

A thermodynamic formulation of the psychrometer constant

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Abstract. This work aimed to present an exact formulation of the psychrometer constant based on thermodynamics reasoning, rather than the usual empirical approach. Such a thermodynamic psychrometer constant sets a common ground for expected values independently of the type of construction and operating conditions of a psychrometer. The assumption that one is dealing with a perfect gas mixture conveys the exact solution to a simpler expression for the thermodynamic psychrometer constant with a relative deviation within -2% for a thermodynamic wet-bulb temperature less than 60°C and standard pressure, which suffices for field applications. It has also been found that the thermodynamic psychrometer constant is strongly dependent on the thermodynamic wet-bulb temperature, weakly dependent on the dry-bulb temperature and pressure dependent too. The formulation is compared with experimental data and reproduces very closely the results for an adiabatic saturation psychrometer. Experimental data on constants of several psychrometer types tend to scatter around the exact solution, except for the World Meteorological Organization psychrometer, which gives consistently lower values than the ones from the present formulation. In addition, an analysis of the error introduced when one mistakes the temperature reading of a regular psychrometer for the thermodynamic property is carried out. A typical difference between these two temperatures is around $\pm 0.4^\circ\text{C}$ for air at room temperature. Finally, speculations regarding means of improving psychrometer design in order to obtain the thermodynamic wet-bulb temperature directly from the instrument are presented.

Keywords: psychrometry, psychrometer, moist air, humidity, wet-bulb temperature

1. Introduction

A psychrometer is an inexpensive instrument for obtaining the thermodynamic properties of moist air that is relatively simple to operate. It is widely used in field measurements and, if certain precautions are taken, it can provide reasonable estimations for moist air within the range of normal atmospheric air conditions. A basic instrument consists of a pair of conventional glass thermometers, one of which has its sensing bulb covered with a wet cloth wick, usually made of cotton. Upon establishing a flux of air around the wet-bulb thermometer, a temperature drop will be observed. The ultimate temperature represents the equilibrium condition of the simultaneous heat- and mass-transfer process that is taking place between the wet wick and the airstream. The surrounding environmental temperature will also take part in the heat-transfer process if a proper radiation shield is not provided. The equilibrium temperature is suggestively named the wet-bulb temperature, whereas the other, unprepared thermometer furnishes the dry-bulb temperature or simply the air temperature as a preferred designation throughout this paper. The two temperature readings are usually combined through the

following empirical equation:

$$x_v = x_v^+ - A(T - T^+) \quad (1)$$

where A is the psychrometer constant in $^\circ\text{C}^{-1}$, T is the air temperature in $^\circ\text{C}$, T^+ is the wet-bulb temperature in $^\circ\text{C}$, x_v is the mole fraction of water vapour at T and x_v^+ is the saturation mole fraction of water vapour at T^+ . The psychrometer constant and the temperatures are the key parameters that one needs to know in order to obtain the thermodynamic properties of moist air. The psychrometer constant is usually obtained in careful laboratory experiments, such as those carried out by Greenspan and Wexler (1968), Wylie (1979), Wylie and Lalas (1981a, b, 1992), Fan (1987) and Visscher (1995), just to refer to the work of a few researchers of the last three decades. Experimentation in this field is not new, however. As reported by Wylie (1979), in 1830 Belli noticed that the air speed influenced the wet-bulb temperature reading and proposed that a sufficiently high air speed should be provided. The origin of the instrument itself goes back to the latter part of the 18th century (Wylie 1979). A value of A around $6.70 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ is generally accepted and psychrometer tables were constructed on the basis of that value (Schurer 1981), but other values are also in current use, depending on the psychrometer construction principle

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and operating conditions. Manufacturers also have their own psychrometer constants.

Controversy over the accurate value of the psychrometer constant is not new. The literature includes many experimental and theoretical works dealing with the problem and new claims are constantly being made. Wylie (1979) and Wylie and Lalas (1981a, b, 1992) have concluded that the conventional value for A was 8% too high, a conclusion that was also shared by Schurer (1981) and Fan (1987). The World Meteorological Organization (WMO) reference psychrometer (Wylie and Lalas 1981a, 1992) is based on Wylie and Lalas' lower constant values. It has also been found that the psychrometer constant is not really a constant magnitude and wet-bulb temperature-dependence formulae have been proposed (Ferrel 1885, Wylie and Lalas 1981a, 1992, Sonntag *et al* 1989). Recent experiments carried out by Visscher (1995) with several types of psychrometer and bulb geometry at various air speeds revealed important findings. Consistently, he found distinctly higher values for the psychrometer constant than the ones expected from Wylie and Lalas' formulation. Moreover, Visscher found a very strong dependence of A on the construction and operating principle of the psychrometer itself. Thus, the psychrometer constant is not universal but rather is closely associated with a particular type of instrument. In addition, Visscher obtained a 3% higher value for a WMO reference psychrometer and a 5% lower value for the Assmann-type aspirated psychrometer, which is also accepted as a reference psychrometer by the WMO.

Psychrometer modelling is usually based on the empirical laws of simultaneous mass and energy transfer and its analogy, given that the mass-transfer process is assumed to occur at a low rate. Threlkeld (1962) outlined a parametric study of the influence of the sensing-bulb size, heat-transfer coefficient, radiation effects and air velocity upon the psychrometer constant. Later, Wylie and Lalas (1981a, b) conducted a more detailed theoretical and experimental formulation in a similar fashion.

This paper is an attempt to establish a theoretical background for the theory of the psychrometer constant based on thermodynamics reasoning in opposition to the traditional empirical approach. A psychrometer-constant formulation for a psychrometer whose wet-bulb temperature matches the thermodynamic wet-bulb temperature, which is defined in the next section, is proposed. The ASHRAE's psychrometric diagrams and tables (ASHRAE 1997) are based on the thermodynamic wet-bulb temperature. Szumowski (1982) pointed out the fact that A should be independent of the psychrometer type and suggested that psychrometric tables should be based on an adiabatic saturation psychrometer. It is only possible to achieve a construction-independent psychrometer constant if the constant itself is a thermodynamic property of the moist air.

Previous analytical formulations were carried out by Threlkeld (1962) and by Greenspan and Wexler (1968). However, the present analytical formulation is more general and is exact for the real mixture of dry air and water vapour presented in section 2.2. In addition, a simplified analysis that is valid for a mixture of perfect gases is also presented in section 2.3. The dependence on pressure of a psychrometer is discussed in section 2.4. In section 3, the present

theory is compared with experimental data available in the literature. Next, some comments on the errors encountered in obtaining the thermodynamic wet-bulb temperature, considering that this magnitude is usually mistaken for the wet-bulb temperature, are presented. Section 4.2 presents some speculations regarding improvements to psychrometer design. Finally, conclusions of this work and suggestions for further investigation are presented in section 5.

2. The theoretical formulation

2.1. The adiabatic saturation process—the adiabatic saturation psychrometer

The adiabatic saturation of a moist air flux occurs in an ideal device called the adiabatic saturation psychrometer, wherein an incoming airstream is brought into contact with liquid (solid) water and, upon leaving the apparatus, the air is saturated with water vapour. The whole process is adiabatic in the sense that there is no heat exchange with the environment. The air saturation reaches a temperature named the thermodynamic wet-bulb temperature T^* . From the laws of conservation of energy and mass, it is straightforward to show that

$$h + (\omega^* - \omega)h_L^* = h^* \quad (2)$$

where h is the specific enthalpy in kJ kg^{-1} dry air, ω is the humidity ratio in $\text{kg water vapour per kg dry air}$ of the entering airstream, which is at T ; ω^* is the humidity ratio in $\text{kg water vapour per kg dry air}$; h_L^* is the specific enthalpy of liquid (solid) water; and h^* is the specific enthalpy in kJ kg^{-1} dry air of the air flux leaving the apparatus at T^* . It is important to note that, in order to maintain steady-state conditions, the adiabatic saturation psychrometer must be supplied with liquid (solid) water at T^* to replace water lost by evaporation. The usefulness of the adiabatic saturation psychrometer analysis is that there is only one temperature (T^*) that resolves equation (2) for a given thermodynamic state of the air (Harrison 1965) at constant pressure. Therefore, T^* is a thermodynamic property of the moist air and consequently one is assured that it has a unique value for a given moist air state. T^* is known as the thermodynamic wet-bulb temperature. Alternatively, T^* is also called the adiabatic saturation temperature by other authors. For the sake of simplicity, a star superscript will always refer to a wet-bulb-temperature-dependent property in this paper.

2.2. The exact formulation of the psychrometer constant

A psychrometer should indicate the thermodynamic wet-bulb temperature T^* , rather than the wet-bulb temperature T^+ . The former is a thermodynamic property, whereas the latter results from a mass- and heat-transfer process and depends on many parameters, such as the air-flow velocity and direction, bulb geometry, radiation and conduction effects and so on. Therefore, the wet-bulb temperature depends on the psychrometer construction and the operating conditions. In order to establish the thermodynamic psychrometer constant, first let equation (1) be rewritten in the following form:

$$x_v = x_v^* - A_\theta(T - T^*) \quad (3)$$

where, x_v is the mole fraction of water vapour, and A_θ is the thermodynamic psychrometer constant. Note that the term *thermodynamic* and the subscript θ were added to distinguish A_θ from ordinary psychrometer constants. Here, the properties are dependent on the thermodynamic wet-bulb temperature (star superscript) in contrast to the ones depending on the wet-bulb temperature (plus-sign superscript) in equation (1). Now, let the humidity ratio ω be defined as

$$\omega = \frac{M_v}{M_a} \frac{x_v}{1 - x_v} \quad (4)$$

where M_a and M_v are the relative molecular masses of dry air and water vapour, whose values are 28.9645 and 18.015 34, respectively; x_v has already been defined; and ω is the ratio of the water-vapour content to the dry air present in the moist air. It is straightforward to show that the difference between ω and ω^* is given by

$$\omega^* - \omega = 0.621\,98 \frac{x_v^* - x_v}{(1 - x_v)(1 - x_v^*)}. \quad (5)$$

Finally, combining the equation of the adiabatic saturation psychrometer (2) and the above equation via the humidity-ratio difference, the final psychrometer equation (3) is obtained, where A_θ is given by

$$A_\theta = \frac{(1 - x_v^*)(1 - x_v)}{0.621\,98(T - T^*)} \frac{h^* - h}{h_L^*}. \quad (6)$$

The following observations are pertinent. (i) The near-constant value of the psychrometer constant A_θ at room temperature is a fortuitous incident, as is shown below. (ii) The present formulation of the psychrometer constant is independent of the instrument type and sets a common ground for expected instrument constants. (iii) Its validity spans the whole range of temperature, pressure and humidity as long as a realistic equation of state is provided.

Figure 1 depicts the thermodynamic psychrometer constant as a function of the thermodynamic wet-bulb temperature for selected constant-temperature curves at normal pressure. It is clear from that graph that A_θ is weakly dependent on T and strongly dependent on T^* , predominantly in the higher temperature range. However, the polynomial-type relationship comes to a nearly flat curve for thermodynamic wet-bulb temperatures of around 30 °C and below, down to 0 °C. In the higher range of T^* , the constant drops considerably. For instance, the constant can be as low as $2.5 \times 10^{-4} \text{ °C}^{-1}$ at $T^* = 90 \text{ °C}$. For $T^* < 0 \text{ °C}$, values of the thermodynamic psychrometer constant tend to cluster around an average value of $5.68 \times 10^{-4} \text{ °C}^{-1}$, independently of the wet-bulb temperature. These features can be well explained in the context of the assumption of perfect gas behaviour, which is presented in the following section. Sample calculations for selected values of T and T^* at normal pressure are shown in table 1. In table 1, one can also see other necessary properties, including the enhancement factor f , the specific enthalpies of moist air and the specific liquid enthalpy at T^* . The exact values of A_θ are also shown. It is noteworthy that A_θ is very dependent on rounding errors and slight differences in values in table 1 could be found if calculations were performed with fewer significant figures.

The enhancement factor is shown in parentheses below the temperature in the first column of table 1. The last two columns present approximated values of the psychrometer constant and their deviations from exact values, which are discussed in the next section. A problem occurs in the vicinity of the saturation limit, for both the numerator and the denominator of equation (6) vanish. The author studied the trends of the graphs and extrapolated the limit of T^* as it approached T . Alternatively, one can apply the L'Hôpital rule of calculus and obtain that limit numerically. Computations of the real thermodynamic properties were carried out using a computer program (Simões-Moreira 1998, 1999) developed using the concept of the enhancement factor, which was originally proposed by Goff and Gratch (1945) and updated by Hyland and Wexler (1983). Thermodynamic properties, such as specific enthalpies and mole fractions, were also obtained using Wexler and Hyland's property correlations. Samples of the enhancement factor from the computer program matched exactly the published data of Hyland and Wexler (1983). Temperature values should not be affected by using either the IPTS-68 or ITS-90 scale, since there is no difference between these two temperature scales in the range of interest (−10 to 60 °C).

2.3. The formulation of the approximated psychrometer constant for a mixture of perfect gases

The above exact formulation can be simplified for the moist air when it behaves as a mixture of two thermally and calorifically perfect gases—water vapour and dry air. It turns out that this assumption is fairly correct within the usual measuring uncertainty ranges, which are about 0.5% under controlled laboratory conditions and higher for field measurements. On the other hand, the assumption of a perfect gas mixture leads to a very narrow deviation from the behaviour of a real gas mixture in a broad range of temperatures (see figure 8.1 of Threlkeld (1962), for instance), which suffices for most environmental process applications. The relative deviation from the real behaviour is less than 0.7% for relevant saturated psychrometric properties in the temperature range −50 to 50 °C. Therefore, it is generally of the same order as the usual measuring uncertainties.

Using the simplification that moist air is a mixture of two perfect gases, equation (2) may be rewritten as

$$\omega^* - \omega = \frac{C_{pu}}{h_{LV}^*} (T - T^*) \quad (7)$$

where h_{LV}^* is the specific enthalpy of vaporization in kJ kg^{−1} at T^* and C_{pu} is the specific heat at constant pressure of the moist air, defined by

$$C_{pu} = C_{pa} + \omega C_{pv} \quad (8)$$

where C_{pa} and C_{pv} are the specific heats at constant pressure in kJ kg^{−1} °C of dry air and water vapour, respectively. C_{pu} is taken at the air temperature. For room and moderately higher temperatures, it is reasonable to assume that $C_{pa} = 1.006 \text{ kJ kg}^{-1} \text{ °C}^{-1}$ for dry air and $C_{pv} = 1.805 \text{ kJ kg}^{-1} \text{ °C}^{-1}$ for water vapour (ASHRAE 1997). Actually, C_{pa} is nearly

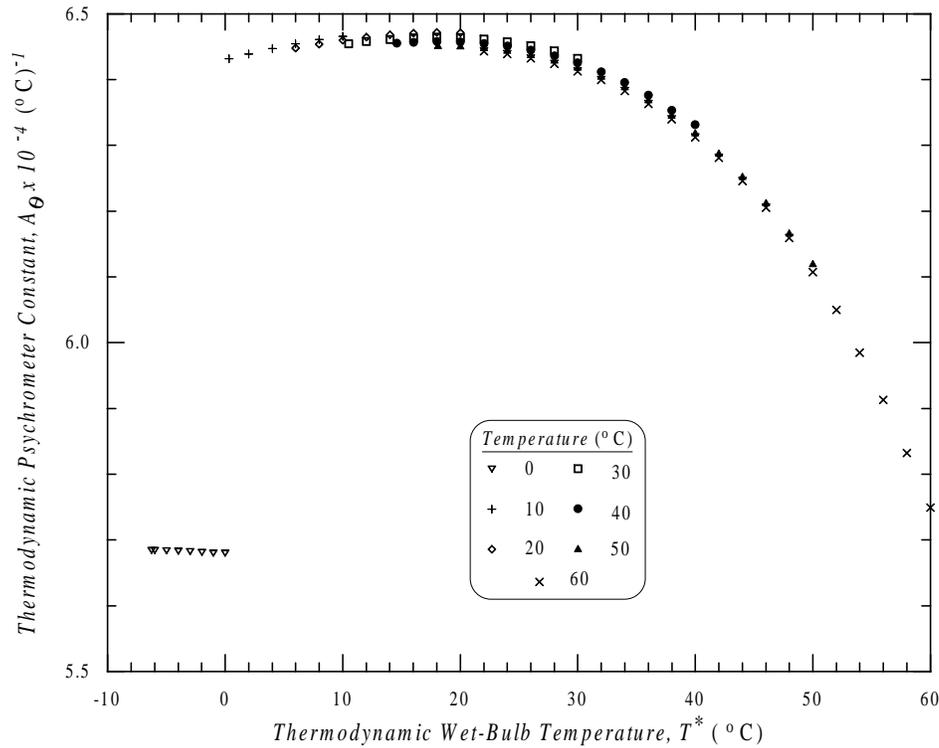


Figure 1. The thermodynamic psychrometer constant as a function of the thermodynamic wet-bulb temperature for several constant-temperature curves (at normal pressure).

constant in the usual working range of temperatures (-10 to 60 °C), which is not quite true for C_{pv} . However, the participation of C_{pv} is weighted by ω , so that in the end its temperature dependence will not influence the overall results very much. The above derivation also implied that the perfect-gas-mixing rule is valid for the specific enthalpy of moist air, i.e. $h = h_a + \omega h_v$, where h_a and h_v are the specific enthalpies of the dry air and water vapour, respectively. By substituting equation (7) into equation (2), after a few manipulations, one obtains

$$A_{PG} = \frac{C_{pu}(P - P_v)(P - P_v^*)}{0.62198h_{LV}^*P^2}. \quad (9)$$

In the above derivation, the perfect-gas approximation $x_v = P_v/P$ was used, where P_v is the partial pressure of water vapour in kPa and P is the total pressure in kPa. In this context, the psychrometer equation (1) can also be cast into a more familiar form, given by

$$P_v = P_v^* - A_{PG}P(T - T^*). \quad (10)$$

Table 1 presents selected values of the approximated psychrometer constant, A_{PG} , and their deviations from the actual values for the same set of data as that which was analysed in the exact formulation. The thermodynamic properties necessary to obtain A_{PG} were calculated using the perfect-gas relations for moist air. The calculations were performed for dry and saturated air as well as for an intermediate moisture content, using the same T and T^* as those used in the exact formulation. Figure 2 indicates the relative deviation of A_{PG} from A_θ as a function of the thermodynamic wet-bulb temperature at normal pressure

for several curves of constant temperature. As before, the thermodynamic wet-bulb temperature plays the most important role. Relative deviations, ε , are within -2% for the whole displayed thermodynamic wet-bulb temperature range, independently of the temperature. Also, the deviation is within -0.4% for $T^* < 30$ °C. The relative deviation in the range 30 °C $< T^* < 50$ °C is within -1% as long as the air is not too close to saturation. An interesting observation is that, for dry air, the maximum relative deviation is -0.17% for the extended range and it is negligibly small ($< -0.08\%$) for $T^* < 30$ °C. On extrapolating the curves, one will find that the deviation grows quickly as the thermodynamic wet-bulb temperature is increased beyond 60 °C. The exact formulation should be used in such cases.

The strong dependence of the thermodynamic psychrometer constant on the thermodynamic wet-bulb temperature can more easily be understood in the light of equation (9). One can easily recognize that the constant is almost entirely dominated by the reciprocal of the specific enthalpy of vaporization at T^* for a low water vapour content. At this point it is also clear that there must exist a discontinuity at 0 °C (more specifically, at the triple point), which accounts for the sudden change in the specific enthalpy of vaporization owing to the shift from a liquid–vapour to a solid–vapour equilibrium system. This explains the jump in A_θ from about 5.68×10^{-4} on the ice side to 6.42×10^{-4} °C $^{-1}$ on the liquid–water side seen in figure 1. The elementary equation still reveals that there is a pressure dependence of A_θ and this topic is further examined in the next section. Of course, an outstanding advantage of using the perfect-gas-mixture formulation is that of considerably less calculational effort being needed. Besides that,

Table 1. Sample calculations of the exact and the approximated thermodynamic psychrometer constant.

T (°C)	RH (%)	Adiabatic saturated air			h_L^* (kJ kg ⁻¹)
		T^* (°C ⁻¹)	x_v^*	h^* (kJ kg ⁻¹ dry air)	
0 (1.0039)	0	-6.28	0.003 568	-0.772	-346.5
	49.75	-3.0	0.004 717	4.336	-339.7
	100	0.0	0.006 055	9.473	-333.4
10 (1.0039)	0	0.35	0.006 210	10.065	1.47
	44.54	5.0	0.008 645	18.638	20.9
	100	10.0	0.012 167	29.351	41.9
20 (1.0041)	0	5.81	0.009 147	20.260	24.3
	51.60	14.0	0.015 840	39.367	58.7
	100	20.0	0.023 176	57.553	83.8
30 (1.0044)	0	10.50	0.012 584	30.534	44.0
	49.99	22.0	0.026 209	64.657	92.2
	100	30.0	0.042 088	100.001	125.8
40 (1.0048)	0	14.56	0.016 424	40.887	61.1
	52.98	31.0	0.044 566	105.363	129.9
	100	40.0	0.073 218	166.673	167.6
50 (1.0053)	0	18.10	0.020 582	51.317	75.8
	50.95	39.0	0.069 399	158.499	163.3
	100	50.0	0.122 530	275.330	209.3
60 (1.0058)	0	21.22	0.024 990	61.822	88.9
	49.18	47.0	0.105 396	236.747	196.8
	100	60.0	0.197 975	460.850	251.1

T (°C)	RH (%)	Moist air			A_{PG} (10 ⁻⁴ °C ⁻¹)	Relative deviation ε (%)
		x_v	h (kJ kg ⁻¹ dry air)	A_θ (10 ⁻⁴ °C ⁻¹)		
0 (1.0039)	0	0	0	5.68	5.68	-0.02
	49.75	0.003 013	4.699	5.68	5.68	-0.05
	100	0.006 055	9.473	5.68	5.68	-0.08
10 (1.0039)	0	0	10.059	6.43	6.43	-0.04
	44.54	0.005 419	18.595	6.45	6.45	-0.08
	100	0.012 167	29.351	6.47	6.46	-0.12
20 (1.0041)	0	0	20.121	6.45	6.44	-0.06
	51.60	0.011 960	39.221	6.47	6.46	-0.15
	100	0.023 176	57.553	6.47	6.46	-0.23
30 (1.0044)	0	0	30.185	6.45	6.45	-0.08
	49.99	0.021 040	64.346	6.46	6.45	-0.24
	100	0.042 088	100.001	6.43	6.41	-0.40
40 (1.0048)	0	0	40.253	6.46	6.45	-0.11
	52.98	0.038 789	104.855	6.42	6.39	-0.39
	100	0.073 218	166.673	6.33	6.28	-0.74
50 (1.0053)	0	0	50.326	6.45	6.44	-0.14
	50.95	0.062 433	157.687	6.33	6.29	-0.60
	100	0.122 530	275.330	6.12	6.05	-1.22
60 (1.0058)	0	0	60.405	6.44	6.43	-0.17
	49.18	0.097 359	235.528	6.18	6.13	-0.91
	100	0.197 975	460.850	5.75	5.63	-2.00

the loss of accuracy is quite acceptable for most of the field applications and working temperatures (see section 4.1).

2.4. The dependence of the psychrometer constant on pressure

Psychrometers are often employed in several places at different altitudes. Therefore, it is essential the dependence of the psychrometer constant on pressure be established. Given the complexity of the equation of state of moist

air, the analysis is carried out first for the approximated thermodynamic psychrometer constant A_{PG} and then the results are checked against actual values from the exact formulation. It is a matter of obtaining a simple closed analytical form for the dependence of the constant on pressure. By applying Taylor's series to equation (9) and truncating the expansion at the first term, one obtains

$$A_{PG} \simeq A_{PG}^o + (P - P^o) \left(\frac{\partial A_{PG}}{\partial P} \right)_{T, T^*, P=P^o} \quad (11)$$

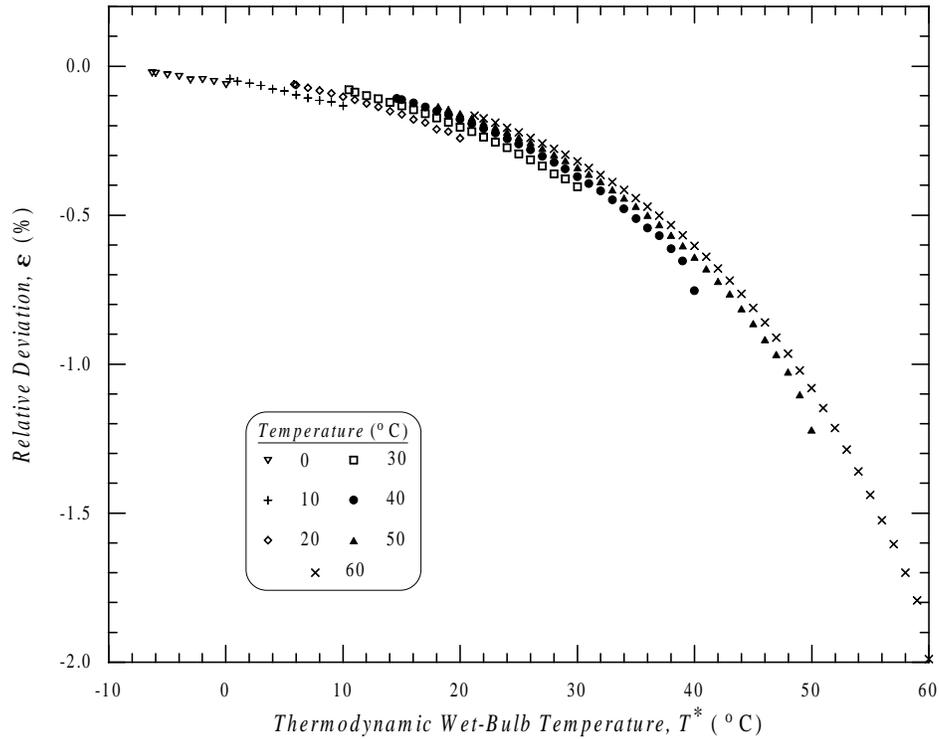


Figure 2. The relative deviation of exact and approximated solutions of the thermodynamic psychrometer constant as a function of the thermodynamic wet-bulb temperature for several constant-temperature curves (at normal pressure).

where the superscript o indicates a known reference condition (standard pressure, for instance). The partial derivative is obtained from equation (9):

$$\left(\frac{\partial A_{PG}}{\partial P}\right)_{T, T^*, P=P^o} = \frac{A_{PG}^o}{0.62198P^o} \times \left[\omega^{o*} + \omega^o \left(1 - \frac{\omega^o P^o C_{pv}}{P_v C_{pu}^o} \right) \right]. \quad (12)$$

Now, on defining the actual pressure deviation, ε_p , and substituting in the approximated perfect-gas relations, one obtains

$$\begin{aligned} \varepsilon_p &= \frac{A_\theta - A_\theta^o}{A_\theta^o} \approx (P - P^o) \frac{1}{A_{PG}^o} \left(\frac{\partial A_{PG}}{\partial P} \right)_{T, T^*, P=P^o} \\ &= 1.60777 \left(\frac{P - P^o}{P^o} \right) \left[\omega^{o*} + \omega^o \left(1 - \frac{\omega^o P^o C_{pv}}{P_v C_{pu}^o} \right) \right]. \end{aligned} \quad (13)$$

The actual deviations relative to the exact formulation for saturated air are shown in figure 3. As before, a very strong dependence on the thermodynamic wet-bulb temperature is found, in contrast to a weak temperature dependence. Maximum relative deviations of about ± 0.02 are anticipated for pressures in the range of ± 10 kPa around normal pressure. The relative deviation can increase to -0.09 for $P = 70$ kPa, predominantly at high values of T^* (around 60°C). Calculations for dry air indicated that the relative deviations are essentially the same. The relative deviations are also pressure-correction factors. Then, from figure 3, corrections for the pressure dependence can be calculated according to

$$A_\theta = (1 - \varepsilon_p) A_\theta^o. \quad (14)$$

Checking the approximated expression for the perfect-gas mixture has shown that it slightly overpredicts the relative deviation (about -0.02) under the worst conditions, which occur generally at high values of the thermodynamic wet-bulb temperature.

3. A comparison with experimental results

3.1. The adiabatic saturation psychrometer

There are very few data available on adiabatic saturation psychrometers, because experimental work tends to be more concentrated on conventional psychrometers (see the next section). Greenspan and Wexler (1968) carried out careful experiments with an adiabatic saturation psychrometer and found very good agreement with the predictions of the perfect-gas-mixture theory. The experiments were conducted only for dry air at temperatures in the range $24\text{--}37^\circ\text{C}$, which resulted in (thermodynamic) wet-bulb temperatures ranging from nearly 8 to 13°C . Also, the pressure was approximately normal. By analysing figure 1, one can see that Greenspan and Wexler's experimental conditions fell right on the flat portion of the curve (with an average constant of about $A_\theta = 6.45 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$). A comparison of their results with the present formulation is shown in table 2. The first three columns indicate the measured or tested properties, i.e. the pressure, temperature and thermodynamic wet-bulb temperature. For all cases $x_v = 0$ (dry air). The actual thermodynamic wet-bulb temperatures (column 4) were calculated using the enhancement-factor concept and the computer program mentioned above (Simões-Moreira 1998). The differences between their measured thermodynamic

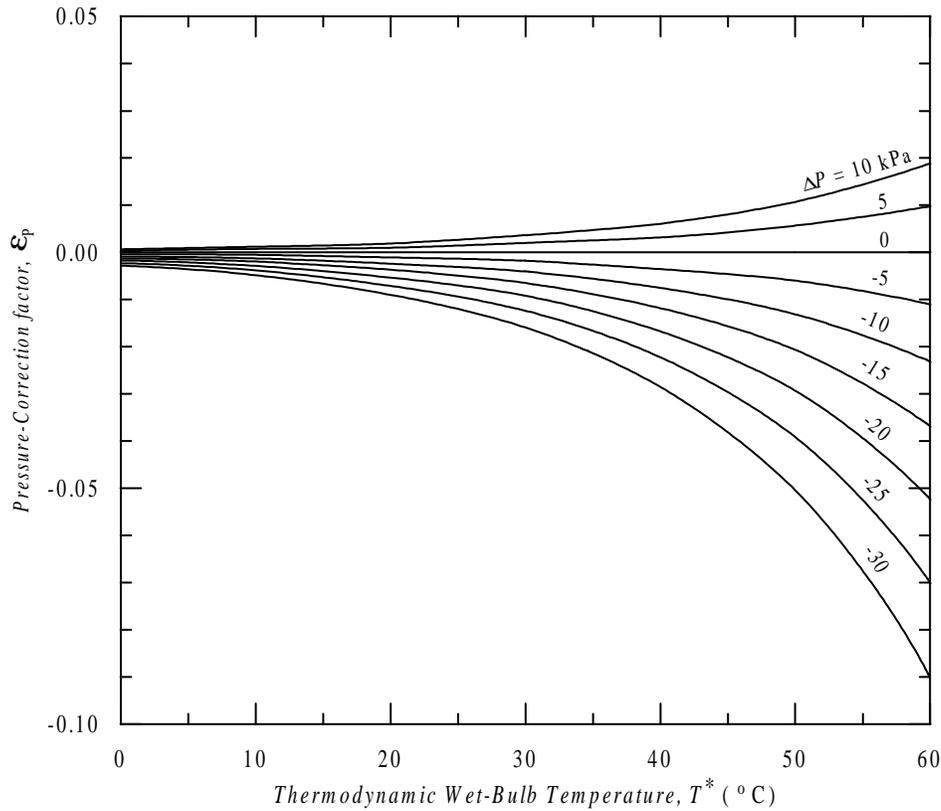


Figure 3. The pressure-correction factor for the thermodynamic psychrometer constant as a function of the thermodynamic wet-bulb temperature for several pressure variations around normal pressure ($\Delta P = P - P^o$).

wet-bulb temperature and the theoretical ones are nearly negligible. The vapour mole fraction and the thermodynamic psychrometer constant, A_θ , were also calculated using the mentioned computer program. The actual thermodynamic constant, A_c , was obtained from equation (3) for the case of $x_v = 0$ with the x_v^* and the measured ΔT (i.e. $T - T^*$) shown in table 2. Finally, the constant relative deviations for the several cases studied are presented in the last column. In theory, these two values should be exactly equal and the relative deviation should be zero. The small deviation may well be related to experimental difficulties in obtaining an actual adiabatic saturation process, since their experimental uncertainties were quite small. Greenspan and Wexler's experiments fully confirm the present formulation of the thermodynamic psychrometer constant.

3.2. Conventional psychrometers

In contrast to the adiabatic saturation psychrometer, there are significant published data on several types of psychrometer. Extensive work was done by Wylie and Lalas to establish a psychrometer standard for the WMO, which was summarized in a technical note (Wylie and Lalas 1992). Details of the construction of the WMO psychrometer are also presented in their report. Despite their great effort, there are still some disputes over accurate values of the psychrometer constant. Visscher (1995) carried out tests in order to compare different types of psychrometer and various operating conditions. He analysed 20 psychrometer configurations. In all cases the air was at a relative humidity (RH) of 10% and a temperature

of $T = 23^\circ\text{C}$. For this air state, a thermodynamic psychrometer constant equal to $6.46 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ is expected from the present formulation for a normal barometric pressure. Figure 4 shows data for all cases tested by Visscher together with experimental uncertainties. The data were extracted from table 2 of his paper. The labels next to the data point stand for psychrometer types and the numbers in parentheses indicate the order of appearance of the instrument in his original table (refer to his paper for further details). The theoretical value is a full vertical line starting at the horizontal axis. It is remarkable that the several psychrometer constants of Visscher's experiments are distributed around the thermodynamic psychrometer constant, A_θ , calculated from the present theory. All of the psychrometer constants fell within 5% relative deviation of A_θ , except that for the ASL psychrometer type.

Figure 4 also presents Fan's six experimental data on the WMO psychrometer. Nominally, three data were obtained with air at a RH of 48% and three data at a RH of 41%. In all situations, the temperature was held constant at 23°C and the pressure was nearly standard pressure. Actual pressures are shown in table 2 of his paper. The broken vertical line is for A_θ calculated from the present theory, which resulted in a value of the thermodynamic psychrometer constant equal to $6.47 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. In all cases, the values derived from Fan's experiments were lower than the predictions of the present formulation.

Wylie and Lalas (1992) have published extensive experimental data on the WMO psychrometer. In all

Table 2. A comparison of the thermodynamic psychrometer constant with experimental results of Greenspan and Wexler (1968) obtained with an adiabatic saturation psychrometer.

Measured or tested property			Present formulation				Relative deviation (%)
P (kPa)	T (°C)	T^* (°C)	T^* Calc ^a (°C)	x_v^* ^b	A_θ ^c (10^{-4} °C^{-1})	Actual A_c ^d (10^{-4} °C^{-1})	
97.45	24.80	7.87	7.843	0.010 933	6.447	6.458	0.16
97.37	27.73	7.81	7.804	0.010 913	6.447	6.450	0.04
98.16	24.62	7.80	7.816	0.010 835	6.448	6.442	-0.10
98.06	24.65	7.82	7.822	0.010 850	6.448	6.447	-0.02
100.02	25.11	8.24	8.194	0.010 911	6.450	6.468	0.27
99.43	37.07	13.25	13.25	0.015 372	6.454	6.453	0.00
99.48	37.01	13.19	13.232	0.015 346	6.454	6.442	-0.17
99.43	37.02	13.27	13.231	0.015 353	6.454	6.464	0.17
99.78	37.02	13.23	13.264	0.015 332	6.454	6.445	-0.14
100.00	24.98	8.15	8.132	0.010 867	6.450	6.457	0.11
100.05	25.02	8.16	8.155	0.010 878	6.450	6.452	0.03
100.08	25.02	8.18	8.157	0.010 877	6.450	6.459	0.14
100.08	25.08	8.22	8.185	0.010 898	6.450	6.464	0.21
100.07	25.17	8.25	8.226	0.010 929	6.450	6.459	0.14
98.18	24.88	8.07	7.939	0.010 924	6.448	6.499	0.78
98.29	24.85	8.10	7.934	0.010 908	6.448	6.512	0.99

^a Obtained using P , T , and RH 0% as input data for the computer program (Simões-Moreira 1998).

^b Same as in (a).

^c Calculated from equation (6) with properties calculated by the computer program mentioned in (a).

^d Actual value calculated from equation (3) for $x_v = 0$, measured temperatures and x_v^* (values shown in the table).

situations, their psychrometer constants are lower than those predicted by this theory, just as the constants obtained by Fan and Visscher with the WMO psychrometer are lower than the theoretical ones mentioned above. In section 4.1, further discussion on the implications of the observed discrepancy between the psychrometer constant and this theory regarding the wet-bulb temperature reading from a regular psychrometer is presented.

4. Comments on the thermodynamic wet-bulb temperature calculation and improvement of psychrometer design

4.1. The error due to the usual thermodynamic wet-bulb temperature approximation

In this section, the error δT is introduced when the thermodynamic wet-bulb temperature is mistaken for the psychrometer temperature reading. Assuming that a given psychrometer has A_m as a constant, then combining the psychrometer equations (1) and (3) for the same x_v yields

$$x_v^* - x_v^+ + A_\theta(T^* - T^+) = (A_\theta - A_m)(T - T^+). \quad (15)$$

Recalling that the vapour mole fraction is given by $x_v = f P_v / P$ (saturated air), the above equation takes the following final form:

$$\delta T = T^* - T^+ = \frac{(A_m - A_\theta)/A_\theta}{1 + \frac{f}{P A_\theta} \left(\frac{dP_v}{dT} \right)_{T=T^+}} (T - T^+) \quad (16)$$

where the truncated Taylor series has been used to expand the difference in partial pressure ($P_v^* - P_v^+$) around the

temperature difference ($T^* - T^+$) for constant values of P and f , i.e.

$$x_v^* - x_v^+ = \frac{f}{P} (P_v^* - P_v^+) = \frac{f}{P} (T^* - T^+) \left(\frac{dP_v}{dT} \right)_{T=T^+}. \quad (17)$$

Note that δT has the same sign as the difference between A_m and A_θ . Equation (16) allows one to obtain the thermodynamic wet-bulb temperature directly from a wet-bulb temperature reading if the psychrometer constant is known. Reiteration would be necessary since A_θ depends on the unknown temperature T^* . However, negligible error will be introduced if A_θ is taken at T^+ . Closer examination of equation (16) shows that the following (i) If $T^* < 15^\circ\text{C}$, then both terms in the denominator are of the same order (i.e. ≈ 1). Thus, a relative difference of $\pm 5\%$ between the two psychrometer constants will result in a maximum error of about $\delta T = \pm 0.4^\circ\text{C}$ for a temperature depression of 15°C . Returning to figure 4, the 5% range would include all psychrometers tested by Visscher, except the ASL type. (ii) In Fan's experiments the maximum relative deviation between constants was about -4% , which gives a maximum error of about $\delta T = 0.15^\circ\text{C}$ for a temperature depression of 8°C . (iii) In general terms, an elementary estimation of the order of magnitude of the temperature deviation gives $\delta T \approx 0.05(T - T^+)/(1 + P_v^+)$. In this derivation, the usual approximation of Clapeyron's equation has been used, i.e. $dP/dT \approx P_v^+ h_{LV}/(RT^2)$, with $h_{LV} \approx 2450 \text{ kJ kg}^{-1}$, $R = 0.4615 \text{ kJ kg}^{-1} \text{ K}^{-1}$, and $T \approx 300 \text{ K}$. It was also assumed that $(A_m - A_\theta)/A_\theta \approx 0.05$, $A_\theta = 6.5 \times 10^{-4} \text{ °C}^{-1}$, $P \approx 100 \text{ kPa}$, $f \approx 1$ and P_v^+ in kPa. For the typical working temperature range, P_v^+ is of the order of 1 kPa and higher. Thus, the temperature difference is at most about 2.5% or less of the

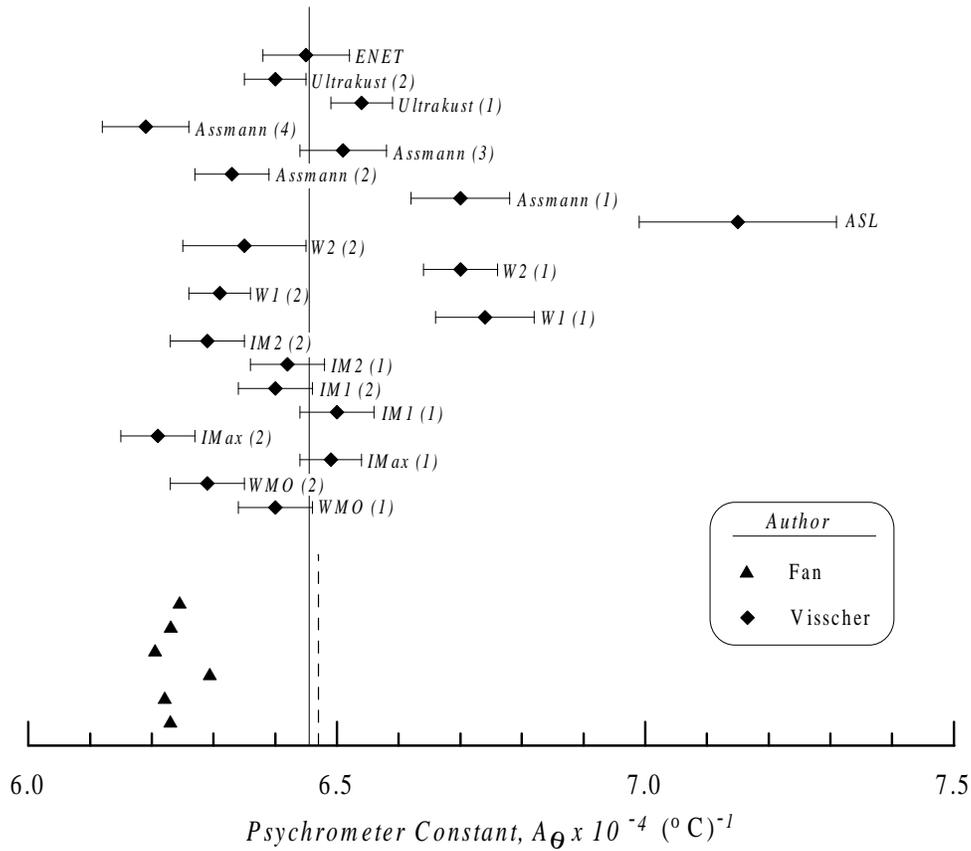


Figure 4. Constants of several types of psychrometers and their theoretical values; full line, the theoretical constant for Visscher’s experiments ($6.46 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$); and broken line; the theoretical constant for Fan’s experiments ($6.47 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$).

temperature depression ($T - T^+$). Typically, the temperature error will be less than $0.4 \text{ }^\circ\text{C}$ for usual psychrometers, such as those tested by Visscher at room temperature. Errors in the freezing zone are about of the same order as those in the case of moderate temperatures. In this region, the following values can be used: $h_{SV} \approx 2800 \text{ kJ kg}^{-1}$, $T \approx 270 \text{ K}$, $(A_m - A_\theta)/A_\theta \approx 0.05$, $A_\theta = 5.7 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$, $P \approx 100 \text{ kPa}$, $f \approx 1$ and $P_v^+ = 0.5 \text{ kPa}$. These values yield temperature differences of about 3% of the temperature depression. (iv) The perfect-gas assumption yields a maximum relative deviation of -2% for the thermodynamic psychrometer constant, under the worst conditions studied (see figure 2). Therefore, the temperature error can be approximated by $\delta T \approx 0.02(T - T_{PG}^*)/(1 + P_{vPG})$. Such an error will be, at most, $\delta T = -0.2 \text{ }^\circ\text{C}$, which is almost imperceptible with many conventional mercury thermometers. In the perfect-gas context, δT is the difference between the actual and the approximated thermodynamic wet-bulb temperatures.

4.2. Psychrometer standardization and design improvement

The thermodynamic state of a gaseous mixture such as moist air is determined by three independent properties. In dealing with psychrometric processes, two natural and easily measured properties are usually chosen; namely the temperature and pressure. Thus, if any other additional property is selected, then the thermodynamic state of the moist air will be

fully determined. On the basis of this principle, strong arguments in favour of using an instrument whose constant is the thermodynamic psychrometer constant discussed in this paper can be made. Firstly, the wet-bulb temperature reading from this psychrometer will also be the thermodynamic wet-bulb temperature. Thus, the two temperature readings from the instrument together with the local pressure will be sufficient to obtain promptly the remaining moist-air properties from a table, chart or computer program. Secondly, most importantly, the psychrometer constant will be instrument independent. Consequently, a standard or an acceptance procedure could be established in order to certify a given psychrometer. Acceptance or rejection of a new psychrometer could be based on how well the instrument constant reproduced the thermodynamic psychrometer constant within specific ranges of temperature, humidity and pressure. Finally, an extended measuring range of temperature and pressure would be immediately available since no restriction is imposed on the moist-air-state calculation by the exact formulation, except at the saturation line.

The remaining question is that of whether a practical psychrometer for the use in the field can be built. A laboratory adiabatic saturation psychrometer has been built (Greenspan and Wexler 1968). One interesting point is that, with some psychrometers, the wet-bulb temperature indication is lower than the thermodynamic wet-bulb temperature (see figure 4) and, as a consequence, their constants are lower than the theoretical ones too. This gives one the possibility of using

some process or technique to achieve the desired increase of T^+ and eventually a T^* reading. One may consider several possibilities. (i) One could just partially shield the wet-bulb thermometer against radiation in such a way that the incident environmental radiation would just be enough to increase T^+ to T^* . (ii) Alternatively, one could use a shielding material with a suitable emissivity coefficient. (iii) One could also think of heating up the wick to achieve the necessary temperature increase. The heating system could be active (controlled electrical resistance) or passive (allowing partial heat conduction flux to the wick, for instance). (iv) Finally, one could attempt to design a practical adiabatic saturation psychrometer for use in the field, which is possibly the best solution. In any case, a more detailed study must be conducted and this will be the subject of a future publication.

5. Conclusions

This paper presented an exact formulation of a psychrometer constant on the basis of thermodynamics principles. The exact theory was then simplified for the case in which moist air behaves as a mixture of two perfect gases, without considerable loss of accuracy for the usual range of environmental temperature. The relative deviation of the thermodynamic psychrometer constant is within -0.4% for $T^* < 30^\circ\text{C}$. In the thermodynamic wet-bulb temperature range $30\text{--}50^\circ\text{C}$ the relative deviation is less than -1% as long as the air is not too close to saturation. For dry air the maximum relative deviation is -0.17% for the extended range and it is negligibly small ($< -0.08\%$) for $T^* < 30^\circ\text{C}$. The present study also showed that the constant is pressure dependent and a correction factor is provided. The exact formulation compared very well with published experimental data on adiabatic saturation psychrometers, which confirmed the theory. Data on conventional psychrometer constants have a tendency to scatter about the central value that is theoretically predicted by this formulation. An exception to this rule is the WMO psychrometer, which has consistently lower psychrometer constants than those predicted by the present formulation. An error analysis was carried out for the situation wherein one mistakes the thermodynamic wet-bulb temperature for the psychrometer reading. Finally, the paper ended with a list of suggestions regarding how to build a psychrometer that would furnish the actual thermodynamic wet-bulb temperature. This is a design challenge, which requires some degree of ingenuity and a fundamental understanding of the simultaneous heat- and mass-transfer process occurring in the wet wick. It will be the subject of a further publication.

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