45	—	5	—
(16) Other manufactures	—	—	31	29	60	2	62	31	31	—	—
(17) Urban households	—	—	52	71	123	*	123	71	52	—	—
(18) Rural households	—	—	1	3	4	*	4	3	1	—	—
(19) Estate households	—	—	*	1	1	*	1	*	*	—	—
(20) Tea and rubber	—	—	22	31	53	-15	38	24	14	—	—
(21) Other agriculture	—	—	11	17	28	-6	22	15	7	—	—
(22) Food processing	—	—	16	22	37	5	42	25	17	—	—
(23) Other manufactures	—	—	33	36	69	3	72	37	35	—	—
(24) Services	—	—	9	10	19	*	9	10	9	—	—
(25) Urban labour	—	—	17	21	38	-4	34	19	15	—	—
(26) Rural labour	—	—	1	1	2	-1	1	*	1	—	—
(27) Estate labour	—	—	—	32	132	-1	131	31	1	—	100
(28) Urban households	100	—	—	77	77	-9	68	68	—	—	—
(29) Rural households	—	—	—	3	3	-1	2	2	—	—	—
(29) Estate households	—	—	—	—	—	—	—	—	—	—	—

\* Absolute value less than 1.

O-L, Open-loop. C-L, Closed loop.

effects that link accounting and fixed-price multipliers, given the assumed fixed-price model.

The first three rows of Table 5 show the decomposition of an injection of 100 units into the tea and rubber sector on itself. There are virtually no multiplier effects since tea and rubber are not large items in endogenous expenditures for any group. In Table 4, tea is shown to be an inferior good, and this fact leads to a fixed-price multiplier that is smaller than the accounting multiplier. In contrast, the second and third rows of Table 5 show that an injection into Tea and Rubber has a transfer effect on the sectors Other Agriculture and Food Processing which derives from the input-output inverse  $(\mathbf{I} - \mathbf{A}_{33})^{-1}$ . Also, there are substantial closed-loop effects: the extra income in Tea and Rubber is spent in ways which result, through the closed-loop, in extra demands on Other Agriculture and Food Processing. These extra demands are larger with the fixed-price multipliers than with the accounting multiplier. This is because, in Table 4, leakages are a decreasing fraction of income for the estate households who are the main element in the first link of the multiplier chain which starts with an injection into Tea and Rubber.

The next three rows of Table 5 (rows 4 to 6) show how households are affected by injections into Tea and Rubber. The open-loop effects primarily benefit estate households. But the lack of linkage of this sector to the rest of the economy implies negligible closed-loop effects for them: the closed-loop effects essentially benefit urban and rural households. The closed-loop effects are again greater according to the fixed-price model than they are according to the accounting multipliers.

Rows 7 to 12 of Table 5 show similar results for an injection into the production sector Other Agriculture. Engel's Law as captured in Table 4 is now sufficiently strong for the fixed-price multipliers to be less than the accounting multipliers in rows 7 and 8. In row 9, the fact that Other Manufactures are superior goods leads to a relatively large increase in the multiplier as we move from  $\mathbf{M}_a$  to  $\mathbf{M}_c$ . It can be noted that the closed-loop effects on household incomes in rows 10 to 12 are very similar to those in rows 4 to 6 and in rows 16 to 18. Similarly, the closed-loop effects on other activities in rows 7 to 9 are essentially replicated when the initial injection is into Services, as in rows 13 to 15, or into Tea and Rubber, as in rows 2 and 3.

In Table 4, leakages in aggregate are a declining fraction of income for rural and estate households, largely because of the food composition of Sri Lanka imports. The results for urban households in rows 19 to 29 of Table 5 are therefore more typical of what might be found in economies with a higher degree of self-sufficiency in basic foods and where institutions like Sri Lanka's free rice rations are non-existent. These last examples show that the fixed-price multipliers are smaller than accounting multipliers as a general result. Exceptions are for the superior elements of demand, viz. Other Manufactures and Services. It is to be noted that an injection into the Urban Household sector does next to nothing for the Estate sector, but the impact on rural activities and incomes is considerable.

## VI. CONCLUSIONS

In this paper we have been concerned with the structure of simple models from various perspectives. From one perspective we have been concerned with the sequential extension of models from a simple Leontief input-output base, to open-loop models, and hence to closed loops. The novelty here is to consider simultaneously the three possible open-loop models in a triangular system, and to show how the multiplier matrix for a model at one stage in this chain of development is obtained as the product of a new multiplier matrix and the multiplier matrix which maintained at the previous stage. This illustrates how complexity in model formulation can be built up sequentially. More innovative is our separate recognition of factors, institutions and activities, with each being disaggregated into several types so that household income distribution, the structure of production and the factorial distribution of income are all interwoven in the scheme.

From another perspective, the analysis here illustrates the approach to model building which starts with a SAM and hence with the structure of an economy at some base date.<sup>1</sup> The accounting multipliers described here give insight into the anatomy of this structure in terms of transfer effects and the full circular and cross-effects between different parts of the economy, corresponding to the circular flow of income which characterises the multiplier process. Our analysis shows that this decomposition of structure can be derived directly from accounting balances. An integral part of it is to show how the structure of production and income distribution are interrelated, and how they derive from the structure of exogenous demand and the distribution of assets.

The analysis also shows the extent to which initial structure is important in determining the impact of changes in demand. Under the assumption that prices are fixed, incremental changes will follow a different pattern to that of the accounting balances only in so far as income elasticities for the outlays of endogenous accounts differ from their average value of unity.

The fixed-price multipliers discussed here represent only a single step beyond the structure of accounting balances. Subsequent steps could embrace the interaction of price changes and shifts in exogenous demand, including variations in the exchange rate and in factor prices. Similarly, investment demands might be modified in the light of savings patterns, capacity utilisation and the flow of funds. These and other developments could, in principle, be built on the foundations laid in this paper, which therefore constitute a beginning. But the starting point and the first step are important. The way they are formulated here is in terms of the accounting structure of the circular flow of income and its modification by allowing for income effects. These already cover a wide class of models in actual use, which our analysis extends by embracing the distribution of income among different types of households and the structure of asset ownership, both among these household types and between them and other

<sup>1</sup> This is, of course, the tradition of the Cambridge Growth Project. See Cambridge, Department of Applied Economics (1962-74).

institutions. Thus, for example, our results on fixed-price multipliers could have been presented as the results of a model with the following specifications: (i) Leontief technology for intermediate inputs, with complementary imports at prices which are set exogenously; (ii) Cobb–Douglas production functions, with firms setting prices as a constant mark-up on material costs and hiring factors so as to minimise variable costs; (iii) wages set so as to clear labour markets; and (iv) household consumption patterns given by linear expenditure systems. Such assumptions are among the variants with which the results in Table 3 are consistent. It seems to us much more helpful to see these results for what they are, namely, a SAM structure modified by allowing consumer demand elasticities to be different from unity. Not least, this brings out the importance of structure, as given by the SAM, in determining results, and the incremental adjustments which follow from allowing behaviour to be different at the margin from what it is on average.

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