CHAPTER 8 Land Use in the Monocentric City

The outcome of the city will depend on the race between the automobile and the elevator, and anyone who bets on the elevator is crazy.
Frank Lloyd Wright

This chapter uses the concepts developed in Chapter 7 to discuss land rent and land use in the monocentric or core-dominated city. The monocentric city was the dominant urban form until the early part of the 20th century. In the monocentric city, commercial and industrial activity is concentrated in the central core area. During the last 70 or 80 years, most large metropolitan areas have become multicentric, with suburban subcenters that complement and compete with the central core area. This chapter explains the market forces behind the development of the monocentric city, and Chapter 10 discusses the market forces behind the transformation of monocentric cities to multicentric ones.

Why study the monocentric city? Although few of today's large cities are monocentric, the analysis of the monocentric city is important for four reasons. First, the monocentric city was the dominant urban form until the early part of the 20th century, so urban history is largely a history of the monocentric city. Second, many of today's small- and medium-sized cities are still monocentric. Third, to understand the transition from the traditional monocentric city to the modern multicentric city, one must understand the forces behind the development of the monocentric city in the first place. Fourth, many of the lessons from the monocentric model can be extended to the modern multicentric city.

The discussion of the various land users in the monocentric city proceeds from the city center outward. The first section derives the bid-rent functions of three business sectors (manufacturers, office firms, retailers), and uses the bid-rent functions to discuss land-use patterns in the central business district. The second section examines the location choices of households, deriving the bid-rent function for residential land. The third explains why employment is concentrated in the city center, that is, why the city is monocentric. The fourth section derives the residential bid-rent function under a more realistic set of assumptions than those of the simple monocentric model. The final two sections deal with empirical issues, addressing three questions. First, why do poor households tend to locate in the central city, while wealthy households tend to locate in the suburbs? Second, how rapidly does land rent fall as distance to the city center increases? Third, what is the relationship between population density and distance to the city center?

The traditional monocentric city has the transportation technology of the 19th century. The monocentric model has four key assumptions:

1. **Central export node.** All manufacturing output is exported from the city through a railroad terminal at the city center (a central export node).
2. **Horse-drawn wagons.** Manufacturers transport their freight from their factories to the export node by horse-drawn wagons.
3. **Hub-and-spoke streetcar system.** Commuters and shoppers travel by streetcar from the residential areas to the central business district (CBD). The streetcar lines are laid in a radial pattern: the lines form spokes that lead into the CBD (the hub).
4. **Agglomerative economies.** The office industry is dependent on face-to-face contact: employees from different office firms meet in the city center to transact business.

Under these assumptions, the city center is the focal point of the entire metropolitan area: manufacturers are oriented toward the export node; office firms are oriented toward the central market area; retailers are oriented to the hub of the streetcar system; and households are oriented toward employment and shopping opportunities in the central core area.

Commercial and Industrial Land Use

This section discusses land rent and land use in the central business district (CBD) of the monocentric city. Three activities occupy the central core area: manufacturers, office firms, and retailers.

The Bid-Rent Function of Manufacturers

Suppose that manufacturers in the monocentric city produce baseballs. The firms in the baseball industry have the following characteristics:

1. **Production.** Firms produce baseballs with land, labor, capital, and raw materials. Every firm produces B tons of baseballs per month.
2. **Fixed prices.** The prices of baseballs and nonland inputs (labor, capital, and raw materials) are determined in national markets, so firms take these prices as given. The prices are the same at all locations in the city.
3. **Competitive markets.** There is free entry into the industry. In equilibrium, each firm makes zero economic profits (normal accounting profits).

4. **Baseball freight cost.** Baseballs are shipped by horse-drawn wagon from the factory to the central railroad terminal, where they are exported to other cities.
5. **Raw material freight cost.** Raw materials are imported to the city by rail. The intracity freight cost of raw materials is small enough to ignore.
6. **Factor substitution.** Baseball makers engage in factor substitution: as the price of land increases, firms substitute nonland inputs for land.

For the purposes of choosing a location within the city, baseball firms are market-oriented. The intracity freight cost of raw materials is assumed to be negligible, and the costs of other inputs (capital and labor) are assumed to be the same at all locations within the city. Because input costs are the same throughout the city, a firm's location decision is based on access to its market. Once a firm decides to locate somewhere in the city, the relevant market is the destination of baseballs *within* the city, that is, the central railroad terminal.

The firm's profit equals total revenue less the cost of inputs, intracity freight, and land. Total revenue is the price of baseballs (P_b) times the quantity produced (B). C is the cost of all nonland inputs (capital, labor, raw materials). If freight cost is t per ton per mile, total freight cost for a location u miles from the export node is t times B times u. If R is land rent per acre and T is the acreage of the factory site, the profit at a location u miles from the export node is

$$\pi = P_b \cdot B - C - t \cdot B \cdot u - R \cdot T \tag{8–1}$$

All markets are perfectly competitive, so the firm's economic profit is zero. According to the leftover principle, the firm is willing to pay its landowner the excess of total revenue over the cost of nonland inputs and freight cost. As shown in Chapter 7, the expression for the bid rent is derived by setting $\pi = 0$, adding $R \cdot T$ to both sides of the profit equation, and dividing by T. The firm's bid rent *per acre* is the pre-rent profit divided by land consumption:

$$R = \frac{P_b \cdot B - C - t \cdot B \cdot u}{T} \tag{8–2}$$

The **bid-rent function** indicates how much the typical firm is willing to pay *per acre* for different production sites in the city. Table 8–1 shows how to compute the bid rent for different distances from the city center, given values for P_b, B, C, t, T, and u. The bid-rent function, shown in Figure 8–1, is negatively sloped because freight cost increases as the firm moves away from the city center.

The bid-rent function is convex because firms engage in factor substitution. As the firm approaches the city center with its higher land cost, the firm substitutes nonland inputs (capital and labor) for land, producing the same tonnage of baseballs with less land and more of the other inputs. From Table 8–1, the firm uses one acre of land and $1,400 worth of other inputs for a location three miles from the city center, and 0.40 acres and $2,600 worth of nonland inputs for a location near the city center. The bid-rent function of the typical baseball firm will be used to represent the bid-rent function of the entire manufacturing sector.

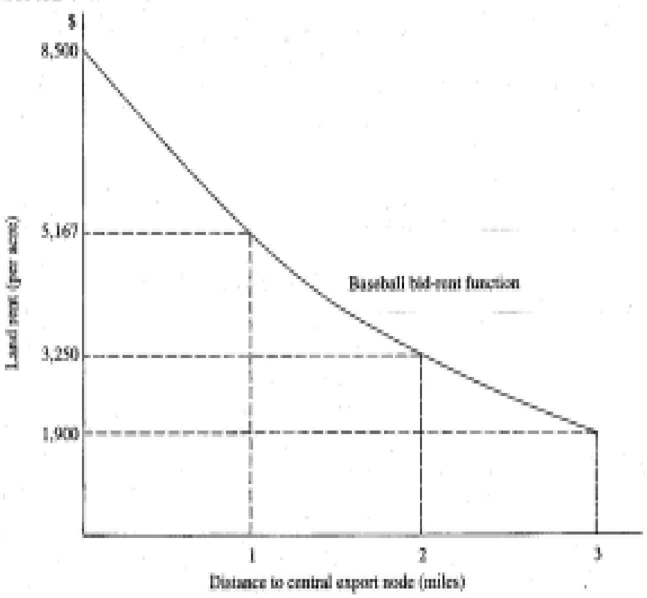
TABLE 8–1 The Bid-Rent Function of Baseball Manufacturers

Miles from Export Node	*Size of Site (acres)*	*Total Revenue*	*Nonland Costs*	*Freight Costs*	*Pre-Rent Profit*	*Rent per Acre*
0	0.4	$6,000	$2,600	$ 0	$3,400	$8,500
1	0.6	6,000	2,000	900	3,100	5,167
2	0.8	6,000	1,600	1,800	2,600	3,250
3	1.0	6,000	1,400	2,700	1,900	1,900

Assumptions:

1. Output = 6 tons of baseballs
2. Price = $1,000 per ton
3. Transport cost = $900 per mile
4. Pre-rent profit = Total revenue − Nonland cost − Freight cost
5. $\text{Rent} = \dfrac{\text{Pre-rent profit}}{\text{Size of factory site}}$

FIGURE 8–1 Bid-Rent Function of Baseball Firms



The bid-rent function of baseball firms is negatively sloped because freight costs increase as the distance to the city's central export node increases. It is convex (not linear) because firms substitute nonland inputs for land as the price of land increases.

The Bid-Rent Function of Office Firms

Although firms in the office sector provide a wide variety of goods and services, office firms share two important characteristics. First, they gather, process, and distribute information. Because information becomes obsolete quickly, office firms must be able to collect and distribute it rapidly. Second, office firms rely on face-to-face contact in the collection, processing, and distribution of information. For example, accountants explain and interpret the information in accounting reports. The loan officers of banks meet with prospective borrowers to appraise their creditworthiness. The investment advisors of finance firms meet with clients to assess their attitudes toward risk and their investment inclinations. In general, office firms rely on speedy face-to-face contact in collecting and distributing information. In contrast to baseball makers, who throw their baseballs into the back of a horse-drawn wagon, office firms transmit their output in the minds and briefcases of their employees.

Suppose that office firms in the monocentric city provide financial services. The finance industry has the following characteristics.

1. **The office.** Every finance firm is based in an office. The nonland cost of the office (capital and labor cost) is C per month. The "output" of the firm is investment consultations, and each firm produces A consultations per month.
2. **Travel to city center.** The manager of each firm travels from the office to the city center (the hub of the streetcar system) to consult with clients. Every consultation requires one trip to the city center.
3. **Fixed prices.** The prices of financial advice and nonland inputs are determined in national markets, so finance firms take the prices as given. The prices are the same at all locations in the city.
4. **Competitive markets.** There is free entry into the industry. In equilibrium, all firms make zero economic profits (normal accounting profits).
5. **Factor substitution.** Finance firms engage in factor substitution: as the price of land increases, they substitute nonland inputs for land.

The travel cost of an office firm equals the opportunity cost of the manager's travel between the office and the clients in the city center. Suppose that the manager takes t minutes to walk one block, and the wage is W per minute. If the office is u blocks from the city center, the travel cost per consultation is

$$TC = t \cdot W \cdot u \qquad (8\text{–}3)$$

For example, if $t = 3$ minutes per block and $W = \$4$ per minute, travel cost per consultation per block is $12, so a firm located 10 blocks from the city center incurs a travel cost of $120 per consultation. If the firm provides A consultations per month, the monthly travel cost for a location u blocks from the city center is

$$TC = t \cdot W \cdot A \cdot u \qquad (8\text{–}4)$$

If $A = 200$ consultations per month, the monthly travel cost is $2,400 for a location one block from the city center, $4,800 for a location two blocks from the city center, and so on.

The firm's total profit is total revenue less the cost of nonland inputs, land, and travel. If the price of a consultation is P_a, rent per acre is R, and the firm occupies T acres of land, profit is

$$\pi = P_a \cdot A - C - R \cdot T - t \cdot W \cdot A \cdot u \quad (8\text{–}5)$$

Since markets are perfectly competitive, economic profit is zero in equilibrium. According to the leftover principle, the firm is willing to pay the landlord the excess of total revenue over nonland cost. The bid rent *per acre* of land is the pre-rent profit divided by the amount of land consumed:

$$R = \frac{P_a \cdot A - C - t \cdot W \cdot A \cdot u}{T} \quad (8\text{–}6)$$

The bid-rent function indicates how much the office firm is willing to pay for different office sites. Table 8–2 shows how to compute the bid rent for different distances from the city center, given values for P_a, A, C, t, W, T, and u. The bid-rent function, shown in Figure 8–2, is negatively sloped because travel cost increases as the firm moves away from the city center.

The bid-rent function is convex because office firms engage in factor substitution. As the firm approaches the city center, it substitutes nonland inputs (capital and labor) for the relatively expensive land, producing the same number of consultations with less land and more of its nonland inputs. In other words, office firms near the city center occupy taller buildings. The bid-rent function of the typical office firm will be used to represent the bid-rent function of the entire office sector.

Land Use in the Central Business District

In the monocentric city, manufacturers and office firms are oriented toward the central business district. Manufacturers are attracted by the central export node, and office firms cluster around the city center to facilitate face-to-face contact. How is CBD land allocated between the two activities?

Figure 8–3 shows the bid-rent functions of manufacturers (R_m) and office firms (R_o). Figure 8–3 also shows the bid-rent function of city residents (R_h), which is derived later in the chapter. Because land is allocated to the highest bidder, office firms outbid manufacturers for land within u_o miles of the city center, generating an office district with a radius of u_o miles. Manufacturers outbid office firms and residents for land between u_o and u_m miles of the city center, generating a manufacturing district with a width of $(u_m - u_o)$ miles. The central area of the city is occupied by the office industry because the office bid-rent function is steeper than the manufacturing bid-rent function.

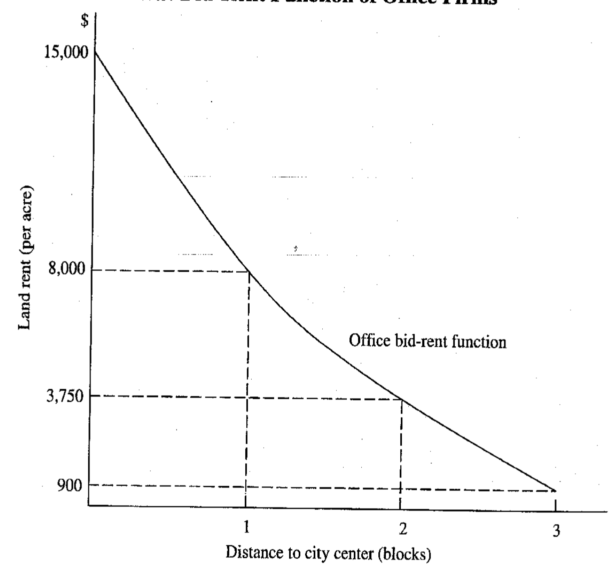
The office bid-rent function is relatively steep because the office industry has relatively high transportation costs. Office firms rely on frequent face-to-face contact, using high-priced financial consultants to transport their output to clients in the city center. In contrast, the manufacturing industry ships its output by horse-drawn wagon, so its transport costs are relatively low. In the numerical examples shown in Tables 8–1 and 8–2, the baseball manufacturer has monthly freight costs of \$900 per mile, while the office firm has monthly travel costs of \$2,400 *per block*. The

TABLE 8–2 The Bid-Rent Function of Office Firms

Blocks from City Center	*Size of Site (acres)*	*Total Revenue*	*Nonland Cost*	*Freight Cost*	*Pre-Rent Profit*	*Rent per Acre*
0	0.4	\$9,600	\$3,600	\$ 0	\$6,000	\$15,000
1	0.6	9,600	2,400	2,400	4,800	8,000
2	0.8	9,600	1,800	4,800	3,000	3,750
3	1.0	9,600	1,500	7,200	900	900

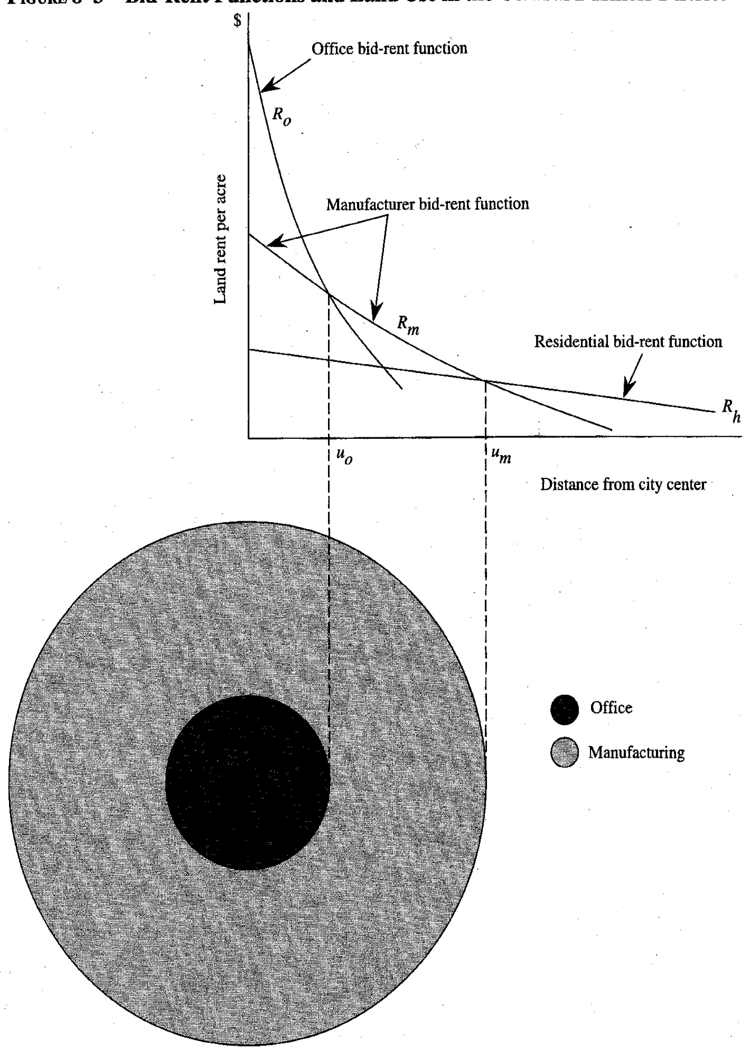
Assumptions:
1. Output (A) = 200 consultations
2. Price = \$48 per consultation
3. Travel time (t) = 3 minutes per block
4. Opportunity cost (W) = \$4 per minute
5. Travel cost = $t \cdot A \cdot W$ = \$2,400 per block
6. Pre-rent profit = Total revenue − Nonland cost − Travel cost
7. Rent = $\frac{\text{Pre-rent profit}}{\text{Size of office site}}$

FIGURE 8–2 The Bid-Rent Function of Office Firms



The office bid-rent function is negatively sloped because travel costs increase as distance to the central marketplace increases. It is convex because firms substitute nonland inputs for land as the price of land increases.

FIGURE 8–3 Bid-Rent Functions and Land Use in the Central Business District



The office industry has a relatively steep bid-rent function because the travel cost of people exceeds the travel cost of freight. The office industry outbids manufacturers for land near the city center. Central land is occupied by the activity with the most to gain from proximity (decreased transportation costs).

bid-rent functions are negatively sloped because of transportation cost, so the larger the transportation cost, the larger the slope.

Does the land market allocate land efficiently? In the terms used by land developers, is land allocated to its "highest and best use"? The office industry, with its higher transportation costs, occupies the land closest to the city center. This allocation is efficient because the office industry has the most to gain from proximity to the city center. To explain, suppose that a finance firm one block from the city center swaps locations with a manufacturer three blocks (one-fifth of a mile) from the city center. The land swap increases the finance firm's travel costs by $7,200 (three blocks times $2,400 per block), but decreases the baseball firm's freight costs by only $180 (one-fifth mile times $900 per mile). Office travel costs increase by more than freight costs decrease, so total transportation costs increase. The market allocation, which gives central land to the office industry, minimizes total transportation costs.

Location of Retailers

Where in the monocentric city do retailers locate? Central place theory, which is used in Chapter 5 to explain the regional distribution of retailers, is also applicable to the intracity distribution of retailers. Retailers carve up the city into market areas, with the size of the market areas determined by scale economies, per capita demand, and transportation costs.

Large Scale Economies—Glove Sellers. Consider first an activity for which scale economies are large relative to per capita demand, for example, a glove store. The glove market has the following characteristics:

1. **Single glove store.** The scale economies of glove selling are exhausted only at outputs that are large *relative* to the total demand for gloves, so there is a single glove store in the city. The efficient size for the glove seller is 5,000 pairs of gloves per month, and total demand for gloves is 5,000 per month.
2. **Glove consumers.** Consumers are distributed uniformly throughout the city.
3. **Perfect competition.** Although there is a single glove seller, entry into the glove market is not very costly. Given the threat of entry, economic profit is zero.

The retailer's profit at a particular location is the excess of total revenue over total cost. Suppose that the profit margin (price less average cost) is constant. If P_g is the price of gloves, G is the quantity sold, and AC_g is the average cost, pre-rent profit is

$$\pi = G \cdot (P_g - AC_g) \qquad (8\text{–}7)$$

For example, if P_g is $9 and AC_g is $5, the profit margin is $4. If G is 5,000, total profit would be $20,000.

At what location will the glove store earn the most profit? If the profit margin is constant, the firm maximizes total profit by maximizing sales volume (G). As shown in Chapter 5, sales volume is maximized at the center of the market area (the median location) for the simple reason that the central location is accessible to the most consumers. Since the glove store sells to people throughout the city, profit is maximized at the city center. The benefit of the central location is reinforced by the hub-and-spoke streetcar system of the monocentric city, which delivers suburban commuters and shoppers to the city center.

Because there is free entry into the glove-selling business, the glove store makes zero economic profit. Competition for the best glove-selling site bids up the price of land to the point at which economic profit is zero. If the glove store refuses to pay its economic profits to the landowner, the landowner will rent the site to another glove seller.

Moderate Scale Economies—Hat Sellers. Consider next an activity for which scale economies are moderate relative to per capita demand, for example, hat stores. The hat industry has the following characteristics:

1. **Five hat stores.** The scale economies of hat selling are moderate *relative to* the total demand for hats, so there are five hat stores in the urban area. The efficient sales volume for each hat store is 4,000 hats per month, and the total demand is 20,000 hats per month.
2. **Hat consumers.** Consumers are distributed uniformly throughout the monocentric city.
3. **Perfect competition.** Entry into the hat market is not very costly. Given the threat of entry, economic profit is zero.

The pre-rent profit of an individual store is total revenue less total cost. If H is the number of hats sold, P_h is the price of hats, and AC_h is the average cost, total profit is

$$\pi = H \cdot (P_h - AC_h) \qquad (8\text{–}8)$$

For example, if P_h is \$8 and AC_h is \$5, the profit margin is \$3. If H is 4,000, total profit would be \$12,000.

According to the simple version of central place theory, hat sellers divide the city into five equal market areas, and each hat seller locates at the center of a market area. This result can also be expressed in terms of bid-rent functions. The bid-rent function of a particular hat store depends on where the other stores locate. Locations close to other hat stores have lower sales volume, so the hat seller is willing to pay less in rent. The bid-rent function of the hat industry has five peaks, one at the center of each market area.

Will hat sellers adopt the location pattern predicted by the simple version of central place theory? The simple version of the theory assumes that unit travel cost (the cost per mile) is the same in all directions. In the monocentric streetcar city, this assumption is violated: the hub-and-spoke streetcar system collects people along the suburban spokes and delivers them to the central "hub." Travel along the spokes into the city is cheaper than travel between the spokes, so a trip from a house in the suburbs to a downtown hatter may be easier than a trip to a suburban hatter. If so, most—if not all—of the hat sellers locate in the downtown core area.

The tendency for hat sellers to cluster in the core area is reinforced by **shopping externalities** (explained in Chapters 2 and 5). If hats from different stores are **imperfect substitutes**, hat consumers travel to several stores to compare hats, and shopping cost is lower if the stores are clustered. Since the core area (the hub of the streetcar system) is accessible to the entire urban area, hatters are likely to cluster near the city center. If hats and gloves are **complementary goods,** consumers save on shopping costs if hat stores are near the glove store in the city center. In terms of central place theory, hat sellers compromise on their central place locations to exploit two types of shopping externalities: the externalities from comparison shopping and the externalities from one-stop shopping. Given the hub-and-spoke streetcar system, the retail clusters are likely to be in the downtown core area.

Residential Land Use

This section uses a simple model of the residential sector to explore residential land use in the monocentric city. According to the leftover principle, the bid rent for residential land equals the excess of total revenue over total cost. The first step in the analysis of residential land rent examines the revenue side of housing production. The housing-price function shows the relationship between housing prices and distance to the city center.

The simple model of the residential sector has a number of simplifying assumptions. Later in the chapter, each of these assumptions will be dropped.

1. One member of each household commutes to a job in the central business district (CBD).
2. Noncommuting travel is insignificant.
3. Public services and taxes are the same at all locations.
4. Air quality is the same at all locations.
5. All households have the same income and tastes for housing.
6. The opportunity cost of commuting time is zero.

The first four assumptions make the CBD the focal point of city residents. All jobs are in the CBD, while all the other things that people care about (public services, taxes, air quality) are distributed uniformly throughout the city. Given the fifth assumption, the choices of the "typical" household can be used to represent the choices of all households in the city. The sixth assumption means that the simple model ignores the time costs of commuting.

The Housing-Price Function

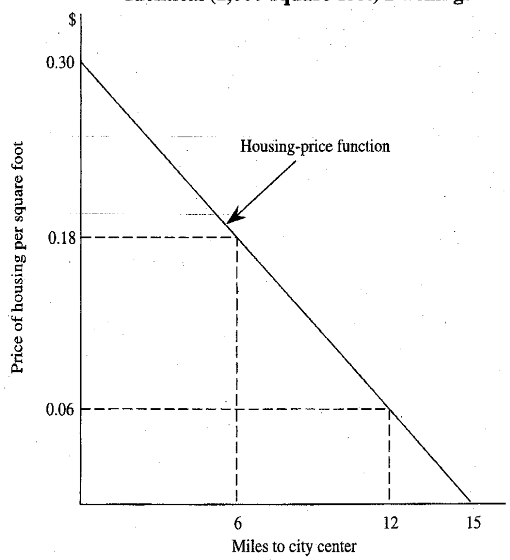
As explained in Chapter 14 (Why Is Housing Different?), the price of housing is usually defined as the price per unit of housing service. For the purposes of this chapter, the *price of housing* is defined as the price *per square foot* of housing per month. If a household rents a 1,000-square-foot house for \$250 per month, the price of housing is 25 cents per square foot (\$250 divided by 1,000 square feet). The **housing-price function** indicates how much a household is willing to pay for dwellings at different locations in the city. There are two types of housing-price functions, linear and convex.

Linear Housing-Price Function: No Consumer Substitution. Figure 8–4 shows the housing-price function for a simple case described by the following set of assumptions.

1. **Identical dwellings.** Every dwelling in the city has 1,000 square feet of living space.
2. **Fixed budget.** The typical household has a fixed budget of $300 per month to spend on commuting and housing costs.
3. **Commuting cost.** The monthly costs of commuting are $20 per mile per month: the household pays $20 per month in commuting costs for a residence one mile from the city center, $40 per month for a residence two miles from the city center, and so on.

How much is the household willing to pay for dwellings at different locations in the city? At the city center, commuting costs are zero, so the household can spend its entire $300 budget on housing. For a 1,000-square-foot house, the price is 30 cents per square foot. At a distance of six miles from the center, commuting costs

FIGURE 8–4 Housing-Price Function for a City with Identical (1,000-square-foot) Dwellings



The price of housing drops from 30 cents per square foot at the city center to 6 cents per square foot 12 miles from the city center. The price increases as commuting cost decreases (as distance to the center decreases), making households indifferent among all locations within the city.

are $120, so the household has $180 left to spend on housing (18 cents per square foot). In Figure 8–4, the slope of the function is two cents per mile.

The negatively sloped housing-price function is necessary for **locational equilibrium**. Locational equilibrium occurs when all households are satisfied with their location choices, that is, no household wants to change its location.

To explain why the equilibrium housing-price function is negatively sloped, suppose that the function starts out as a horizontal line. If the price of housing is 15 cents per square foot throughout the city, the household can get a 1,000-square-foot dwelling anywhere in the city for $150 per month. Suppose that a household starts out in a dwelling 10 miles from the city center. Because a move toward the city center decreases commuting costs without affecting rent, the household will move closer to the city center. Other households have the same incentive to move closer to the center. As the demand for housing near the city center increases, the price of housing near the center increases; as the demand for suburban housing decreases, the price of suburban housing decreases. In other words, the movement of households toward the city center transforms a horizontal housing-price function into a negatively sloped function.

The equilibrium housing-price function makes residents indifferent among all locations because differences in commuting costs are offset by differences in housing costs. The good news from a one-mile move toward the city center is that commuting costs decrease by t (commuting cost per mile). The bad news is that housing costs increase by ΔP (the change in the price per square foot) times H (housing consumption in square footage). The household will be indifferent between the two locations if the decrease in commuting costs equals the increase in housing costs:

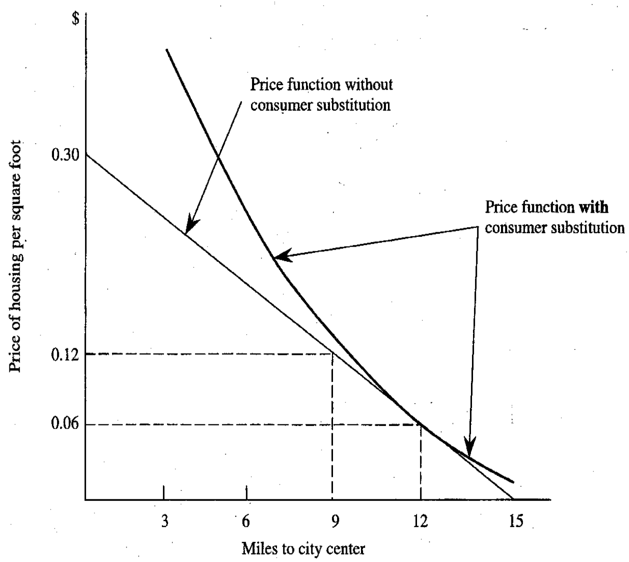
$$t = -\Delta P \cdot H \tag{8–9}$$

If t = $20 and H = 1,000 square feet, the household will be indifferent between the two locations if the price of housing increases by two cents per square foot. In Figure 8–4, the price of housing increases by two cents per mile as the household moves toward the city center.

Convex Function: Consumer Substitution. The housing-price function in Figure 8–4 is linear because the city's dwellings are identical: everyone lives in a 1,000-square-foot house, regardless of the price of housing. A more realistic assumption is that housing consumption depends on the price of housing. In other words, households obey the law of demand, decreasing the quantity demanded as the price increases. As a household moves toward the city center, it pays a higher price for housing, so it occupies a smaller dwelling. As the relative price of housing increases, the household substitutes nonhousing goods (pizza, hot dogs, stereo equipment) for housing.

Figure 8–5 shows the effects of consumer substitution on the housing-price function. The assumed pattern of housing consumption is shown below the graph. Suppose that a household moves from a distance of 12 miles (where the price of housing is 6 cents per square foot) to 9 miles. If housing consumption is fixed at 1,000 square feet, the household would be willing to pay an additional $60 for housing (the decrease in commuting cost), or 6 cents more per square foot. Because its housing consumption drops from 1,000 square feet to 750 square feet, the household is

FIGURE 8–5 Housing-Price Function with and without Consumer Substitution



As the household approaches the city center, the price of housing increases. If the household substitutes other goods for housing, the housing-price function is convex, not linear.

Assumed Consumption Pattern:

Distance to city center (miles)	3	6	9	12
Housing consumption (square feet)	400	600	750	1,000

willing to pay more than an additional 6 cents per square foot to offset the decrease in commuting costs. In general, as a household moves toward the high-priced city center, it occupies smaller dwellings, requiring progressively larger increases in the price *per square foot* of housing to offset the fixed \$20 per mile decrease in commuting costs. The lesson from Figure 8–5 is that if consumers obey the law of demand, the housing-price function is convex, not linear.

The slope of the housing-price function can be expressed in simple algebraic terms. Since both the price of housing (P) and housing consumption (H) vary with distance to the city center (u), the trade-off between commuting and housing costs can be rewritten as

$$\Delta u \cdot t = -\Delta P(u) \cdot H(u) \tag{8–10}$$

At a given location (u), the change in commuting cost (the change in u times the transport cost per mile) equals the change in the housing price times housing consumption. The equation can be rearranged to show the slope of the housing-price function:

$$\frac{\Delta P(u)}{\Delta u} = -\frac{t}{H(u)} \tag{8–11}$$

In the numerical example, t is \$20 and $H(9)$ is 750, so the slope of the housing-price function at $u = 9$ is \$0.0267 (20/750), compared to a slope of \$0.02 under the assumption of fixed housing consumption. Since $H(6)$ is 60, the slope at $u = 6$ is \$0.033 (20/600). As the household moves toward the city center, housing consumption decreases, increasing the slope of the housing-price function.

How rapidly does the price of housing decrease as distance to the city center increases? The *housing-price gradient* is defined as the percentage change in the price of housing per mile. Dividing both sides of (8–11) by P,

$$\frac{\Delta P/P}{\Delta u} = -\frac{t}{H(u) \cdot P(u)} \tag{8–12}$$

In words, the housing-price gradient equals transport cost per mile divided by housing expenditures. If the full cost of commuting (including monetary and time costs) is \$1 per round-trip mile, the monthly commuting cost (for 20 workdays per month) is \$20 per round-trip mile. If the household spends \$500 per month on housing, the rent gradient is 4 percent per mile (20/500).

The Residential Bid-Rent Function

The **residential bid-rent function** indicates how much housing producers are willing to pay for land at different locations in the city. According to the leftover principle, housing producers are willing to pay land rent equal to the excess of total revenue over total cost. There are two types of bid-rent functions, one that occurs if housing is produced with fixed factor proportions, and one that occurs if housing firms engage in factor substitution.

The Bid-Rent Function with Fixed Factor Proportions. Consider first the possibility that housing is produced with fixed factor proportions. The characteristics of firms in the housing industry are as follows:

1. **Production.** Each firm produces Q square feet of housing, using land and other inputs. Once the firm erects a building, it can be used as a single dwelling (with Q square feet of space), or divided into x units, each of which has (Q/x) square feet of living space.
2. **Nonland cost.** Firms use K worth of nonland inputs for each building.
3. **Fixed factor proportions.** Each firm produces its Q square feet of housing with T acres of land and K worth of other inputs, regardless of the price of land.
4. **Housing prices.** The housing-price function is negatively sloped and convex.
5. **Perfect competition.** The markets are perfectly competitive, so the firm makes zero economic profits.

According to the leftover principle, the bid rent for land equals the excess of total revenue over total nonland cost. Total revenue equals the price of housing (P) times Q, and total cost is nonland cost (K) plus land cost (R times T). Since P varies with the distance to the city center (u), the bid rent for land is

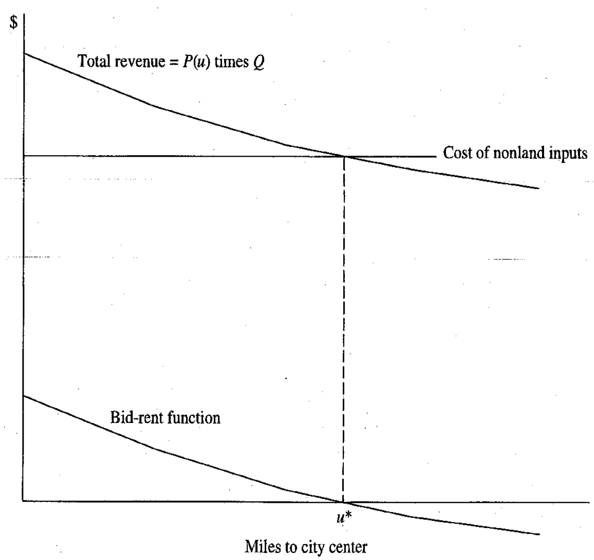
$$R(u) = \frac{P(u) \cdot Q - K}{T} \quad (8\text{–}13)$$

If the price of housing decreases as u increases, the residential bid-rent function is negatively sloped.

Figure 8–6 maps the residential bid-rent function. The horizontal line is nonland cost per acre, assumed to be the same at all locations. Since the bid rent equals total revenue less nonland cost, the bid-rent function lies below the revenue function, with the distance between the two equal to the cost of nonland inputs. At u^*, total revenue equals nonland cost, so the bid rent for land is zero. The bid-rent function is convex because the housing-price function is convex.

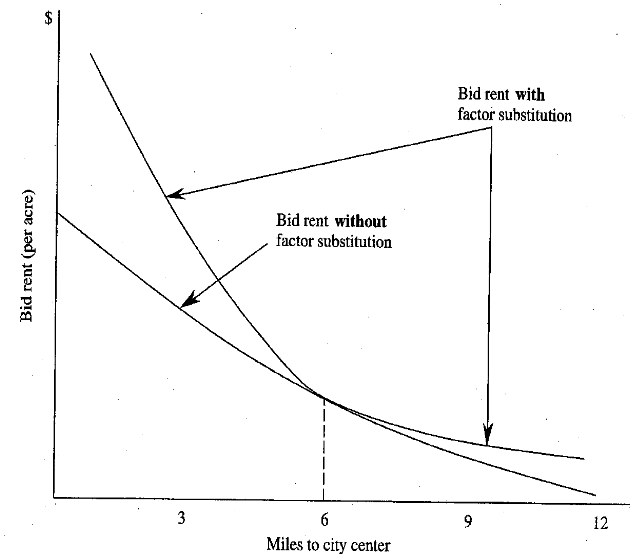
The Bid-Rent Function with Factor Substitution. The bid-rent function shown in Figure 8–6 is based on the assumption that housing is produced with fixed factor proportions. Housing firms use the same input combination at all locations, regardless of the price of land. What happens if firms substitute other inputs for land as the price of land increases?

FIGURE 8–6 Housing-Price and Bid-Rent Function



The bid rent of the housing firms equals total revenue per acre less the cost of nonland inputs. Total revenue (the price of housing times square footage produced) decreases as the distance to the city center increases because the housing-price function is negatively sloped. The cost of nonland inputs is the same at all locations. The bid-rent function is convex because the housing-price function (and the revenue function) is convex. At u^*, the cost of nonland inputs equals total revenue, so the bid rent equals zero.

FIGURE 8–7 The Residential Bid-Rent Function with Factor Substitution



Factor substitution (substituting nonland inputs for land as the price of land increases) increases the convexity of the bid-rent function.

Figure 8–7 shows the bid-rent functions for inflexible and flexible housing producers. The inflexible firm uses the same input combination throughout the city. In contrast, the flexible firm substitutes nonland inputs for land as the price of land increases, building progressively taller buildings as it approaches the city center. The flexible rent function lies above the inflexible rent function at every location except $u = 6$. At this location, the input combination of the inflexible firm is, by chance, the efficient combination, so the two builders use the same input combination. While the inflexible firm's input ratio is efficient for $u = 6$, it is inefficient for other locations (too low for locations closer to the city center and too high for locations farther from the city center). For all locations except $u = 6$, the flexible firm produces housing for a lower cost and thus outbids the inflexible firm.

Summary: The Convex Bid-Rent Function. There are two lessons from the analysis of residential land rent. First, the bid-rent function is negatively sloped because the housing-price function is negatively sloped. Second, the bid-rent function is convex because of both consumer substitution (which makes the housing-price function convex) and factor substitution (which increases the convexity of the rent function).

How rapidly does the price of residential land decrease as distance to the city center increases? The **rent gradient** is defined as the percentage change in land rent (or market value) per mile. The gradient depends on (1) the housing-price gradient

TABLE 8–3 Land-Rent Gradient

	Location	
	A	*B*
Distance to city center (miles)	4	5
Market value of housing ($)	150,000	144,000
Land value ($)	30,000	24,000
Capital value ($)	120,000	120,000

Assumptions:
1. At location A, land value is 20 percent of the market value of housing.
2. Housing-price gradient is 4 percent per mile.

and (2) the relative importance of land in the production of housing. In Table 8–3, the housing-price gradient is 4 percent (a one-mile move away from the city center decreases the market value of housing by 4 percent, from $150,000 to $144,000) and the value of land is assumed to be 20 percent of the total property value at location A. Because the price of capital is the same at all locations, land absorbs the entire $6,000 decrease in market value, dropping from $30,000 to $24,000, a 20 percent decrease. Since the market value is simply the present value of the annual rental income (annual rent divided by the interest rate), the rent gradient (percentage change in land rent per mile) is 20 percent, or five times the housing-price gradient.

The relationship between the housing-price gradient and the rent gradient can be stated algebraically as

$$\text{Rent gradient} = \frac{1}{\text{Land's share of house value}} \cdot \text{Housing-price gradient} \quad (8\text{–}14)$$

The smaller the land's share of house value, the larger the percentage decrease in land rent needed to absorb a given decrease in the price of housing. For example, if land's share of house value is 10 percent, the rent gradient is 10 times the housing-price gradient.

Residential Density

How does population density vary within the monocentric city? Table 8–4 shows how to compute population density at different locations in the city. The first step is to compute the lot size (the amount of land occupied per household). Lot size increases with distance to the city center for two reasons:

1. **Consumer substitution.** The price of housing decreases as the distance to the city center increases, and households respond to lower housing prices by consuming more housing. In Table 8–4, housing consumption rises from 1,404 square feet for a household 0.20 miles from the city center (location A) to 3,000 square feet for a household four miles from the center (location B).

TABLE 8–4 Population Density at Different Locations

	Location	
	A	*B*
Distance to city center (miles)	0.2	4.0
Housing consumption (square feet)	1,404	3,000
Land per square foot of housing (square feet)	0.33	2.2
Lot size (square feet)	468	6,600

2. **Factor substitution.** The price of land decreases as the distance to the city center increases, and housing firms respond to lower land prices by using more land per unit of housing. In Table 8–4, at a distance of 0.20 miles from the city center, every square foot of living space comes with 0.33 square feet of land. In other words, people live in three-story apartment buildings. At a distance of four miles from the center, the amount of land per square foot of housing is 2.20: households live in one-story houses with lot sizes 2.2 times the "footprint" of the house.

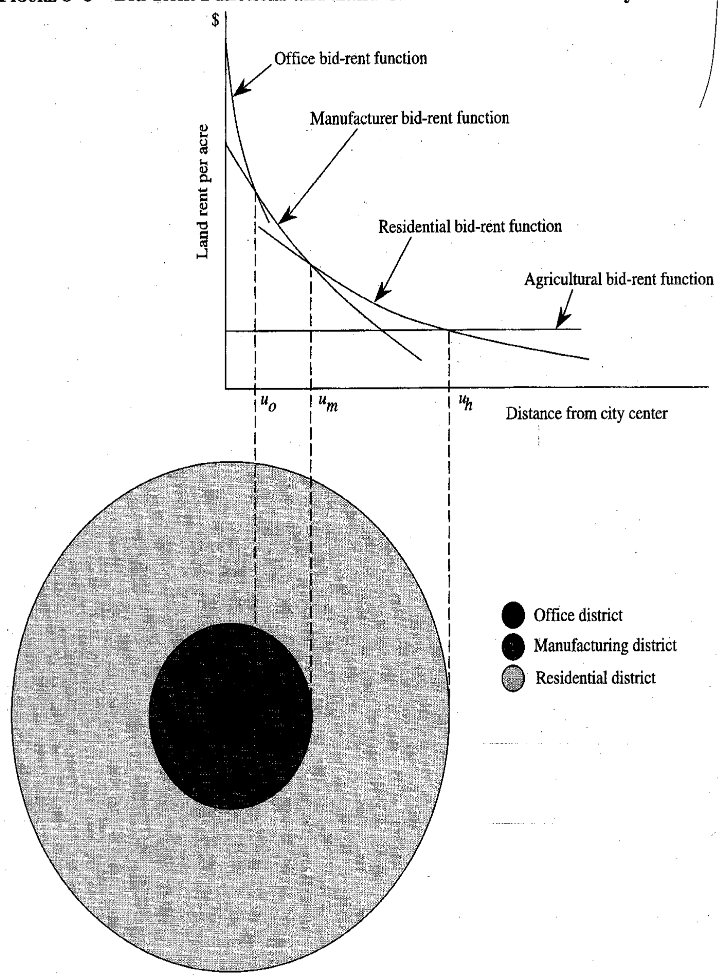
Lot size equals housing consumption (in square feet of living space) times the amount of land per unit of housing. Because of consumer substitution and factor substitution, the lot size increases as distance increases: a household located 0.20 miles from the center uses only 468 square feet of land (sharing the 1,404 square feet under the three-story apartment building with two other households), while a household located 4 miles from the center uses 6,600 square feet of land. In this example, residential density at a location 0.20 miles from the city center is about 14 times the density 4 miles from the center.

Land Use in the Monocentric City

Figure 8–8 shows the land-use pattern of the monocentric city. The office bid-rent function intersects the manufacturing function at a distance of u_o miles from the city center, so the office district is a circle with radius u_o miles. The manufacturing bid-rent function intersects the residential function at a distance of u_m miles from the city center, so the manufacturing district is a ring of width $(u_m - u_o)$ miles. The residential bid-rent intersects the agricultural bid-rent function at a distance of u_h miles, so the residential district is a ring of width $(u_h - u_m)$ miles. The retail bid-rent function is omitted from this diagram in the interests of simplicity. As explained earlier, most retailers congregate at the center of the monocentric city.

Activities are arranged according to their transportation costs: the higher the transportation cost, the closer to the city center. As explained in Chapter 7, the activity with relatively high transport costs has a relatively steep bid-rent function, and thus locates closer to the marketplace. In the monocentric city, the market is the city center, where office workers meet with clients and manufacturers load their output

FIGURE 8–8 Bid-Rent Functions and Land Use in the Monocentric City



onto ships or trains. The office sector, with the highest transport costs and thus the steepest bid-rent function, occupies land closest to the center. Manufacturing, with the next highest transport costs and thus the next steepest bid-rent function, occupies the next ring of land. The residential sector, with relatively low transport costs and thus a relatively flat bid-rent function, occupies the land farthest from the city center.

This spatial arrangement has two interesting features. First, office firms occupy the central area of the CBD. As explained earlier in the chapter, office firms have

relatively high transport costs and thus a relatively steep bid-rent function because office output is transmitted by office workers, while manufacturing output is transported by horse-drawn wagon.

The second feature of the monocentric city is that employment is concentrated in the CBD, not distributed throughout the city. Why do all the manufacturers and office firms locate in the CBD? To explain this monocentric location pattern, consider a baseball firm that is considering a move from the CBD to a suburban location. What are the trade-offs associated with a move to the suburbs?

1. **Higher freight costs.** The firm will be farther from the central export node, so it will pay higher freight costs.
2. **Lower wages.** The firm will be closer to its work force, so workers will commute shorter distances. The wage compensates workers for commuting costs: the longer the commuting distance, the higher the wage. When the firm moves closer to its work force, it decreases its workers' commuting costs, so the firm can pay a lower wage.

The firm's location decision is determined by the outcome of a tug-of-war. On one side is the central export node, which pulls the firm toward the CBD. On the other side is its suburban work force, which pulls the firm toward the low-wage suburbs.

In the monocentric city, the tug-of-war was won by the CBD because the cost of moving freight was large relative to the cost of moving workers. Freight traveled by horse-drawn wagon, a relatively slow and expensive travel mode. In contrast, workers traveled by streetcar, a relatively fast and inexpensive travel mode. Although wages were lower in the suburbs, the wage differential was relatively small because commuting by streetcar was fast and efficient: workers demanded a relatively small premium to commute to CBD jobs. If a firm moved to the suburbs, the small savings in wages would be more than offset by an increase in the cost of shipping freight to the central export node. In the monocentric city, it was cheaper to bring the workers from the suburbs to the central-city factory than to bring the output from a suburban factory to the central export node.

The same analysis applies to office firms. Although a move from the CBD to the suburbs would decrease wages, it would also increase the costs of travel between the office and the central market area: managers would spend more time traveling between the office and clients in the city center. Given the frequency of travel to clients in the central market area, the increase in travel costs would dominate the savings in wages. In the monocentric city, it was cheaper to bring the workers from the suburbs to a central-city office than to bring the output (in the minds and briefcases of managers) from a suburban office to clients in the city center.

Relaxing the Assumptions

The simple monocentric model has a number of unrealistic assumptions about residential land use. This section derives the residential bid-rent function under more realistic sets of assumptions. The assumption that the city is monocentric is maintained: all employment is assumed to be in the CBD.

Changes in Commuting Assumptions

The simple model is based on a number of simplifying assumptions about commuting. What happens if these assumptions are dropped?

1. **Time cost of commuting.** The simple model assumes that the only cost of commuting is a monetary cost, that is, money spent on cars (for gasoline and maintenance) or public transit (bus tickets). In fact, commuting time comes at the expense of work or leisure, so there is an opportunity cost associated with commuting. The unit cost of commuting (t) is actually the monetary *and* time costs per mile of travel. Studies of commuting behavior suggest that most people value commuting time at between one third and one half the wage rate. For a worker with a wage of $10, the time cost of commuting is between $3.33 and $5.00 per hour. Commuting costs are discussed in greater detail in chapters 19 (Autos and Highways) and 20 (Mass Transit).
2. **Noncommuting travel: uniform distribution of destinations.** The simple model assumes that noncommuting travel is insignificant. This assumption is unrealistic because households travel to different destinations within the city for shopping and entertainment. Suppose that shopping and entertainment destinations are distributed uniformly throughout the urban area. For example, the household commutes northward to a job in the city center and also travels north to a museum, south to a grocery store, west to a disco, and east to the shore. If the frequency and distance of travel to the four sites are about the same, any change in residence causes a relatively small change in total noncommuting travel time. If the household moves south, the cost of museum travel increases, but the cost of the grocery travel decreases. If the household travels in all directions for shopping and entertainment, noncommuting costs usually offset one another, and it is appropriate to focus on commuting as the primary factor in the location decision.
3. **Noncommuting travel: concentrated destinations.** Consider next the possibility that shopping and entertainment sites are concentrated rather than dispersed. Suppose that members of a household travel to the city center for work, shopping, and entertainment. As the household moves closer to the city center, it saves on travel costs for commuting, shopping, and entertainment, so the savings in travel cost are relatively large and the housing-price function is relatively steep. In general, the more frequent the travel to the city center, the steeper the housing-price function and the residential bid-rent function.
4. **Two-earner households.** The simple model assumes that a single person from each household commutes to the city center. Suppose that all the households in a city are suddenly transformed into two-earner households. What happens to the housing-price function? If two members of each household commute to the CBD and have the same commuting cost per mile, a household that moves closer to the city center experiences double the savings in commuting costs. In equilibrium, the housing-price function and the bid-rent function will be steeper in the two-earner city, reflecting the greater savings associated with living closer to central-city jobs.

Variation in Tastes for Housing

The simple model assumes that every household has the same tastes for housing. The housing-price function of the "typical" resident is used to represent the housing-price function of the entire city. Suppose that there are two types of households in the city, large and small, and that all households have the same income. The "tastes" for housing are dictated by the number of children: the small household lives in a small dwelling, and the large household lives in a large dwelling.

Where in the city will the two types of households live? Since land is rented to the highest bidder, the division of residential land between the two household types is determined by the bid-rent functions of the two groups. As shown in Chapter 7, the land user with the steeper bid-rent function occupies land closer to the city center. Since the residential bid-rent function is determined by the housing-price function, the user with the steeper housing-price function has a steeper bid-rent function.

Which household has the steeper housing-price function? The expression for the housing-price function is

$$\frac{\Delta P(u)}{\Delta u} = \frac{-t}{H(u)} \tag{8–15}$$

If the two households have the same commuting cost per mile (t), the small household has a steeper housing-price function because it consumes less housing (smaller H). Because the small household consumes a smaller amount of housing (in square feet), it takes a larger change in the price of housing per square foot to compensate for an increase in commuting cost.

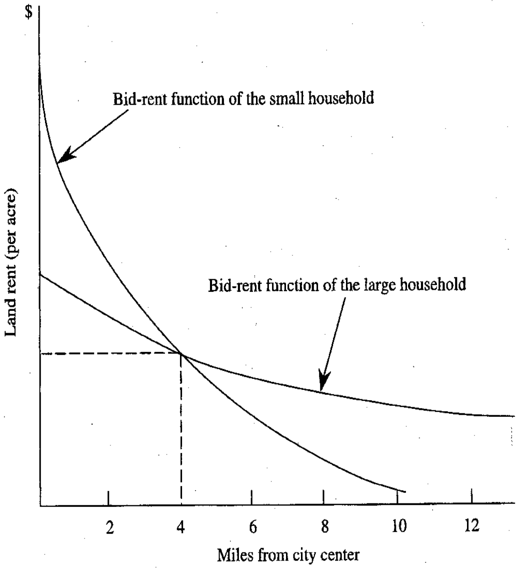
Figure 8–9 shows the bid-rent functions for the two households. The bid-rent function of the small household is steeper because its housing-price function is steeper. The two functions intersect at a distance of four miles from the city center, so small households occupy dwellings within four miles of the city center, and large households occupy dwellings outside the four-mile radius. Large households occupy low-price suburban housing because they live in large houses and thus have more to gain from inexpensive suburban housing.

Spatial Variation in Public Goods and Pollution

The simple model has a number of assumptions that make the city center the focal point of the city. Except for jobs in the CBD, all the things that people care about (public services, taxes, pollution, amenities) are distributed uniformly throughout the city. What happens if these things are not distributed uniformly?

Public Goods and Taxes. Taxes and public services vary within a metropolitan area. Suppose that the quality of public schools varies within the city, but the cost

FIGURE 8–9 Bid-Rent Function and Family Size



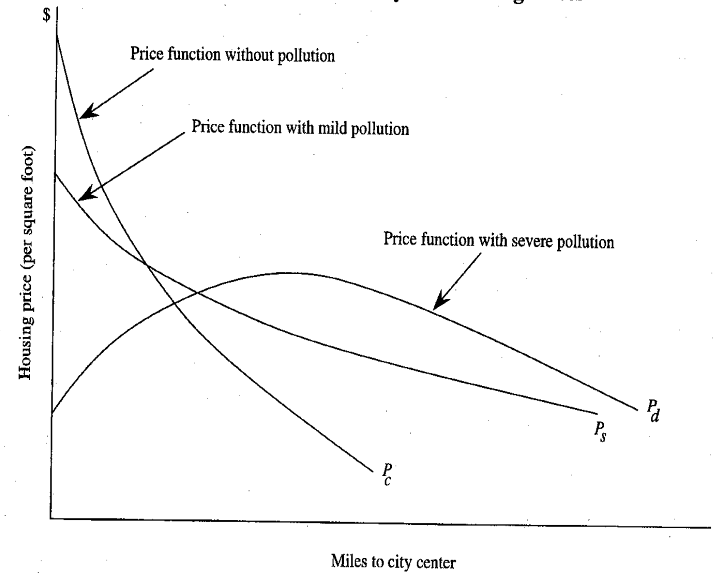
The slope of the bid-rent function decreases as housing consumption increases, so large households, who live in large houses, have relatively flat bid-rent functions. Large families have the most to gain from low suburban housing prices, so they outbid small households for suburban housing.

of schools (tuition and taxes) is the same throughout the city. In equilibrium, the price of housing is higher in the communities with better schools. Parents pay for better public schools indirectly: instead of paying higher taxes, they pay more for housing and residential land. Similarly, the prices of housing and land are higher in communities with lower crime rates. The same argument applies to variation in taxes. If two communities have the same level of public services but one community has higher taxes, the price of housing is higher in the low-tax community.

Pollution and Amenities. The simple model assumes that environmental quality is the same at all locations in the city. To explain the effects of pollution on housing and land prices, suppose a polluting factory moves into the center of a previously clean city. If the smoke and smell from the factory are heaviest in the central area of the city, the factory decreases the relative attractiveness of dwellings near the city center, decreasing the price of housing. In addition, the factory increases the relative attractiveness of more remote dwellings, increasing the price of suburban housing.

Figure 8–10 shows the effects of the polluting factory on the housing-price function. P_c is the price function in the absence of pollution (the clean city), and P_s is the price function with a small amount of pollution. The pollution from the central-city factory decreases the slope of the housing-price function. As a household moves toward the city center there are costs (more pollution) as well as benefits (lower

FIGURE 8–10 Pollution in the Central City and Housing Prices



Air pollution from a central-city factory decreases housing prices near the city center and increases housing prices far from the city center. The more severe the pollution, the greater the change in housing prices.

commuting costs), so in the polluted city, the price of housing increases less rapidly as one approaches the city center. If the city has a high level of pollution, the housing-price function may be positively sloped, as shown by P_d. In this case, central-city pollution is so obnoxious that the advantages of a central-city dwelling (lower commuting costs) are dominated by its disadvantages (greater exposure to pollution). As a result, people are willing to live near the city center only if they are compensated in the form of lower housing prices.

Changes in the housing-price function cause similar changes in the residential bid-rent function. A relatively flat housing-price function (P_s) generates a relatively flat bid-rent function. Similarly, a positively sloped housing-price function (P_d) generates a positively sloped residential bid-rent function.

The same arguments apply to locations that have positive locational attributes (amenities) such as scenic views or access to parks. If people get utility from scenic views or park access, they are willing to pay more for dwellings that provide such amenities.

Income and Location

In U.S. cities, the wealthy tend to locate in the suburbs, and the poor tend to locate near the city center. In other words, average household income increases as one moves away from the city center. Because the most expensive land is near the city center, this location pattern is puzzling: why should the poor occupy the most expensive housing and land? There are several theories of this observed pattern of income segregation. The first is based on the simple monocentric model, and the others are based on extensions of the monocentric model.

Trade-off between Land and Commuting Costs

According to the simple monocentric model, a household chooses the location that provides the best trade-off between land costs and commuting costs. One theory of income segregation, developed by Alonso (1964) and Muth (1969), suggests that central locations provide the best trade-off for the poor, while suburban locations provide the best trade-off for the wealthy.

Table 8–5 shows the trade-offs between land costs and commuting costs for a household with the following characteristics:

1. The household takes the residential land-rent function as given. The second column of the table shows the land rent (per month per acre) for different locations, and the third column shows the changes in land rent for one-mile moves away from the city center.
2. Land consumption by the household is 0.20 acres, regardless of location (the fourth column in the table).
3. Commuting cost is $40 per round-trip mile per month.

The marginal benefit of distance, defined as the decrease in the household's land cost from a one-mile move outward, equals the decrease in land rent times land consumption. For example, a one-mile move away from the city center decreases land rent per acre by $700 and decreases land cost by $140 (0.20 times $700). The marginal benefit decreases as we move down the table because land rent falls at a decreasing rate: the land-rent function is convex. The marginal cost of a one-mile move outward, defined as the increase in commuting cost, equals the commuting cost

TABLE 8–5 Trade-Offs between Land Cost and Commuting Cost

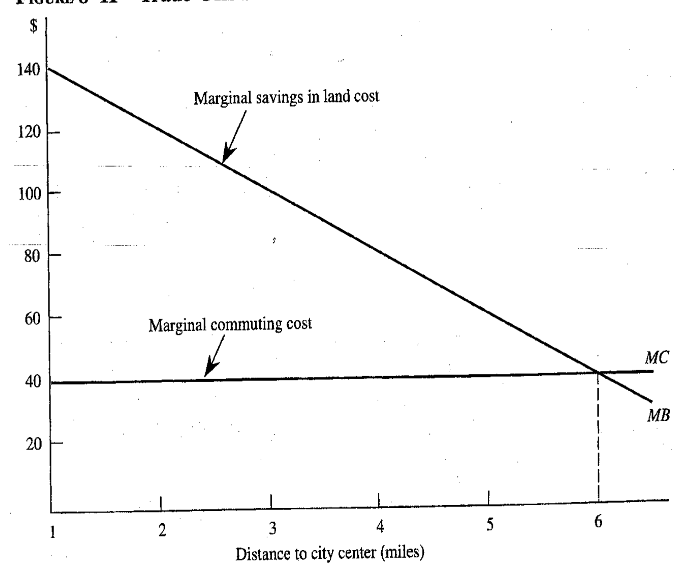
Distance to City Center (miles)	*Land Rent per Acre*	*Decrease in Land Rent*	*Land (acres)*	*Marginal Benefit*	*Marginal Cost*
0	$3,800				
1	3,100	$700	0.2	$140	$40
2	2,500	600	0.2	120	40
3	2,000	500	0.2	100	40
4	1,600	400	0.2	80	40
5	1,300	300	0.2	60	40
6	1,100	200	0.2	40	40
7	1,000	100	0.2	20	40

per mile per month ($40). Suppose that the household tentatively decides to live in the city center. Given the numbers in the table, a one-mile move outward would decrease land cost by more than it would increase commuting cost ($140 versus $40), so a central-city location is clearly inferior to a location one mile from the city center.

The optimum location is where the marginal benefit from a one-mile move outward (the savings in land cost) equals the marginal cost (the increase in commuting cost). In Table 8–5, the optimum location is six miles from the city center. At any location closer to the center, the marginal benefit exceeds the marginal cost, so the household will be better off at a more distant location. At six miles, the marginal benefit equals the marginal cost.

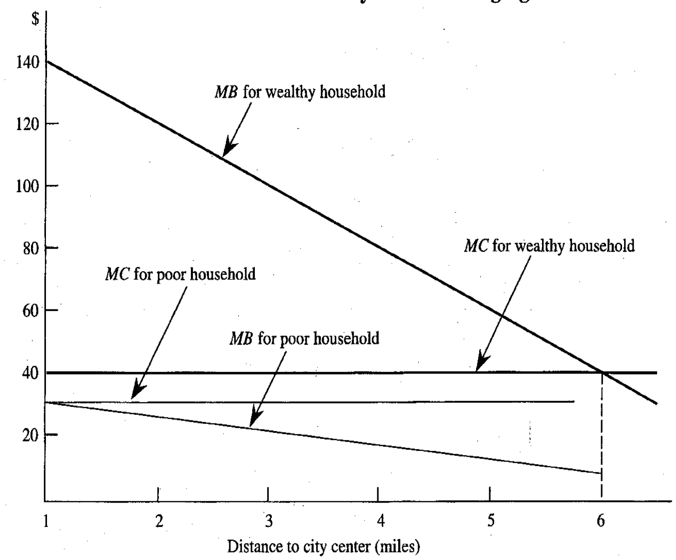
Figure 8–11 shows the benefit and cost curves from Table 8–5. The optimum location is where the marginal-benefit curve intersects the marginal-cost curve. The position of the marginal-benefit curve is affected by the household's land consumption: the larger the lot, the larger the benefit associated with lower land rent at more remote locations. In Figure 8–11, an increase in land consumption shifts the benefit curve upward, increasing the optimum distance. The position of the cost curve is determined by commuting costs: an increase in commuting costs shifts the cost curve upward and decreases the optimum distance.

FIGURE 8–11 Trade-Offs between Land Cost and Commuting Cost



The optimum location is where the marginal benefit of distance (*MB*) equals the marginal cost (*MC*). The marginal benefit equals the decrease in land rent times land consumption. The marginal-benefit curve is negatively sloped because the land-rent function is convex. The marginal cost equals the increase in commuting cost per mile.

FIGURE 8–12 The Alonso-Muth Theory of Income Segregation



If the income elasticity of demand for land is large relative to the income elasticity of commuting cost, the gap between the marginal-benefit curves will be larger than the gap between the marginal-cost curves. Therefore, the poor live near the central city, and the wealthy live in the suburbs.

Under what circumstances will the poor locate near the central city, while the wealthy locate in the suburbs? Figure 8–12 shows the benefit and cost curves underlying the Alonso-Muth theory of income segregation. Consider the location choice of a poor household with the following characteristics:

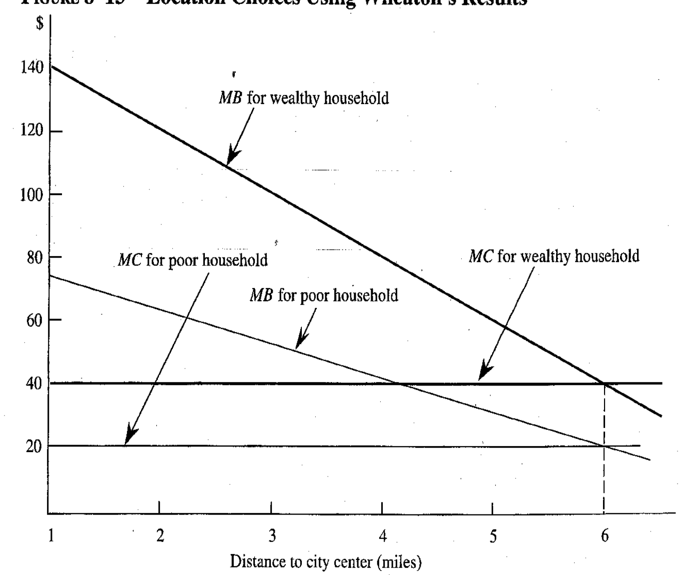
1. The household has one fifth the income of the wealthy household whose characteristics are shown in Table 8–5.
2. The poor household consumes one fifth as much land as the wealthy household (0.04 acres). In other words, land is a normal good, with an income elasticity of demand (the percentage difference in land consumption divided by the percentage difference in income) equal to 1.0.
3. The commuting cost of the poor household is 70 percent the commuting cost of the wealthy household ($28 per month per mile). The poor household has a lower commuting cost because it has a lower wage and thus a lower opportunity cost of commuting.

In Figure 8–12, the optimum location for the wealthy household is six miles from the city center, and the optimum location for the poor household is one mile from the center.

What are the assumptions underlying this theory of income segregation? The benefit and cost curves in Figure 8–12 are drawn under the assumption that the income elasticity of demand for land is large relative to the income elasticity of commuting cost (the percentage difference in commuting cost divided by the percentage difference in income). Although both land consumption and commuting cost increase with income, the increase in land consumption is relatively large. Therefore, the gap between the two marginal-benefit curves is larger than the gap between the marginal-cost curves, so the poor occupy central-city housing.

Wheaton (1977) provides empirical evidence that questions the validity of the Alonso-Muth model of income segregation. His results suggest that the income elasticity of demand for land equals the income elasticity of commuting cost. Therefore, an increase in income shifts the benefit and cost curves upward by about the same amount (in percentage terms). In Figure 8–13, the poor household (with one fifth the income of the wealthy household) has half the land consumption and half the commuting cost of the wealthy household. There is a 50 percent gap between the benefit curves of the two households, and the same gap between the cost curves, so the optimum location for both households is six miles from the city center. This result suggests the observed locational pattern (poor central-city residents and wealthy suburbanites) cannot be explained by the trade-off between commuting cost and land cost. Wheaton's results suggest that one must look beyond the simple monocentric model to explain the observed pattern of income segregation.

FIGURE 8–13 Location Choices Using Wheaton's Results



If the income elasticity of demand for land equals the income elasticity of commuting cost, the gap between the marginal-benefit curves equals the gap between the marginal-cost curves (in percentage terms). Therefore, the simple monocentric model predicts that location choices are unaffected by income: both households in the example pick a location six miles from the city center.

Other Explanations

Some alternative explanations of income segregation are based on factors excluded from the simple monocentric model.

1. **New suburban housing.** Suppose that the utility generated from a particular dwelling decreases over time as the dwelling deteriorates and becomes obsolete. In other words, people get less utility out of an older house because it is less fashionable, has higher maintenance costs, and is equipped with fewer modern gadgets. The wealthy, who demand high-quality housing, occupy new housing instead of used housing. As an urban area grows, it expands outward, and developers build new housing for high-income households in the peripheral areas. The poor are left with old houses in the central city.
2. **Fleeing central-city problems.** As explained later in the book, poverty contributes to three urban problems. First, crime rates are higher among the poor, in part because the poor face a relatively low opportunity cost of committing crime. Second, fiscal problems are more likely in a jurisdiction with a large fraction of low-income citizens. Third, students from poor families have relatively low achievement levels and pull down the achievement levels of other students. To escape these problems, wealthy households flee to the suburbs, leaving large concentrations of poor households behind.
3. **Suburban zoning.** As explained in Chapter 11 (Land-Use Controls and Zoning), suburban governments use zoning to exclude low-income households. Therefore, only the wealthy have the opportunity to escape the problems of the central city.

Income and the Residential Bid-Rent Function

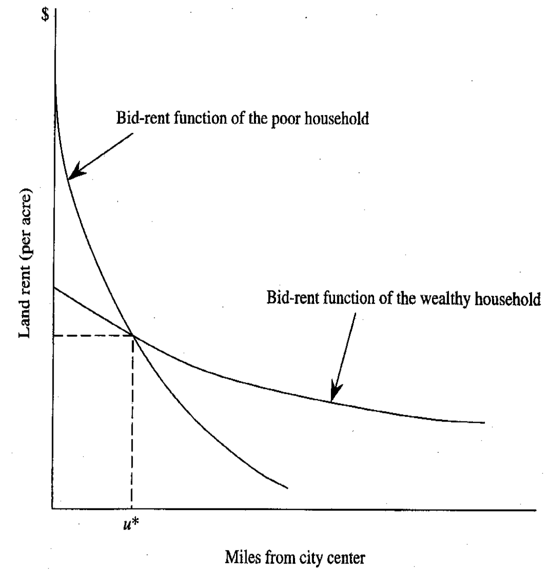
The issue of income segregation can also be explained with the housing-price function and the residential bid-rent function. As explained earlier in the chapter, the activity with the steeper bid-rent function occupies land closer to the city center. Since the residential bid-rent function is determined by the housing-price function, the poor occupy central land if they have a steeper housing-price function. Figure 8–14 shows two residential bid-rent functions, one for low-income housing and one for high-income housing. The low-income function is steeper because the poor have a steeper housing-price function.

Why do the poor have a steeper housing-price function? In the simple monocentric model, the expression for the slope of the housing-price function is

$$\frac{\Delta P}{\Delta u} = -\frac{t}{H(u)} \tag{8–16}$$

An increase in income increases both t (the opportunity cost of commuting) and H (housing consumption), so rising income has an ambiguous effect on the slope of the

FIGURE 8–14 Bid-Rent Function and Income



The wealthy have a flatter bid-rent function, so the poor occupy central land (land less than u^* miles from the city center). The wealthy have a flatter bid-rent function because they are sensitive to crime, pollution, and the quality of schools, and the central cities have more crime and pollution, and inferior schools.

housing-price function. If the increase in H exceeds the increase in t (if the income elasticity of demand for housing exceeds the income elasticity of commuting cost), the wealthy have a flatter housing-price function and a flatter residential bid-rent function. This is the Alonso-Muth theory of income segregation.

Wheaton's results suggest that the slope of the housing-price function is independent of income. If H and t increase at the same rate as income increases, the two housing-price functions have the same slope, so the two residential bid-rent functions have the same slope. Therefore, if the poor have a relatively steep bid-rent function, it is not because of the trade-off between commuting costs and housing costs.

The alternative explanations of income segregation suggest that the slope of the residential bid-rent function is affected by other factors. Specifically, if central cities have higher taxes, inferior schools, and more pollution and crime, households are willing to pay more for housing and land in the suburbs. In other words, the problems of the central city decrease the slope of the bid-rent function. If the income elasticities of demand for safety, clean air, and education are relatively large, the bid-rent function of wealthy households will be flatter than the bid-rent function of poor households. In other words, if the wealthy are willing to pay much more than the poor for safety, clean air, and superior education, wealthy households will outbid poor households for land in areas that are relatively safe and clean and provide high-quality education. Wasylenko (1984) summarizes the empirical evidence supporting these alternative explanations of income segregation.

Policy Implications

These alternative theories of income segregation suggest that public policy can affect the location choices of wealthy and poor households. A housing policy that encourages the renovation of central-city housing stock may cause some high-income households to return to the central city. Policies that decrease poverty decrease crime rates, reduce fiscal problems, and improve central-city schools, encouraging high-income households to live in the central city. Similarly, policies that address the crime and education problems directly increase the relative attractiveness of central-city locations. Finally, policies that control exclusionary zoning allow the poor to move to the suburbs.

Empirical Estimates of Rent and Density Functions

This section discusses empirical studies of land rent and land use in the monocentric city. The studies are based on data from the early 20th century, during the heyday of the monocentric city. Chapter 10 (Suburbanization and Modern Cities) discusses land-use patterns in modern cities. One of the key questions in Chapter 10 is: how "monocentric" are modern cities? In other words, are the patterns of land rent and land use in modern cities roughly consistent with the patterns predicted by the monocentric model?

Estimates of the Land-Rent Function

A number of researchers have estimated the relationship between land rent and distance to the city center. Mills (1969) used data collected by Homer Hoyt to estimate the relationship between land *value* and distance. As explained in Chapter 7, land value is the present value of land rent, so it's easy to make the translation from value to rent. Mills assumes the following relationship between value and distance:

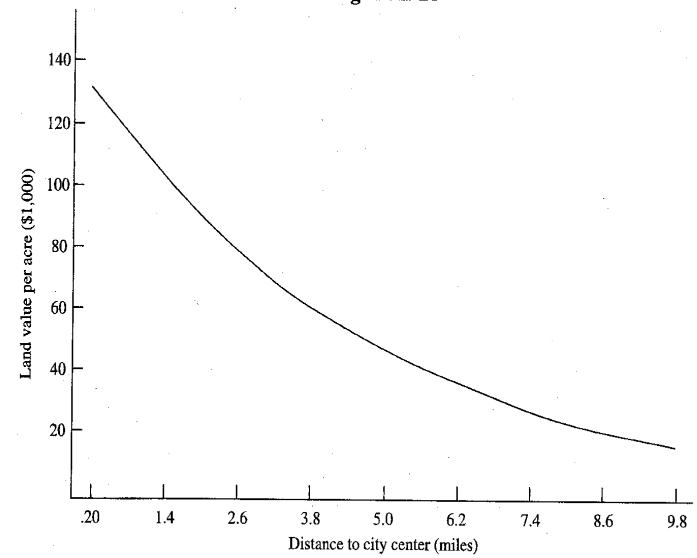
$$V(u) = B \cdot e^{-c \cdot u} \qquad (8\text{–}17)$$

where

$V(u)$ = Value of land u miles from the city center
B = parameter to be estimated from the data
e = Base of the natural logarithm
c = parameter to be estimated from the data

Figure 8–15 shows the estimated relationship for Chicago in 1928, when the city was monocentric. The value of land drops from about $140,000 per acre at the city center to about $114,000 at 1 mile from the center, to about $17,000 at 10 miles from the city center. The value of land falls by 21 percent per mile, that is, the **rent gradient** is 21 percent.

FIGURE 8–15 Value of Land in Chicago in 1928



SOURCE: Edwin S. Mills, "The Value of Urban Land," in *The Quality of the Urban Environment,* ed. H. Perloff (Washington, D.C.: Resources for the Future, 1969).

Estimates of the Population-Density Function

How does population density vary within the monocentric city? The density function describes the relationship between population density and distance to the city center. Mills (1972) has estimated the density functions for 18 metropolitan areas for different years. The assumed relationship between density and distance is

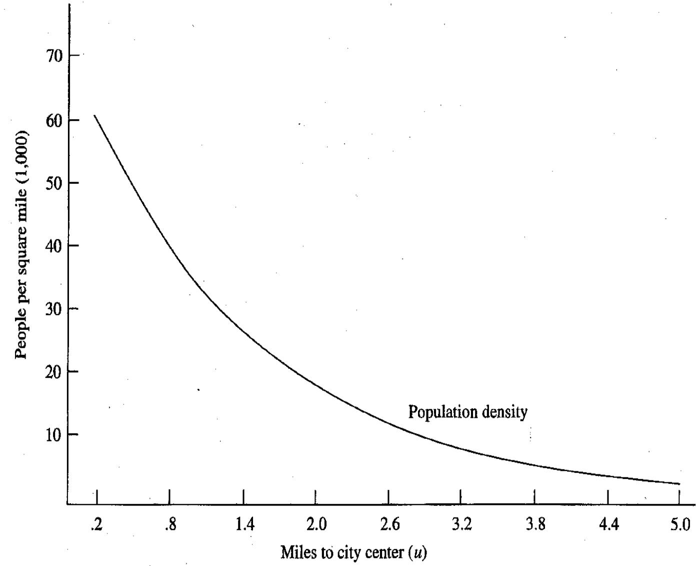
$$D(u) = A \cdot e^{-g \cdot u} \qquad (8\text{–}18)$$

where

$D(u)$ = Population density u miles from the city center (people per square mile)
A = Parameter to be estimated from the data
e = Base of the natural logarithm
g = Parameter to be estimated from the data

Figure 8–16 shows the estimated relationship for Baltimore in 1920, when the city was monocentric. Population density drops from about 60,000 people per square mile at a distance of 0.20 miles from the city center, to about 34,000 at a distance of 1 mile, to about 4,200 at a distance of 4 miles. The density gradient, defined as the percentage change in population density per mile, is about 70 percent.

Figure 8–16 Population Density in Baltimore in 1920



The density function is negatively sloped because (1) housing consumption increases with u (a result of declining housing prices) and (2) land per unit of housing increases with u (a result of declining land prices).
Source: Edwin S. Mills, *Studies in the Structure of the Urban Economy* (Baltimore: Johns Hopkins, 1972).

Summary

1. The monocentric city has the following characteristics:
   a. All manufacturers export their output through a central export node.
   b. Manufactured goods are transported within the city by horse-drawn wagon.
   c. Office workers travel by foot from offices to a central market area to exchange information.
   d. Commuters and shoppers travel on a hub-and-spoke streetcar system.
2. The manufacturing bid-rent function is negatively sloped because transport cost is lower near the export node. It is convex because of factor substitution.
3. The office bid-rent function is negatively sloped because travel costs are lower near the central market area. It is convex because of factor substitution.
4. Transport costs in the monocentric city are relatively high for office firms, so the office bid-rent function is relatively steep and office firms occupy the central area of the city.

5. The location choices of retailers depend on scale economies, per capita demand, and shopping externalities. In the monocentric city, the hub-and-spoke streetcar system makes central locations accessible to the entire urban area, causing most retailers to locate there.
6. In the simple monocentric model, all employment is in the CBD, and other things that residents care about (e.g., public services, taxes, pollution) are the same at all locations within the city.
7. The housing-price function shows the price of housing (per square foot of living space) for different locations in the city. The function is negatively sloped because commuting costs increase with the distance to the city center. It is convex because of consumer substitution: as the price of housing rises, consumers substitute other goods for housing.
8. The residential bid-rent function shows the amount housing producers are willing to pay for residential land at different locations in the city. It is negatively sloped because the housing-price function is negatively sloped. It is convex because of consumer substitution (which makes the housing-price function convex) and factor substitution.
9. The density function shows the number of people per acre for different locations in the city. It is negatively sloped for two reasons.
   a. Housing prices are higher near the city center, so housing consumption is lower (fewer square feet of living space per household).
   b. Land prices are higher near the city center, so the amount of land per square foot of housing is lower.
10. Activities in the monocentric city are arranged according to their transportation cost: the higher the transportation cost, the closer to the city center.
    a. The city center is occupied by the office sector rather than the manufacturing sector because office output is transmitted by high-cost office workers, while manufacturing output is transported by horse-drawn wagon.
    b. Employment is concentrated in the CBD because the cost of commuting (from the suburbs to the CBD factories and offices) is low relative to the cost of moving output (from the suburbs to the city center). Freight travels by horse-drawn wagon, a relatively slow and expensive travel mode. In contrast, workers travel by streetcar, a relatively fast and inexpensive travel mode.
11. If the assumptions of the simple monocentric model are dropped, the housing-price function and the residential bid-rent change.
    a. The opportunity cost of commuting is the value of forgone work or leisure time. The typical commuter values commuting time at between one third and one half of his or her wages. The unit cost of commuting is the sum of time and monetary costs.
    b. Noncommuting travel to the CBD increases the slopes of the housing-price and bid-rent functions because total travel costs increase more rapidly as the household moves away from the CBD.

c. The two-earner household has a relatively steep housing-price function if both earners commute to the CBD.

d. The prices of housing and land are higher in communities with superior local public goods and low taxes.

e. The prices of housing and land are higher in communities with clean air, scenic views, and access to parks.

12. The slopes of the housing-price and land-rent functions depend on housing consumption. A household that occupies a relatively large house has a relatively flat housing-price function and a relatively flat bid-rent function, so the household lives relatively far from the city center. Such a household has more to gain from inexpensive suburban housing.

13. In U.S. cities, the wealthy tend to locate in the suburbs, and the poor tend to locate near the city center. One theory of income segregation suggests that the location choices of wealthy and poor households are based on different trade-offs between commuting and land costs. This theory has been refuted by empirical evidence, suggesting that the observed pattern of segregation is caused by other factors, such as the demand for new suburban housing, the desire to escape central-city problems, and exclusionary zoning in the suburbs.

Exercises and Discussion Questions

1. In the city of Trekburg, manufacturers have two options for intracity freight. They can use a conventional transportation system (the truck) or a matter transmitter, which instantly transports the output from the factory to the central export node ("Beam it over, Scotty"). A transmitter can be rented for C per year, and running the machine is costless. The transmitter can transport output up to a distance of two miles. All manufacturing output goes through the export node.

   a. Draw the bid-rent function for a firm that uses the matter transmitter, and label it M.

   b. On the same graph, draw the bid-rent function for a firm that uses the truck, and label it T.

   c. Will every manufacturer use the matter transmitter? If not, where will the firms using the truck be located?

2. Consider a traditional 19th-century monocentric city with a CBD radius of one mile. In 1869, buildings at the edge of the CBD are four stories tall. In 1870, all the buildings in the CBD are destroyed by an earthquake, and the mayor of the city announces that the maximum building height in the rebuilt city will be four stories. In addition, business development will be confined to a circle with a one-mile radius, that is, the size of the CBD is fixed. Suppose that the city is small enough that events in the city do not affect the equilibrium prices of its export goods (office or manufacturing goods).

   a. Draw the business bid-rent function before the earthquake and label it R°. Draw the business bid-rent function after the earthquake (with the height restrictions) and label it R^{*}.

   b. Explain any differences between the two bid-rent functions.

   c. Suppose that the city is large enough that events in the city affect the price of its exports. Will the height restrictions increase or decrease the price of the goods? What are the implications of the change in the price of goods on the business bid-rent function?

3. Complete the following table, given the following assumptions:

   i. The office firm produces 100 consultations per month.

   ii. The consultation fee is \$75.

   iii. Travel time is five minutes per block.

   iv. The opportunity cost of travel time is \$3 per minute.

   v. Every consultation requires one trip to the city center.

Distance to City Center (miles)	*Size of Site (acres)*	*Total Revenue*	*Nonland Cost*	*Travel Cost*	*Pre-Rent Profit*	*Rent per Acre*
0	0.40		\$3,600			
1	0.70		2,000			
2	0.90		1,200			
3	1.00		900			

4. Consider an office firm with the following characteristics: the wage of executives is \$120 per hour, and the executive takes four minutes to walk one block (eight minutes to make a round trip); the price of output is \$150, and the firm produces 50 consultations (requiring 50 trips to the city center); at a location four blocks from the city center, the firm occupies a one-acre site and spends \$1,000 on nonland inputs.

   a. What is the travel cost per block?

   b. How much is the firm willing to pay for land four blocks from the city center?

   c. Given the available information, is it possible to compute how much the firm is willing to pay for land one block from the city center? If not, what additional information do you need?

5. Consider two monocentric cities: Rigid City, where office firms produce with fixed factor proportions, and Flexville, where office firms produce with variable factor proportions. In each city, the CBD is a circular area with a radius of one mile, and all land in the CBD is used for office space. At the edge of the CBD there is 5,000 square feet of office space per acre, and the bid rent for office land is \$20,000 per year. Suppose that each city imposes an annual tax of \$1 per square foot of office space. Assume that the cities are small enough that the equilibrium price of office services is unaffected by events

in the cities. For each city, draw the office bid-rent function before and after the new office tax. Provide numbers for the bid rents (pre-tax and post-tax) at the CBD edges. Explain any differences in the effects of the tax in the two cities.

6. Depict graphically the effects of the following changes on the division of CBD land between office firms and manufacturers:
   a. The unit freight cost decreases.
   b. The price of office output increases.
   c. The opportunity cost of executive travel decreases.
7. Consider an industry that makes table tennis balls and competes with the baseball makers for land near the central export node. Each table tennis ball firm produces the same amount of output as a baseball firm (five tons of balls), sells for the same price ($160 per ton), and has the same production isoquants. Which activity will locate closer to the export node?
8. Consider a monocentric city in which the unit cost of commuting is $10 per mile per month. A household located eight miles from the city center occupies a dwelling with 1,200 square feet at a monthly rent of $600. Nonland cost per dwelling is $200, and there are four houses per acre.
   a. What is the price (per square foot) of housing at $u = 8$? What is the bid rent at $u = 8$?
   b. Assume that the demand for housing is perfectly inelastic. What is the price of housing at $u = 5$?
   c. Assume that housing firms do not engage in factor substitution. What is the bid rent at $u = 5$?
   d. How would your answers to (*b*) and (*c*) change if the demand for housing is price-elastic and firms engage in factor substitution? Would the prices of housing and land be larger or smaller?
9. Choose one word in each set of parentheses to make the following statements correct, and then explain your choice of words.
   a. "The flatter the demand curve for housing, the (*more, less*) curvature in the housing-price function. In other words, the flatter the housing demand curve, the (*more, less*) convex the housing-price function."
   b. "The flatter the demand curve for housing, the (*more, less*) curvature in the residential bid-rent function."
10. Consider a region with two cities: Lawland (L) and Violateville (V). The two cities differ in their demand curves for housing: consumers in Lawland have negatively sloped demand curves; consumers in Violateville have positively sloped demand curves (consumers in Violateville actually consume more housing as the price of housing increases). Draw the housing-price functions for the two cities (labeled *PL* for Lawland and *PV* for Violateville) under the assumption that $PL = PV$ at a distance of five miles from the city center. Briefly explain any differences between the two housing-price functions.
11. Depict graphically the effects of the following changes on the equilibrium housing-price function:

   a. The workweek is shortened from five days per week to four days per week.
   b. The workers in two-earner households start riding to work together.
12. Suppose that a city restricts the heights of residential structures. The maximum height is four stories, the height that would normally occur at a distance of five miles from the city center. Draw two residential bid-rent functions, one for the city in the absence of height restrictions and one with height restrictions.
13. Between 1940 and 1965, the average household size increased dramatically. Draw two housing-price functions (one for 1940 and one for 1965) and explain the differences between the two functions. Could the increase in household size explain part of the suburbanization that occurred between 1940 and 1965?
14. Consider the example of the trade-offs associated with location choices in Table 8–5. Suppose the poor household (with one fifth of the income of the wealthy household) consumes 0.15 acres of land and has a monthly commuting cost of $15 per mile.
   a. What are the implied income elasticities of land consumption and commuting cost?
   b. What is the optimum distance for the poor household?
   c. What are the implications for income segregation?
15. Suppose that the income elasticity of demand for land is +0.75. The unit commuting cost (cost per mile) is the sum of monetary cost (30 cents per mile) and time cost (opportunity cost). Suppose that the typical commuter earns a wage of $12 and takes 30 minutes to commute 10 miles to work. Every worker values commuting time at half of his or her wage. Can the observed pattern of income segregation be explained by the trade-offs between commuting cost and land cost? If there's not enough information to answer the question, what additional information do you need, and how would you use it?
16. Suppose that the demand for housing (H = square feet of housing space per capita) and the demand for land (T = square feet of land per square foot of housing space) are described by the following equations (P = price of housing, and R = land rent per square foot):

$$H = 1,500 - 500 \cdot P$$

$$T = \frac{1}{10,000} \cdot (15,000 - R)$$

   Compute land consumption per capita for the following locations:
   a. Location A: $P = 1.5$ and $R = 12,000$
   b. Location B: $P = 1.0$ and $R = 10,000$
   c. Location C: $P = 0.3$ and $R = 3,000$
17. In Figure 8–5, the introduction of consumer substitution increases the price of housing at all locations. Comment on the following: "Something is wrong

here. As we move from a world without substitution to a world with substitution, overall demand for housing decreases. Yet Figure 8–5 shows an increase in the price of housing. How can price rise when demand falls?"

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